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Teaching Methodology of Mathematics: A Scientific Approach

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Abstract:
Modern math teaching methodology offers various possibilities for solving the problem of involving students in independent and research work, it develops their problem solving skills and develops their creative thinking processes and skills. One of those possibilities is in the area of scientifi c framework. The foundation of a scientific framework is the principle of science and scientifi c research methods. The article describes science in various segments of math teaching starting with the nature of math to mathematical tasks as an important method in shaping the system of basic mathematical knowledge, abilities and habits in students. In the end, some drawbacks in math teaching are mentioned which occur due to the inappropriate treatment of science in the teaching process.
Key words: *math, teaching math, scientific approach, the science principle, mathematical concept, theorem, problem – task.*

I. INTRODUCTION

Math teaching today primarily takes place within a professional framework. However, teaching math is a complex and demanding process. Even though being professional is a condition for its success, it is not sufficient. The complexity is successfully resolved by relating math to other sciences. That way we get a process which has to take place harmoniously within several frameworks. The main frameworks are *language frameworks, professional frameworks, methodology frameworks, scientific frameworks, pedagogical frameworks* and *psychological frameworks*.

As it is not easy to achieve harmony, occasional slips and weaknesses occur in math teaching which significantly influence the quality of math education. That reflects negatively on the aims of modern math teaching which emphasizes involvement of students in independent and research work, developing skills for problem solving and the development of creative thinking and creative skills.

Modern math teaching methodology offers various possibilities for solving the above mentioned problem. A teacher can find many possibilities within the scientific frameworks. The foundation of scientific frameworks is the *science principle* and *scientific research methods*. These concepts often cause a dilemma. What does a scientific approach mean in math teaching? The aim of this article is to describe that meaning and to give a few postulates and issues which arise in scientific frameworks of math teaching. N.B. a math teacher does not have to be a scientist in order to appropriately and correctly apply the science principle and research methods in math teaching.

THE SCIENCE PRINCIPLE

Didactic principles are the founding ideas and guidelines based on which teaching takes place. The basic characteristic of each principle is contained in the name of the principle itself which math teachers mostly understand. The same applies for the science principle. Nevertheless, the principle should be described in detail.
The science principle in math teaching consists of the appropriate harmony of teaching content and teaching methods on the one hand and the demands and regularities of math as a science on the other hand. That means that a math teacher should introduce students to those facts and form in their thought processes those mathematical occurrences which are scientifically founded today. Math teaching has to be such to enable further broadening and enrichment of content and a natural continuation of math education at a higher level.

It is evident that from the description the principle of science makes a connection between math as a teaching subject and math as a science.

II. SCIENTIFIC METHODS

In the process of learning and becoming involved with the law of nature, scientists apply special methods – *scientific research methods*. Basic methods of scientific thinking and research are: *analyses and synthesis, analogy, abstraction, and concretization, generalization and specialization, induction, and deduction*.

The work of a math teacher in a classroom differs in many respects from the work of a math scientist, but there are also these common characteristics:

In the process of learning the scientist applies the mentioned methods since they are necessary for obtaining new statements, their proof and their link with already known facts and theories. The shortest overview of some mathematical theory has four steps:

A) Stating basic concepts

B) Axiom formulation

C) Introduction of new concepts

D) Deriving and proving a theorem.

In other words, some scientific math area is a formation of axioms, basic concepts, derived concepts and theorems.

In the teaching process, a math teacher helps students to discover and learn new mathematical truths. That knowledge can be obtained in various ways and the bases of all those methods are also concepts and theorems.

TEACHING MATH

From the comparison mentioned we can easily conclude that scientific methods are important for modern math teaching. That is why they are the subject of research in modern math teaching methodology. Through the selection of appropriate problems and through the application of that method a creative teacher can prepare students for work which is very similar to research work, work of a scientist. Plenty of math teaching content can undergo such application thus meeting the science principle in its extent.

What does our teaching practice show in that respect? During the lesson, the math teacher often says: “the analysis shows”, “let’s have a look at some concrete examples”, “analogous it is proven”, “this set of facts induce the conclusion”, “the result of these observations is a generalization”, “through specialization we get the formula”, “mathematical concepts are abstract” etc. Do the students understand these words? How do we check their understanding? Knowledge of the procedures mentioned is often implied and therefore lack an explanation. That is not good.

Students should gradually and appropriately be taught how *to analyze, synthesize, abstract, induce, deduce, generalize, specialize, observe analogies,* regardless of whether they will be seriously involved in math at a later stage. As opposed to the usual acquisition of content, this is a higher level of mathematical education. Mathematical way of thinking is a valuable gain of mathematical education, applicable in many other activities. The words gradual and appropriate are emphasized. If scientific procedures are appropriately and correctly applied, with a necessary feeling for the difficulty of math content and mathematical way of thinking, taking into consideration mathematical abilities of each student, it can be expected that math teaching will be successful. On the contrary, students will have significant difficulties in acquiring the teaching content and with time they can get the wrong impression that math is a more difficult subject than it actually is. Sadly, math books, and consequently the teaching process do not pay sufficient attention to the regularities of the application of scientific procedures. In teaching some math content it can be established that they are wrong from that point of view. The science principle is therefore neglected.

Students’ failures in math and the inadequate knowledge which is displays upon the completion of their education are for the majority part a consequence of the fact that teaching is mostly done at a lower level, where acquisition of content is overemphasized, while the higher level is neglected. The reason for this neglect lies in the fact that for higher level math teaching one needs more demanding scientific methods based on teaching which is heuristic and problem solving. On the other hand, the need for (appropriate) use of scientific methods in math teaching can be explained with the following facts:

Developing math is a *concrete* and *inductive* science, and math itself is an *abstract* and *deductive* science.

What is teaching math in that respect? Teaching math in primary school is also mostly *concrete* and *inductive*. Math teachers arrive at abstract postulations, generalizations by observing concrete objects and concrete examples and through inductive conclusions. This method is familiar and appropriate for students of that age. The inductive procedure is made up of a chain of inductive steps which lead to the understanding of the general. We begin with concrete objects and special cases, inductive conclusions are sequenced by analogy, and the observed facts are generalized. We observe a tight link between *induction with concretization, specialization, analogy* and *generalization*. The advantage of applying induction: implementation of the easier to more difficult principle, simpler to complex, study-ing new abstract concepts and phrases through observation and assessment, guiding students to new concepts, expression of new theorems, etc. The *inductive* approach is important in the development of a student's thought process which on the other hand is necessary for acquiring a lot of content in school math. Among such content are various rules, regularities, formulas, theorems, especially if they are not strictly derived or proven.

The opposite of induction is deduction. The deductive process of thinking and proving, takes place after induction, at a higher level of math teaching and math education.

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An illustration of an appropriate methodological way of teaching mathematical content and the application of scientific methods is finding the sum K_n of all inner angles of a n angle with n sides.

In teaching this teaching unit in the seventh grade of primary school one should start from facts acquired in the previous grade. The first of those facts is a statement about the sum of all inner angles of a triangle: $K_3 = 180^0$. The second fact is the statement about the sum of all inner angles of a square: $K_4 = 360^0 = 2{\cdot}180^0$.

Furthermore, for the sum of all inner angles of a pentagonal a formula should be derived $K_5 = 540^0 = 3{\cdot}180^0$, for the sum of all inner angles of a hexagon a formula should be derived $K_6 = 720^0 = 4{\cdot}180^0$, students should be encouraged to conclude that the formula for a heptagon is $K_7 = 5{\cdot}180^0$, for the octagonal $K_8 = 6{\cdot}180^{0\ \text{etc.}}$. Comparison of formulas should follow. Only after completing all of those steps should be able to cognitively be ready for giving the following general statements:

The sum K_n of all inner angles of a polygon with n sides is given with the formula $K_n = (n - 2){\cdot}180^0$.

Questions such as: what is the sum K_{2008} ? follow.

Let us analyze the described procedure. *Analysis* points to the special part of this topic (triangle, square) which is taught in the previous grade. The first two concrete steps are therefore students' background knowledge and initial *inductive* conclusions. The third and fourth steps are two new *inductive* statements. The fifth and sixth steps are conclusions arrived at by *analogy*, and in the end there is the observation of regularities, *abstraction of concrete* cases and stating the *gener-alization*. In making a statement proof can easily be observed, which *synthesis* is in this case. Upon proving the formula, considerations related to their application have a *deductive* character and are in a tight relation with *specialization*.

In the example described, all 9 basic scientific methods are applied!

III. MATHEMATICAL CONCEPTS

Concept is a form of thought which reflect important characteristics of the objects studied.

The process of formulating a concept is a gradual process. We can roughly describe the process in the following way: The initial and most simple step of be-ing aware of the concept is observation and introduction to *concrete* objects and their *concrete* characteristics related to the concept and sensory awareness – ob-servation. The second step is observing something general and common to ele-ments in the observed group of objects – having an idea about the concept. The third step is pointing out the important characteristic of such objects – formulation and acquisition of the concept.

It is not difficult to recognize some important scientific procedures in the described process: *analysis, synthesis, abstraction* and *generalization*. That means that any concept, including mathematical concepts, after careful *analysis* develop through *abstracting* characteristics of objects which exist in nature and through *generalization*. In that way mathematical concepts, although *abstract concepts*, reflect some characteristics of the real world and in that way contribute to their awareness.

According to that, in teaching mathematical concepts, the teacher realizes *the science principle* if the process of formulating concepts is appropriately implemented (observation, the idea about the concept, formulating the concept) and if he adheres to the rules which must satisfy the definition of a concept (appropriateness, content minimum, conciseness, naturalists, applicability, and contemporariness).

At first glance it can seem that the need for content minimum in the definition is rather rigorous, even when it can easily be accomplished in teaching. That is not the case. A demand has its methodological explanation. Redundant definitions on the one hand burden the student's memory, and on the other hand cause confusion in differentiating definitions and theorems.

The critical place for working on a concept is the transition to that level where the *abstraction* procedure begins, since the transfer from *concrete* to *abstract* is rather difficult for some students.

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One of the characteristics of a concept as a form of thought is that formulating a concept as part of human awareness is inseparable from expressing words or recording or using symbols. This characteristic is especially emphasized in mathematics. The issue of language in teaching math is very sensitive. There can be vagueness and violation of the science principle in this area. As an example we can look at several formulations from math books:

Parallelogram is a quadrilateral whose opposite sides are parallel.

Parallelogram is a quadrilateral whose opposite sides are parallel and congruent, opposite angles is congruent and the angles on the same side are supplement.

The bisector of a length is the set of all points of a plane which are of the same distance from the end points of a length.
An equation in the form $ax^2+bx+c = 0$, where a, b, c are real numbers and a

$\neq$ 0, is called *equation of the second degree* or *quadratic equation*.

The first sentence is a concrete definition of a parallelogram; however it would be even better and more precise in the following form: A quadrilateral whose opposite sides are parallel is called a *parallelogram*.

The second statement is not a definition since it has redundant words and concepts and it is unlikely that all sixth grade students would know how to use it. It actually consists of the first definition and three theorems.

The third sentence causes ambiguity. It can be a definition of the symmetric length of a line; however, since in teaching the usual definition is the symmetry of the length as a line which passes through the midpoint of the length and is perpendicular to it, the mentioned theorem needs to be proven.

The fourth sentence is a concrete, abstract-deductive definition of a quadratic equation.

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At times the *science principle* is realized in agreement about the meaning of a particular concept, the size or object and the explanation why the agreement is introduced. For example, the following questions can cause initial not understanding and dilemmas: Is number 1 a cardinal number or not? What is the point of an empty set? How much is a^o?

Number 1 formally meets the condition for the definition of a cardinal number: it is divisible only by 1 and with itself. However, number 1 is still not part of the set of cardinal numbers. One of the reasons for the agreement is that 1 is not a cardinal number is found in the basic arithmetic theorem according to which any natural number other than 1 can be written in unique way in the form of a product of cardinal factors. If we said that 1 is a cardinal number, that theorem, without other conditions, would not be valid. In that case, we would have e.g. for number 2008 these divisions into cardinal factors $2008 = 2{\cdot}2{\cdot}2{\cdot}251 = 1{\cdot}2{\cdot}2{\cdot}2{\cdot}251$

= $1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 251$ etc. Therefore, the division would not be unique. This would ap-ply to each natural number.

An empty set $\varnothing$ is a set which consists of no elements. This meaning of an empty set would not have much sense if there was no serious scientific argument for it. We find it in the operation set cross section. The demand that cross $A \cap B$ of any two sets A and B is a set, and that means a cross section of disjunctive sets, leads to the need for introducing the concept empty set.

$a^0 = 1$. In school mathematics this equivalence is introduced without explanation. And the explanation is simple. It stems from the rule for dividing the exponent of equal bases: $a^m : a^n = a^{m-n}$ ($m > n$). For $m = n$ the left side of the equivalence is equal to 1, and the right side a^0. In order for the rule to be valid and in that case, the agreement is that $a^0 = 1$.

IV. THEOREMS AND PROOFS

What a theorem is we know. A theorem is a mathematical judgment whose truth is established by proof. A theorem is one of the most important mathematical concepts and its analysis demands special attention of every math teacher. Appropriate teaching of that concept enables faster development of mathematical thinking of a student and better understanding of math itself.

In teaching a theorem the teacher realizes the science principle if he teaches his students to appropriately and precisely formulate a theorem, clearly differentiate assumptions from a theorem statement, formulate a theorem twist, formulate an opposite statement, and if he achieves understanding of the methodology in proving a theorem. Indirect theorem proofs, especially forms such as proof of contraposition and contradiction (*reductio ad absurdum)* create great difficulties for students.

The question posed here is: should a student who will not deal with mathematics in everyday life at a later stage in life, or for whom math will not be of essential importance, know and understand these theorems? The answer can be portended from the following irrefutable truth: *learning how to prove means learning how to judge (reason)*, and that is one of the basic tasks in teaching math. Every person should know how to judge (reason) in life. How else can two different statements be compared, or extract from several statements those that are true, check the correctness of a suspicious proof, disprove someone's opinion, come to the appropriate conclusion about something, etc.? Yes, every student should learn how to prove. That is why education is not complete if a student throughout schooling has not encountered and understood proof for several standard mathematical theorems.

Teaching how to prove presents a great challenge for a math teacher, since it obviously is neither simple nor easy. Especially since a teacher must keep in mind an important fact:

Although math is a *deductive science*, school math is not developed at any teaching level as a strictly deductive system, but remains within the framework model. This especially applies for math teaching in primary school since it is *inductive* for the majority part. Many theorems are taught without proof.

A critical part for carrying out *generalizations* through *inductive* sequences of *concrete* cases is the transfer to the level where the *abstraction* procedure be-gins, since the transfer from *concrete* to *abstract* is even at this point quite difficult for some students.

In the case of theorems the use of words, writing or symbols is important. Accordingly the link between the first, second and third can be read in the follow-ing axiom for the polygon surface:

If polygons P_1 and P_2 *are congruent, then numbers* $p(P_1)$ and $p(P_2)$ *are equal, that is, the following implication applies*

$P_1 \cong P_2 \Rightarrow p(P_1) = p(P_2)$.

TASKS

Contemporary math teaching presupposes different knowledge activities than traditional. Emphasis is given to the development of the ability to work independently with a creative approach to math, and on developing conditions for successful application of acquired mathematical knowledge and abilities. Students' independent work on acquiring knowledge of math is achieved largely through the possibility of appropriately choosing and using teaching tasks. In that way tasks become an important means in forming students systems for basic mathematical knowledge, abilities and habits and aid to the development of their mathematical skills and creative thinking.

A task is a complex mathematical object and its composition is not always easy to analyze. However, in a broader sense we can isolate five of its basic constituents: *conditions, aim, theoretical basis, solution, overview.*

For the topic discussed, the most important constituent is the last one – overview. It offers possibilities of testing new ideas and further directions of students' thoughts. Particular directing can be accomplished by using some of these questions:

Can the manner for solving the problem be made simpler? Can the problem be solved in another way? Have we used the described procedure for finding a solution in some other problem? Can the problem be made simpler? Can the problem be generalized? Can you come up with a similar problem? What is the oppo-site statement? Is the opposite statement valid?

The questions obviously point to *analysis, synthesis, analogy, specialization and generalization*. In seeking answers to those questions particular mathematical skills of students are developed and nourished, and their creativity is lifted to a higher level.

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The example of mathematical content where analysis is important are *school word problems*. Why do such problems pose difficulties to students and teachers to the extent that some teachers avoid them? For the majority part, the explanation lies in the nature of the problems themselves. Each such problem ac-tually consists of two problems: making equations by translating normal language into mathematical lan-guage (the Descartes method), equation solving.

The first one is not always easy, and demands significant mental effort and knowledge of the procedure of analysis, which it is often presupposed that stu-dents know without explanation. This is where the difficulties arise, and the result is often antagonism towards such problems. However, solving equations is very useful since it enables the development of logical thinking, resourcefulness, observation and the ability to independently conduct small research. That is why it is not a good idea to avoid such problems; rather they should be appropriately methodologically explained so as to meet their educational goal.

SHORTCOMINGS

Here are some shortcomings in math teaching observed during teaching practice of math students in the teaching profession and who are very much influenced by the science principle and with the application of scientific methods:

1) Knowledge of mathematical concepts is really confusing. At the beginning of their education in methodology they do not know the principle of defining mathematical concepts, to they introduce everything they know about a concept into the definition (examples, characteristics). In that way, instead of a short, precise and complete definition of a concept they get a redundant text in which the basic point is lost! Such confusion, or one could say ignorance, cannot be a means for successful math teaching. A methodologist should invest a lot of effort into filling the ob-served gaps in students' knowledge.

2) In math teaching, *synthesis* is not often preceded by *analysis*, and that influences the clarity of teaching and understanding the problem thus lowering the value of teaching. Analysis is more or less a necessity in all research and cannot be avoided.

3) Students do not always clearly differentiate between definitions and theorems.

4) In *inductive* teaching an appropriate number of *concrete* and special cases is needed. A math teacher often considers an insufficient number of such cases, so the obtained statements become inconclusive and unclear with the consequence of students' lack of knowledge. Another error by teachers is also present when they do not give a larger number of students the chance to become involved in working out the *inductive* sequence.

5) Generalization is also a critical point in math teaching since the transfer from concrete and individual to general is often difficult for students to grasp. That is why a math teacher is faced with a responsibility to make the transfer for students easier using appropriate methodological procedures and skill.

6) A lot of mathematical content enable generalization, but math teachers often overlook such situations. This is a disservice to students learning math since generalizations are suitable for the development of mathematical thinking in students. This is especially true for gifted children who most likely have mathematical skills for broader studying of math.

7) In math teaching, analogy is not used enough although it is the best means for faster development and acquisition of new mathematical truths.
8) Math teacher creativity is often repressed due to overly relying on the manner of teaching mathematical content in textbooks

V. CONCLUSION

We have already mentioned that a math teacher need not be a scientist in order to appropriately and adequately apply the science principle and scientific methods in teaching. This occurs in math teaching without much interference. Solving a math problem implies some research and development. That is why the teacher has to create the spirit of curiosity in his students, the inclination for inde-pendent mental work and to show them ways to new discoveries. A creative math teacher using creative teaching methods has great chances to develop in his students creative characteristics.

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