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Effects of ion-neutral collisions in the propagation of ion-acoustic solitary waves in non-uniform plasmas

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ABSTRACT:*The effects of ion-neutral collisions on the amplitude and width of ion-acoustic solitary waves are investigated in non-uniform plasma consisting Maxwelian distributed electrons. The fluid model of the plasma for this system is considered. The related fluid equations have been treated by reductive perturbation technique using a space–time stretched coordinate. The modified Korteweg–de–Vries (mKdV) equation has been derived. The soliton solutions are found to be affected by inhomogeneity and lowest order collisional terms.*
KEYWORDS*: Ion–acoustic solitons, non-uniform plasma, mKdV equation, ion-neutral collision.*

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I. INTRODUCTION

Investigations of different properties of ion-acoustic waves are studied widely in plasma dynamics after it was first conceptualized by Washimi and Tanuity[1] through a nonlinear wave equation known as Korteweg- de-Vries(KdV) equation[2]. In uniform plasma, the propagation characteristics of ion-acoustic waves are governed by this KdV equation due to a balance between of nonlinearity and dispersion. In uniform plasma, an ion-acoustic waves travels without change in amplitude, shape and speed. As uniformity is a special condition of non-uniformity, so non-uniformity is quite common in all environment of plasma medium. In non-uniform plasma, the soliton is altered as it propagates, so it is of interest to know how this non-uniformity affects the soliton structure. Non-uniformity may be due to density gradient, temperature gradient or magnetic field etc. The disturbance travels in an non-uniform medium which varies spatially like the unperturbed plasma density in which an ion acoustic wave propagates, is a function of space variable. In a weakly varying medium, the disturbances traveling in one direction are governed by a KdV equation with either variable coefficient or a small additional term, which is a modified form of the original KdV equation. Nishikawa and Kaw [3] (1975) have derived such type of KdV equation describing the propagation of a weakly nonlinear ion acoustic wave in a non-uniform plasma with a density gradient. The various aspects of the propagation of ion-acoustic waves have been intensively studied in non-uniform medium [4-17] that are governed by modified KdV (mKdV) equation. In the present work, we have derived a modified KdV equation describing the ion-acoustic solitary wave in presence of weak ion-neutral collision frequency. The reductive perturbation analyses of fluid equations are carried out by employing a set of ‘stretched coordinates’ appropriate for spatially non-uniform plasma.

II. BASIC EQUATIONS

We have considered an unmagnetised weakly non-uniform ion-neutral collisional plasma. The Boltzmann distribution for electrons, at constant electron temperature and zero ion temperature are assumed. The continuity and momentum equation for this plasma model with Poisson’s equation and electron Boltzmann distribution can be written as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(\mathrm{n}_i \mathrm{u}_i\right) = 0\,, \qquad (1)$$

$$\frac{\partial u_i}{\partial t} + \mathrm{u}\,\frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} = -\mu u_i\,, \qquad (2)$$

$$n_e = n_{i0}\exp\phi\,, \qquad (3)$$

$$\frac{\partial^2 \phi}{\partial x^2} - n_e + \mathrm{n}_i = 0\,, \qquad (4)$$

where n_i and n_e are respectively, the ion and electron densities, u_i is the ion fluid velocity, ϕ is the electrostatic potential, μ is the ion neutral collision frequency and x , t are space and time variables respectively.

Normalizing n_i and n_e by the zero-order ion density at $x = 0$, the quantity u_i is normalized by the ion-acoustic speed and ϕ by $K_B T_e / e$ where K_B , T_e and e are Boltzmann's constant, electron temperature and ion charge respectively. The time t and spatial coordinate x are normalized respectively by the reciprocal of the ion plasma frequency at $x = 0$ and the Debye length at $x = 0$. The ion-neutral collision frequency μ is normalized by ion-plasma frequency at x = 0.

III. DERIVATION AND SOLUTION OF THE MODIFIED KDV EQUATION

The set of stretched coordinates [18], which is appropriate for specially non-uniform plasma is as follows

$$\xi = \varepsilon^{\frac{1}{2}}\left(\frac{x}{\lambda_0} - t\right), \quad \tau = \varepsilon^{\frac{3}{2}} x, \tag{5}$$

where ε is expansion parameter and λ_0 is the phase velocity of the ion-acoustic wave.

Further, because of weak collisional effects, we ordered as

$$\mu = \varepsilon^{\frac{3}{2}}\mu_{0}, \quad \mu_0 = o(1). \tag{6}$$

Since n_{i0} and λ_0 are independent of ξ , we have

$$\frac{\partial n_{i0}}{\partial \xi} = \frac{\partial \lambda_0}{\partial \xi} = 0. \tag{7}$$

Using Eqs. (5) and (6), Eqs. (1) – (4) becomes

$$-\frac{\partial n_i}{\partial \xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial \xi}(n_i u_i) + \varepsilon \frac{\partial}{\partial \tau}(n_i u_i) = 0, \tag{8}$$

$$-\frac{\partial u_i}{\partial \xi} + \frac{u_i}{\lambda_0}\frac{\partial u_i}{\partial \xi} + \varepsilon u_i \frac{\partial u_i}{\partial \tau} + \frac{1}{\lambda_0}\frac{\partial \phi}{\partial \xi} + \varepsilon \frac{\partial \phi}{\partial \tau} = -\varepsilon \mu_0 u_i, \tag{9}$$

and

$$\frac{\varepsilon}{\lambda_0^{\,2}}\frac{\partial^2 \phi}{\partial \xi^2} + \frac{2\varepsilon^2}{\lambda_0}\frac{\partial^2 \phi}{\partial \xi\, \partial \tau} + \varepsilon^3 \frac{\partial^2 \phi}{\partial \tau^2} - \frac{\varepsilon^2}{\lambda_0^{\,2}}\frac{\partial \lambda_0}{\partial \tau}\frac{\partial \phi}{\partial \xi} - n_{i0}e^{\phi} + n_i = 0. \tag{10}$$

The plasma parameters $n_{i,}$ u_i and ϕ are expressed as power series in ε as

$$\left.\begin{aligned} n_i &= n_{i0} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \varepsilon^3 n_{i3} + \dots \\ u_i &= u_{i0} + \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \varepsilon^3 u_{i3} + \dots \\ \phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \dots \end{aligned}\right\}, \tag{11}$$

where n_{i0} are u_{i0} are the plasma parameters in unperturbed state.

From equations (8) – (10), the zeroth-order of ε gives

$$\frac{\partial u_{i0}}{\partial \xi} = 0, \quad \frac{\partial}{\partial \tau}(n_{i0}u_{i0}) = 0 \quad \text{and} \quad \frac{\partial u_{i0}}{\partial \tau} = \mu_{0,} \tag{12}$$

which gives

$$\frac{\partial n_{i0}}{\partial \tau} = -\frac{n_{i0}\mu_0}{u_{i0}} \tag{13}$$

Now using Eqs. (11) into Eqs. (8) – (10), the lowest order of ε together with Eqs. (7), (12) and applying boundary conditions $u_{i0} \to 0, \quad u_{i1} = \phi_1$ and $n_{i0}, \lambda_0 \to 1$ as $|\xi| \to \propto$ we get

$$\left.\begin{aligned} u_{i1} &= P n_{i1} \\ \phi_{i1} &= n_{i0} u_{i1} P \end{aligned}\right\} \quad \text{where} \quad P = \frac{\lambda_0 - u_{i0}}{n_{i0}}$$

and hence we get

$$\left(\lambda_0 - u_{i0}\right)^2 = 1. \tag{14}$$

Now for second order of ε in Eqs. (8) – (10) and eliminating all the second order quantities we get the following modified KdV equation

$$\frac{\partial \phi_1}{\partial \tau} + \alpha \phi_1 \frac{\partial \phi_1}{\partial \xi} + \beta \frac{\partial^3 \phi_1}{\partial \xi^3} + \gamma \phi_1 = 0\,, \tag{15}$$

where

$$\alpha = \frac{1}{\lambda_0{}^2}, \qquad \beta = \frac{1}{2 n_{i0} \lambda_0{}^4}, \qquad \gamma = \frac{1}{\lambda_0}\frac{\partial \lambda_0}{\partial \tau} + \frac{\mu_0}{2\lambda_0 u_{i0}}\,. \tag{16}$$

The solution of Eq.(15) cannot be determined by ordinary methods due to the presence of variable coefficients. So we use a more appropriate method known as sine-cosine method which was first applied by Yan [18] to solve the KdV equation derived in uniform plasma and find the exact soliton propagation in plasmas and also later Yan et al. extended it further for non-uniform plasmas. To use sine-cosine method in Eq.(15), we use the following variable transformations:

$$\zeta = \kappa\left(\xi - U\tau\right),\ \psi\left(\xi,\tau\right) = \Phi\left(\zeta\right), \tag{17}$$

where κ^{-1} is the width of the solitary wave and U is the shift in the velocity when the wave evolves as a soliton, then the Eq. (15) becomes

$$-\kappa U \frac{d\Phi}{d\zeta} + \alpha\kappa\Phi \frac{d\Phi}{d\zeta} + \beta\ \kappa^3 \frac{d^3\Phi}{d\zeta^3} + \gamma\ \Phi = 0\,. \tag{18}$$

Using sine-cosine method, the solution of (18) can be written as

$$\Phi\left(\omega\right) = A_0 + \sum_{i=1}^{p}\left(B_i \sin\omega + A_i \cos\omega\right)\cos^{i-1}\omega\,, \tag{19}$$

where $\dfrac{d\omega}{d\zeta} = \sin\omega$ and A_i, B_i are functions of ζ and ω but they will not appear explicitly as functions of $\sin\omega$ and $\cos\omega$. Also, p is determined by the balance of the leading order of nonlinear to linear terms. As we have a nonlinearity of lower order, so we take p = 2 in our present case. With this, the solution in the form of intermediate variable ω can be written as

$$\Phi\left(\omega\right) = A_0 + A_1\cos\omega + B_1\sin\omega + A_2\cos^2\omega + B_2\cos\omega\sin\omega. \tag{20}$$

To determine the coefficients A_i, B_i, U and k, putting the values of $\Phi\left(\omega\right)$ from Eq. (20) in Eq. (18) and then the coefficients of the various trigonometric identities are put equal to zero. As the odd functions $\sin\omega$, $\cos\omega\sin\omega$ etc. do not play any rule in solution yielding

$B_1 = B_2 = 0$. Finally as by Das et al.[16, `18], we get

$$\left.\begin{aligned} A_0 &= -A_2 = \frac{2\beta\kappa^2}{\alpha} \\ A_1 &= -\frac{40\beta\kappa^5}{\gamma\alpha} \\ k^4 &= \frac{\gamma^2}{2\beta U} \\ U &= \frac{\alpha}{6A_2}\left(3A_1{}^2 + 6A_0A_2 - 4A_2{}^2\right) - \frac{\gamma A_1}{2kA_2} \end{aligned}\right\}. \tag{21}$$

So, from (6.23) the solution of mKdV equation becomes

$$\Phi(\omega)=\frac{2\beta\kappa^2}{\alpha}\sin^2\omega-\frac{40\beta\kappa^5}{\gamma\alpha}\cos\omega. \quad (22)$$

In terms of ϕ_1, this solution becomes

$$\phi_1(\xi,\tau)=\frac{2\beta\kappa^2}{\alpha}\sec h^2\left[\kappa(\xi-U\tau)\right]\pm\frac{40\beta\kappa^5}{\gamma\alpha}\tanh\left[\kappa(\xi-U\tau)\right]. \quad (23)$$

IV. RESULTS AND DISCUSSION

Two parts of the solution given by Eq. (23) gives two forms of soliton structures. The first part which is a soliton structure of the simple KdV equation derived normally in uniform plasma. The second part represents a soliton like tailing structure following the main soliton which is a combined effect of plasma inhomogeneity (term γ) and ion – neutral collision frequency. The main focus of this study is on this second part which contains the terms containing plasma inhomogeneity and ion-neutral collision frequency.

In fig, 1, we have plotted the variation of the amplitude of the soliton against ion – neutral collision frequency μ_0 for three different values of $n_{i0}(=0.2,\ 0.5,\ 0.8)$ with $u_{i0}=0.5$. The ion – neutral collision frequency μ_0 has an effective role on amplitude of the soliton as sharp changes (decrease) occurs for increasing values of μ_0. For increasing values of unperturbed ion number density n_{i0}, the sharpness of decreasing trend of the amplitude decreases for increasing μ_0.

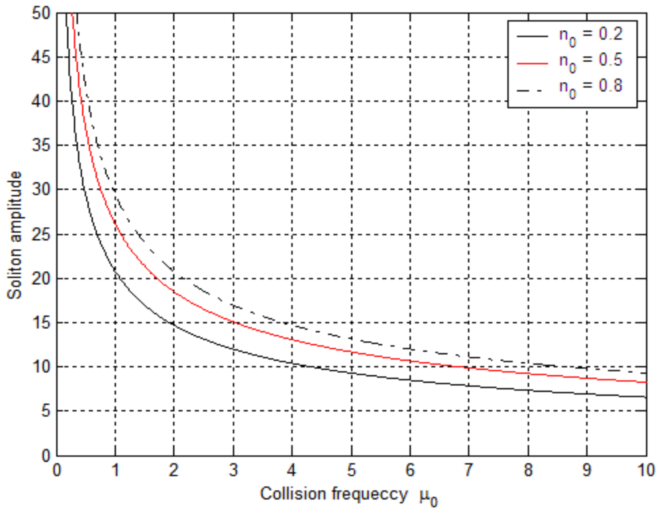

Fig.1: Variation of soliton amplitude against collision frequency μ_0 for three different values of $n_{i0}(=0.2,\ 0.5,\ 0.8)$ with $u_{i0}=0.5$.

In Fig. 2, variations of amplitude of soliton is depicted for increasing ion – neutral collision frequency μ_0. The amplitude of the soliton increases for increasing values of ion – neutral collision frequency μ_0. The rate of increase of amplitude is increased for increasing values of unperturbed ion number density n_0.

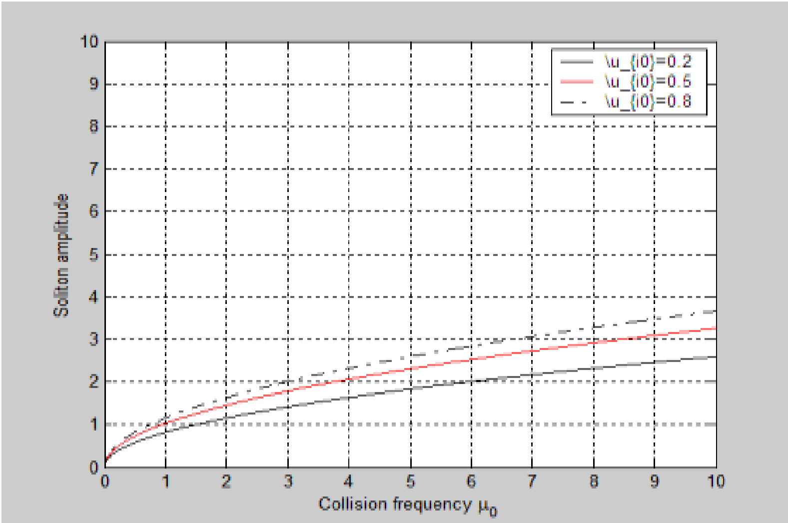

Fig.2: Variation of soliton amplitude against collision frequency μ_0 for three different values of $u_{i0}(=0.2,\ 0.5,\ 0.8)$ with $n_{i0}=0.5$.

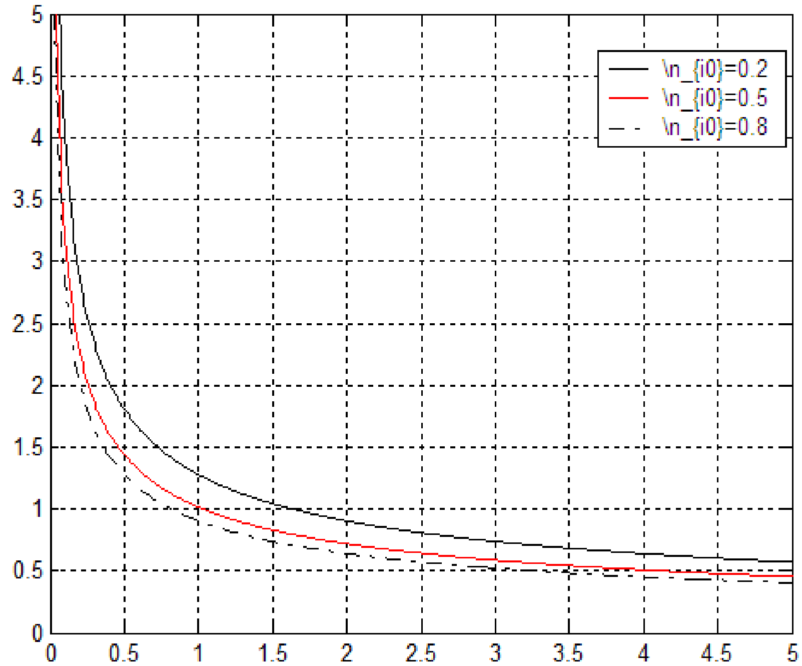

Fig. 3: Variation of soliton width against collision frequency μ_0 for three different values of $n_{i0}(=0.2,\ 0.5,\ 0.8)$ with $u_{i0}=0.5$.

The analysis of the width is depicted in Fig. 3 against ion – neutral collision frequency μ_0 for three different values of $n_{i0}(=0.2,\ 0.5,\ 0.8)$. We see that width decreases for increasing μ_0 with slightly higher rate for greater values of n_{i0}.

V. CONCLUSION

We have derived the modified mKdV equation in the non-uniform plasma containing ion-neutral collision frequency. To study the characteristics of propagation of ion-acoustic solitary waves, we have solved the mKdV equation using sine-cosine method. The solutions which have two forms of soliton structures, the first one

($sech^2$) profile could be observed for simple KdV equation derived in case of uniform plasma where as the second part (tanh) profile due to the presence of plasma inhomogeneity, is a soliton like tailing structure which follows the main soliton.

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