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Blade Tip-Time Measurement System: Design Fundamentals

Roberto Marsili[1,] Tommaso Tocci[1,] Lorenzo Capponi[2,] Gianluca Rossi[1,]
[1]University of Perugia, Department of Engineering, via G. Duranti 93, 06125 Perugia, Italy,
[2]University of Padova, CISAS, Via Firenze 15, 35131 Padova, Italy
Corresponding Author: Roberto Marsili

ABSTRACT: A first study of measurement techniques to determine vibrations of turbomachinery blades by using stationary sensors mounted onthe casing is here developed.Firstly, mathematical model is defined starting from basic physical fundamentals. Then this model is applied to two different measurement set-ups: one with reference sensor and one without. After that, a harmonic study of displacement and velocity is performed. The intrinsic uncertainty of these methods, together with the performances of the measurement chain are defined as well. The analysis of the measurement technique leads to some conclusions about the practical set-up and about theperformances of this methods.
KEYWORDS: Vibrations, Blade, Turbomachinery, Contactless measurement

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I. INTRODUCTION

Contactless measurement of turbomachinery blades vibrations is a very researched topic. Strain gauges are usually applied, however due to difficult operating conditions andtheir intrusiveness, non-contact methods are preferred and used to overcoming these issues, offering a way for monitoring operating conditions up to now impossible through extensimetric techniques. In particular, it comes increased the reliability of the machine-sensor-chain treatment system of the signal, so it is possible to monitor in operation conditions, aiming to a scheduled maintenance. The proposed methods [1][2][3][4][5][6]are based on observation of the transition of the rotor blades in front of referring pointsfixed to the stator of the axial turbomachinery.

Non-contact measurement of the vibrations of rotating parts of machines with stationary sensors is based on the analysis of time intervals and of their variations. These analyses are performed on the signalselectric sensors, properly arranged, which detect the transitioning points of the moving body. A first approach proposed here extends the performance of these methods to the case of measurements on vibrating organs and at the same time in rotation or alternating motion, move these periodicals, but with significant degree of irregularity and illustrates some techniques that they also allowto determine the frequency content of the law of deformation. A first solution proposed requires a sensor placed in a position to detect the transition of a point of the moving body that not undergoes any deformation. Not always it is possible to place a sensor in order to detect the transitionof a point of the moving organ that does not undergo deformation, therefore a second technique is shown which allows also to detect deflection samples of the moving part.

II. FUNDAMENTALS OF THE REFERENCE SENSOR METHOD

In the following a methodology is described which allows the dynamic measurement of the relative displacement law s(t), supposed periodic, between two points of an organ in rotary or alternating motion, expressed in a reference system integral with the organ itself, using non-contact sensors connected to the stator of the machine. From the sensor signals, ideally impulses, the instants of passage of the points of the movable member in front of the sensors themselves are determined.

One of the sensors is used as a reference being arranged so as to detect the passage of a point of the movable member at which the deformation is zero. The other sensors are mounted so as to detect the passage of points that undergo deformation and then measure the delay Δt (or the advance), over time, of this passage, due to the deformation itself; the duration of the passage in front of the sensors is also measured.

All this allows the measurement of a displacement and velocity value to be obtained each time the moving part passes in front of the fixed sensors. Since the frequencies of the law of deformation are generally higher than the frequency of passage of the moving member in front of the sensors, what has been done and a sampling of the s(t) and its first derivative, the speed v(t), in aliasing conditions. But, through the acquired values of the function and its first derivative, we have a sufficient number of information, so it is possible to calculate the harmonics of s(t), therefore the determination of the characteristics of the supposed periodic vibration.

III. SPEED MEASUREMENT PRINCIPLE AND DISPLACEMENTS WITH REFERENCE SENSOR

Consider, for example, a bracket integral with a rotating drum subject to bending deformations as shown inFigure 1. Let ω be the angular rotation speed of the disk.

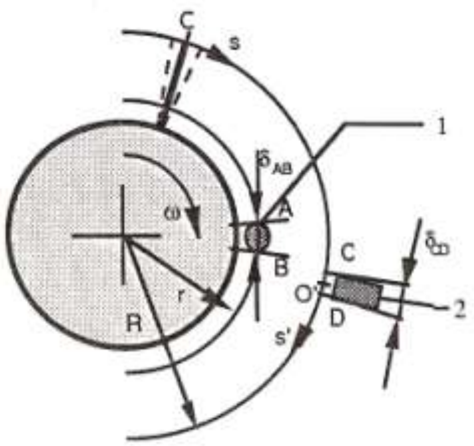

Figure 1 - Reference kinetic scheme

Consider the two fixed reference systems (O',s') and rotating (O,s), both consisting of the curvilinear abscissa on the circumference of radius R. Let O be the rotating point that identifies the position occupied by the top of the undeformed shelf. The value of s then approximates the value of the arrow of the shelf (circumferential component). It is assumed that this arrow varies over time according to a periodic law of the type:

$$s(t) = S\,f(t) \tag{3.1}$$

where f(t) is a periodic function of period T, frequency fundamental V_O = l / T and unitary amplitude and S is the amplitude of the s(t). The function s (t) is the object of the measure. In the fixed reference system, we have, by known laws kinematic:

$$v(t) = v^{'}(t) - v_0{}'(t) \tag{3.2}$$

Being v(t) the speed in the rotating system and v'(t) the absolute speeds. With reference to the assumed kinematic scheme, the measurement of shelf deflection is the measure of the curvilinear abscissa s in the rotating reference system, in the instant in which the top of the shelf passes in front of point C. This measurement can be made starting from the timeΔt_{AC} of exceeding a threshold level S_g of two successive pulses produced by the two sensors 1 and 2 ofFigure 1. Figure 2shows the typical signals.If the shelf does not flex and the angular speed is strictly constant the subsequent Δt_{AC} are all equal to each other.

If, on the other hand, the shelf flexes, the subsequent Δt_{AC} will be different between there. In illustrating the method of measuring displacements and displacements speed, we will make some hypotheses that it isanalyzed in followed with reference to the uncertainties and limits resulting from the hypotheses themselves.It is assumed that the time interval Δt_{AC} does not vary due to the irregularity of the rotation and that the Δt_{AC}' relative to the passage of the shelf in the undeformed configuration is equal to the mean value of the $\Delta t_{AC:}$

$$\overline{\Delta t_{AC}} = \frac{1}{N}\sum_{i=1}^{N} \Delta t_{AC}(i) \tag{3.3}$$

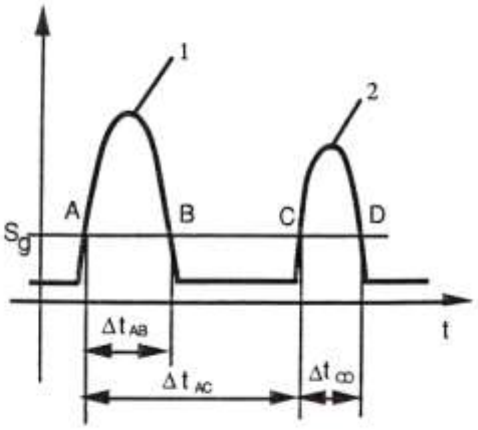

Figure 2 - Typical signals of generic transducers

With these hypotheses, the advances or delays of time due to the oscillation s(t) data from:

$$\Delta\Delta t_{AC} = \Delta t_{AC} - \overline{\Delta t_{AC}} \quad (3.4)$$

Associating the lengths to the times Δt_{AB} and Δt_{CD} respectively δ_{AB} and δ_{CD} along which the sensors see the reference and supply a signal, defined as distances corresponding to the crossing of the threshold S_g as in Figure 2. Assuming that such distances do not vary with the same speeds or due to three accidental causes, it is compute the average speeds respectively between A and B and between C and D:

$$\overline{v_0} = \frac{\delta_{AB}}{\Delta t_{AB}}\frac{R}{r} \quad (3.5)$$

$$\overline{v'} = \frac{\delta_{CD}}{\Delta t_{CD}} \quad (3.6)$$

Such speeds, being the duration of the signals very short, can be assumed at an acceptable instantaneous speed estimate. The value of s is then calculated assuming that it does not vary appreciably in the time Δt_{CD} and that, in this interval of time also the absolute speed value of the top of the shelf remains constant and equal to$\overline{v'}$. With these hypotheses the value of s can be calculated through the relation:

$$s(i) = \frac{\delta_{CD}}{\Delta t_{CD}}\Delta\Delta t_{AC} \quad (3.7)$$

The value of v, assuming that in the Δt_{AC} time also v_0’ remains significantly constant and approximate with$\overline{v_0}$, can for the (3.2)be expressed by:

$$s(i) = \frac{\delta_{CD}}{\Delta t_{CD}} - \frac{\delta_{AB}}{\Delta t_{AB}}\frac{R}{r} \quad [3.8]$$

Use in (3.7) and (3.8) of the instantaneous speeds of rotation allows a considerable extension of the applicability of the method of measurement on machines with higher degrees of irregularity compared to the methods presented in[1][2][3]. Here, in fact, it requires sufficient constancy to the v_0’ only in the Δt_{AC} period, while in the other methods, where $\overline{v'} = \omega R$is evaluated, it must have good constancy of v_0’ during a whole revolution. Through (3.7) and (3.8), a pair of values s(i) and v(i) is calculated at each instant of passage of the object being measured in front of the sensors fixed. Vibration frequencies are generally higher than frequency of passage, for which this sampling is in conditions of "aliasing" if a single pair of sensors is used.

IV. METHOD WITHOUT REFERENCE SENSOR

It is not always possible to install a reference sensor in such a position as to detect the passage of a point of the organ in motion that does not undergo deformation. In these cases, it is possible to install the sensors as shown inFigure 3. Only peaks related to one are taken into account specified blade, for each pair of peaks the interval is calculated of time Δt among the initial instants of the same peaks identified from exceeding a certain threshold level S_g.

It is assumed that the time interval does not change in a way appreciable due to the irregularity of the rotation and that the Δt, relative to the passage of the blade in the undeformed configuration, isequal to the average value of Δt measured over a certain number of revolutions.

The advances or time delays ΔΔt(i) are calculated with respect to the average values. Assuming a strictly constant peripheral speed equal to ωR, the subsequent displacement samples S(i) are obtainable from the following relation:

$$s(i) = \omega R \, \Delta\Delta \mathrm{t(i)} \quad [4.1]$$

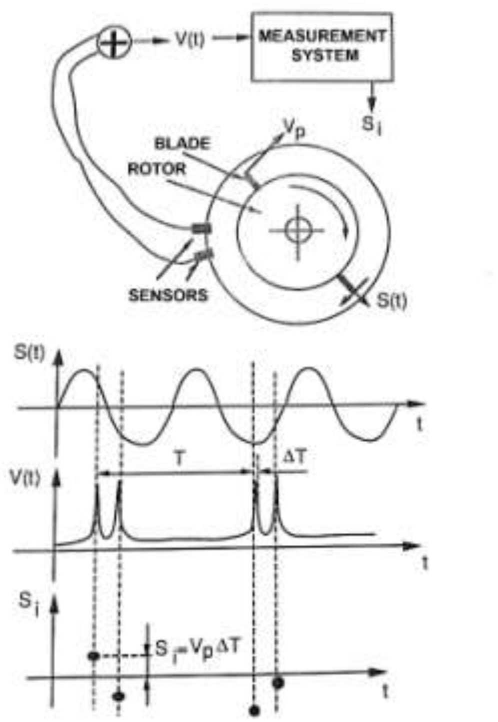

Figure 3 - Measure method without reference sensor

Analyzing instead that the instants of beginning, those of end of the various peaks can still get displacement samples with the same methodology. So for each peak they can have or one pair of displacement samples s and s' value of average displacement equal to (s + s')/2 and a speed value v equal to (s'-s) / t_d, having indicated the duration of the peak with t_d.

To determine the vibration frequency, one of the possible algorithms (in the hypothesis of sinusoidal vibration) is obtained starting from the following considerations:

$$S = A\cos(\omega t) \quad [4.1]$$

$$V = -A\sin(\omega t) \quad [4.2]$$

From the ratio of amplitudes to obtain the frequency sought:

$$\omega = \frac{(V_{max} - V_{min})}{(S_{max} - S_{min})} \quad [4.2]$$

V. VIBRATION HARMONICS CALCULATION

The sampling described produces a succession of displacement and velocity values of the end of the vibrating member with respect to the movable reference system. At each instant of passage, ie at time t(i), a shift value s(i) and so are associated
one of speed v(i).

The relative motion of the blade top can be described by a periodic function over time. In order to determine the fundamental frequency, its harmonics and relative amplitudes of this periodic motion are calculated. A physical phenomenon, such as vibration, can be effectively described by a relatively small number of harmonics, where the energy being almost completely contained in the earliest harmonics.
The two functions s(t) and v(t) can be approximated by a Fourier series development; the development can be limited to M harmonics, as stated:

$$s(t) = \sum_{j=1}^{M} [A_j \cos(2\pi v_0 jt) + B_j \sin(2\pi v_0 jt)] \quad [5.1]$$

$$v(t) = 2\pi v_0 \sum_{j=1}^{M} [-A_j j \sin(2\pi v_0 jt) + B_j j \cos(2\pi v_0 jt)] \quad [5.2]$$

The values of s(t) and v(t) are measured at the instants t(i), not necessarily equal in time; these values s(i) and v(i) are then used to construct a system of nonlinear equations f with unknowns the fundamental frequency V_0 and the 2Mcoefficients A_j and B_j of the subsequent harmonics. Thus, (2M + l) equations are written; to do this, it is necessary to carry out a sufficient series of samples of s(t) and v(t). The number of harmonics can be set based on the knowledge of the physical phenomenon to be measured. Then it isproceeding to the acquisitions of the series s(i) and v(i).It isconstructing the non-linear system and solve it, obtaining the discrete spectrum of the harmonics of the searched vibration. Another methodology may be that of an approach to least squares with a similar formulation to the previous one. The limits of these methodologies have been established in [7].

VI. UNCERTAINTY ANALYSIS

The hypotheses made for the description of the measurement principle of speeds and displacements put some conditions of applicability of the methodology to the study.This condition isquantify, at least to a first approximation, as a function of parameters whose relief is basic in the assumed kinematic model for the system, a rotating disk with a vibrating shelf. These parameters are ω, R, i, v, and S.A first cause of error can be identified in the variation of the Δt_{AC} due to the degree of rotation irregularity defined interms of rotation periods, as shown in (6.1).

$$i = \frac{(T_{max} - T_{min})}{T_{average}} \quad [6.1]$$

As a first approximation, this error can be estimated with the

$$E_i = i\,\Delta t_{AC} \quad [6.2]$$

It is then assumed that v and s do not vary apprezabile in time Δt_{AC}. The maximum deviations deriving from these hypotheses, due to the actual variation, can be estimated from the following relations:

$$E_s = \left[\frac{ds}{dt}\right]_{max} \Delta t_{AC} \quad [6.3]$$

$$E_v = \left[\frac{dv}{dt}\right]_{max} \Delta t_{AC} \quad [6.4]$$

Assumed for s(t) a simple sinusoidal vibration law of the type:

$$s(t) = S\, sin(2\pi vt) \quad [6.5]$$

It had the following relationships:

$$\left[\frac{ds}{dt}\right]_{max} = 2\pi vs \qquad [6.6]$$

$$\left[\frac{dv}{dt}\right]_{max} = 4\pi^2 v^2 s \qquad [6.7]$$

replacing these values in (6.3) and (6.4), it is obtained the:

$$E_s = 2\pi vs\ \Delta t_{AC} \qquad [6.8]$$

$$E_v = 4\pi^2 v^2 s\ \Delta t_{AC} \qquad [6.9]$$

Considering the maximum Δt_{AC} that is obtained when the absolute speed of the top of the shelf is minimal, it has:

$$E_s = 2\pi vs\ \left[\frac{\delta_{AC}}{(\omega R - 2\pi vs)}\right] \qquad [6.10]$$

$$E_v = 4\pi^2 v^2 s\ \left[\frac{\delta_{AC}}{(\omega R - 2\pi vs)}\right] \qquad [6.11]$$

The (6.2), (6.10) and (6.11) identify the conditions of applicability of the method of measurement described; they impose variation limits on the fundamental parameters of the system, in relation to the case-by-case uncertainties quantized as acceptable.

The resolution$\Delta s$of the displacement measurement s, once the applicability of the method has been verified, is given by the $\Delta\Delta t_{AC}$resolution. Starting from:

$$R_s = \frac{\Delta s}{S} \qquad [6.12]$$

and with t_{rs} the resolution in the measure of the times, it has

$$t_{rs} = R_s\ (\Delta\Delta t_{AC})_{max} \qquad [6.13]$$

and being approximately:

$$(\Delta\Delta t_{AC})_{max} = \frac{S}{\omega R} \qquad [6.14]$$

A relation is obtained for the choice of resolution in the measure of time to obtain a desired relative resolution R_s on the measurement of s

$$t_{rs} = R_s\ \frac{S}{\omega R} \qquad [6.15]$$

The resolution on speed measurement depends essentially on $\Delta\Delta t_{CD}$ resolution. Given the hypothesis of validity of (6.5), and similarly to the previous case, it has:

$$t_{rv} = R_v\ (\Delta\Delta t_{CD})_{max} \qquad [6.16]$$

and being approximately:

$$(\Delta t_{CD})_{min} = \frac{\delta_{CD}}{(\omega + 2\pi vs)} \qquad [6.17]$$

$$(\Delta t_{CD})_{max} = \frac{\delta_{CD}}{(\omega - 2\pi vs)} \qquad [6.18]$$

it still has:

$$(\Delta\Delta t_{CD})_{max} = \frac{[(\Delta t_{CD})_{max} - (\Delta t_{CD})_{min}]}{2} \quad [6.19]$$

and therefore from (6.19) is obtained:

$$t_{rv} = 2\pi S\delta_{CD}\frac{R_v}{\omega^2 R^2 - 4\pi^2 v^2 s^2} \quad [6.20]$$

A further cause of uncertainty is given by the variations of the δ_{AB} and δ_{CD} directly related to the choice of the definition criterion

of crossing the $S_g$threshold. In a first analysis it has seen that the aforementioned distances strongly depend on the geometry of the sensor and the organ, therefore on the installation, and also on their relative speed.

VII. CONCLUSIONS

Different techniques have been developed for the measurement of vibrations of moving parts with stationary sensors without contact. On the base of the theoretical analysescarried out and of the study of bibliography some innovative methodologies have been developed. With these methodologies, from the impulsive signals that sensors generate at the passage of the organ it is possible to determine oneseries of sampled values of the periodic laws of vibrations s(t) and its first derivative, the speed v(t). These values allow the mathematical reconstruction of the development through Fourier series of the vibration law, limited to one reduced number of harmonics. Error analysis has highlighted the limits of applicability of the proposed methodologies in relation to operating conditions to the main characteristics of the measurand.

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