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Factori-Difference Labeling Of Some Square Graphs

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ABSTRACT
In this paper, we focus on the factori-difference labeling and apply to some square graphs. A connected graph G is a factori-difference labeling if there exists a bijection f:V(G)→{2,3,… ,p} such that the induced function $g_f: E(G) \rightarrow N$ defined as $g_f(uv) = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and the edges labels are distinct. Graph which produces a factori-difference labeling has a factori-difference graph. We discuss this labeling conditions satisfies to some square graphs of path, cycle, brush, fan, friendship, ladder, wheel, helm, sun let graphs and also find the chromatic number of some square graphs.
KEYWORDS: Chromatic number, Factori-difference labeling, Factori-difference graph, Square graphs.

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I. INTRODUCTION

Graph labeling is one of the most simplicity and very interesting area of the graph theory. Graph labeling which means assign the values to vertices and edges with some conditions. Researchers are very interested and happy to research this graph labeling area. Beginning this area is β-labeling by Alexandar Rosa in late 1960's. A dynamic survey on graph labeling is systematic updated by J. A. Gallian[7] upto 2017 and it is published by Electronic Journal of Combinatorics. Basic definitions are referred to as Frank Harary[5]. An enormous body of literature is available on different types of graph labeling grown around in the last four decades and more than 1000 research papers have been published. Graph labeling are using in many departments namely computer science, engineering, medical line, etc., We discuss this labeling conditions satisfies to some square graphs of path, cycle, fan, brush, friendship, wheel, helm, ladder, sun let graphs and also find the chromatic number of some square graphs.

1. Preliminaries

1.1. Definition[5]

A walk in which no vertex is repeated is called a path P_n. It has n vertices and $n-1$ edges.

1.2. Definition[5]

A closed path is called a cycle $C_n, n \geq 3$. It has n vertices and n edges.

1.3. Definition[10]

A fan graph $F_n (n \geq 2)$ is defined as the graph $K_1 + P_n$, where K_1 is the singleton graph and P_n is the Path on n vertices. It has $n+1$ vertices and $2n-1$ edges.

1.4. Definition[2]

The brush graph $B_n, (n \geq 2)$ can be constructed by path graph $P_n, (n \geq 2)$ by joining the star graph $K_{1,1}$ at each vertex of the path. i.e., $B_n = P_n + nK_{1,1}$. It has $2n$ vertices and $2n-1$ edges.

1.5. Definition[9]

A cycle C_3 with n copies having a common central vertex is called a friendship graph T_n. It has $2n+1$ vertices and $3n$ edges.

1.6. Definition[8]

The cycle graph $C_n, n \geq 3$ joining the complete graph of one vertex K_1 is called the wheel graph $W_n, n \geq 3$. It has $n+1$ vertices and $2n$ edges.

1.7. Definition[1]

The wheel graph W_n by adding a pendant edge at each vertex on the rim of W_n is called the helm graph $H_n, n \geq 3$. It has $2n+1$ vertices and $3n$ edges.

1.8. Definition[3]

The cartesian product of path graphs $P_n \times P_2$ is known as ladder graph $L_n, n \geq 2$. It has $2n$ vertices and $3n-2$ edges.

1.9. Definition[8]

The cycle graph C_n with attaching n pendant vertices at each vertex is called the sun let graph $S_n, n \geq 3$. It has both $2n$ vertices and $2n$ edges.

1.10. Definition[4]

Square of a graph G denoted by G^2 has the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance of 1 or 2 apart in G.

1.11. Definition[6]

A coloring of a graph is an assigned color to its points so that two adjacent points have different colors and also non-adjacent vertices have either same color or any other colors. The chromatic number $\chi(G)$ is defined as the minimum *l* for which a graph G has an *l*-coloring.

II. FACTORI-DIFFERENCE LABELING TO SOME SQUARE GRAPHS

1.12. Definition

A connected graph G is a factori-difference labeling if there exists a bijection $f : V(G) \rightarrow \{2, 3, \ldots, p\}$ such that the induced function $g_f : E(G) \rightarrow N$ defined as $g_f(uv) = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and that edges labels are distinct. A graph which acknowledges a factori-difference labeling produces a factori-difference graph.

1.13. Theorem

The square graph ${P_n}^2, n \geq 3$ of a path graph $P_n, n \geq 2$ is a factori-difference graph.

Proof

Let $P_n, (n \geq 2)$ be a path graph with *n* vertices say $u_1, u_2, \ldots, u_n$ and $n-1$ edges. Let G be the square graph of a path graph ${P_n}^2, n \geq 3$ with *n* vertices and $2n-3$ edges. The successive vertices of square graph of a path graph ${P_n}^2$ are $u_1, u_2, \ldots, u_n$. i.e., $V(G) = V({P_n}^2) = \{u_i / 1 \leq i \leq n\}$ and $E(G) = E({P_n}^2) = \{u_iu_{i+1} / 1 \leq i \leq n-1 \cup u_iu_{i+2} / 1 \leq i \leq n-2$. Also, $V(P_n^2)=n$ and $E(P_n^2)=2n-3$. The maximum degree is $\Delta=n-1, n\leq 54, n\geq 6$ and the minimum degree is $\delta=2$ of the square graph of a path graph $P_n^2, n\geq 3$. Define $f : V P_n^2 \rightarrow \{1, 2, \ldots, n\}$ as follows, $f(u_i) = i+1$ for $1 \leq i \leq n$. Then, the factori-difference labeling conditions $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$ are satisfied. Clearly, vertices and edges labels are distinct. Hence, the function f is a factori-difference labeling for a square graph of a path graph ${P_n}^2, n \geq 3$. Thus, the square graph of a path graph ${P_n}^2, n \geq 3$ is a factori-difference graph. The chromatic number $\chi({P_n}^2), n \geq 3$ of a square graph of a path graph ${P_n}^2$ is the minimum *k*. i.e. $\chi({P_n}^2) = 3$.

1.14. Example

The factori-difference labeling of a square graph of a path graph ${P_n}^2, n \geq 3$ is shown figure 1.

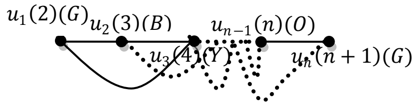

Figure 1. Factori-difference labeling for a graph ${P_n}^2$ and $\chi({P_n}^2) = 3$.

1.15. Remark

The resultant graph of a square graph of a path graph ${P_n}^2$ is a complete graph if $n = 3$.

1.16. Theorem

The square graph ${C_n}^2, n \geq 4$ of a cycle graph $C_n, n \geq 3$ acknowledges a factori-difference graph.

Proof

Let $C_n, n \geq 3$ be the cycle graph with both n vertices and n edges. Let $V(C_n) = \{u_i / 1 \leq i \leq n\}$ and $E(C_n) = \{u_i u_{i+1} \ / \ 1 \leq i \leq n-1\} \cup \{u_n u_1\}$. Let the resultant graph G be a square graph of a cycle graph and it is denoted by ${C_n}^2, n \geq 4$ with n vertices say $u_1, u_2, \dots, u_n$ and $\begin{cases} n+2, n=4 \\ 2n, n \geq 5 \end{cases}$ edges. Here, $V(G) = V\left({C_n}^2\right) = \{u_i / 1 \leq i \leq n\}$ and $E(G) = E\left({C_n}^2\right) = \{u_i u_{i+1} \ / \ 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_i u_{i+2} \ / \ 1 \leq i \leq n-2\} \cup \{u_{n-1} u_1\} \cup \{u_n u_2\}$. Also, $\left|V({C_n}^2)\right| = n$ and $\left|E({C_n}^2)\right| = \begin{cases} n+2, n=4 \\ 2n, n \geq 5 \end{cases}$. The maximum and minimum degree of a square graph of a cycle graph ${C_n}^2, n \geq 5$ are both $\begin{cases} \Delta = n-1 = \delta, n=4 \\ \Delta = 4 = \delta, n \geq 5 \end{cases}$. The bijection mapping function is defined $f : V\left({C_n}^2\right) \rightarrow \{1, 2, \dots, n\}$ as follows, $f(u_i) = i+1$ for $1 \leq i \leq n$. Then the factori-difference labeling conditions are satisfied and that the conditions are every edge $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \neq f\left(e_j\right), i \neq j$. Clearly, every vertices and edges labels are distinct. Thus the function f is a factori-difference labeling for a square graph of a cycle graph ${C_n}^2, n \geq 4$. Hence, the square graph of a cycle graph ${C_n}^2, n \geq 4$ acknowledges a factori-difference graph. The chromatic number $\chi({C_n}^2)$ of a square graph of a cycle graph ${C_n}^2, n \geq 4$ is the minimum *k*. i.e. $\chi\left({C_n}^2\right) = \begin{cases} 3 \text{ if } n \text{ is even} \\ 4 \text{ if } n \text{ is odd} \\ n \text{ if } {C_n}^2 \text{ is complete graph} \end{cases}$.

1.17. Example

The factori-difference labeling of a square graph of a cycle graph ${C_n}^2, n \geq 4$ is shown figure 2.

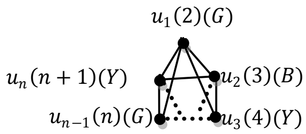

Figure 2. Factori-difference labeling for a graph ${C_n}^2$ and $\chi\left({C_n}^2\right) = \begin{cases} \mathbf{3} \textbf{ if } \boldsymbol{n} \textbf{ is even} \\ \mathbf{4} \textbf{ if } \boldsymbol{n} \textbf{ is odd} \\ \boldsymbol{n} \textbf{ if } \boldsymbol{{C_n}^2} \textbf{ is complete graph} \end{cases}$.

1.18. Remark

The resultant graph of the square graph of a cycle graph ${C_n}^2, n \geq 4$ is a complete graph if $n = 4, 5$.

1.19. Theorem

The square graph ${F_n}^2, n \geq 3$ of a fan graph $F_n, n \geq 2$ admits a factori-difference graph.

Proof

Let the fan graph $F_n, n \geq 2$ be $u_1, u_2, \dots, u_{n+1}$ vertices i.e., $n+1$ vertices and $2n-1$ edges. Let $G = {F_n}^2, n \geq 3$ which means square graph of a fan graph with $n+1$ vertices and $3n-3$ edges. Let $V(G) = V\left({F_n}^2\right) = \{u_i / 1 \leq i \leq n+1\}$ and $E(G) = E\left({F_n}^2\right) = \{u_1 u_i \ / \ 2 \leq i \leq n+1\} \cup \{u_i u_{i+1} \ / \ 2 \leq i \leq n\} \cup \{u_i u_{i+2} \ / \ 2 \leq i \leq n-1\}$. Here, $\left|V({F_n}^2)\right| = n+1$ and $\left|E({F_n}^2)\right| = 3n-3$. The maximum and minimum degree of a square graph of a fan graph ${F_n}^2, n \geq 3$ are $\Delta = n$ and $\delta = 3$. The labeling function is defined $f : V\left({F_n}^2\right) \rightarrow \{1, 2, \dots, n+1\}$ as follows, $f(u_i) = i+1$ for $1 \leq i \leq n+1$. Then the factori-difference labeling conditions are satisfied and that the conditions are every edge $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \neq f\left(e_j\right), i \neq j$. So that, every vertices and edges labels are distinct. Hence, the function f is a factori-difference labeling for a square graph of a fan graph ${F_n}^2, n \geq 3$. Therefore, the square graph of a fan graph ${F_n}^2, n \geq 3$ admits a factori-difference graph. The chromatic number $\chi({F_n}^2)$ of a square graph of a fan graph ${F_n}^2, n \geq 3$ is the minimum *k*. i.e. $\chi\left({F_n}^2\right) = 4$.

1.20. Example

The factori-difference labeling of a square graph of a fan graph ${F_n}^2, n \geq 3$ is shown figure 3.

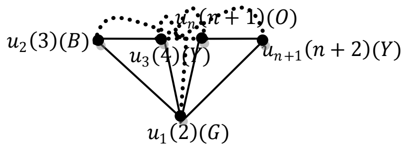

Figure 3. Factori-difference labeling for a graph $F_n{}^2$ and $\chi(F_n{}^2) = 4$.

1.21. Theorem

The square graph $B_n{}^2, n \geq 2$ of a brush graph $B_n, n \geq 2$ is a factori-difference graph.

Proof

Let the brush graph $B_n, n \geq 2$ with $2n$ vertices and $2n-1$ edges. Here, $u_1, u_2, \dots, u_n, u_{n+1}, \dots, u_{2n}$ be the vertices of brush graph. Let $G = B_n{}^2, n \geq 2$ i.e., the square graph of a brush graph with $2n$ vertices and $5n-5$ edges. Let $V(G) = V(B_n{}^2) = \{u_i / 1 \leq i \leq 2n\}$ and $E(G) = E(B_n{}^2) = \{u_i u_{i+1} \,/\, 1 \leq i \leq n\} \cup \{u_i u_{2n+1-i} \,/\, 1 \leq i \leq n\} \cup \{u_i u_{i+2} \,/\, 1 \leq i \leq n\} \cup \{u_i u_{2n-i} \,/\, 1 \leq i \leq n-1\} \cup \{u_i u_{2n+2-i} \,/\, 2 \leq i \leq n\}$. Also, $|V(B_n{}^2)| = 2n$ and $|E(B_n{}^2)| = 5n-5$. The maximum and minimum degree of a square graph of a brush graph $B_n{}^2, n \geq 2$ are $\Delta = \begin{cases} n+1, n=2 \\ n+2, n \geq 3 \end{cases}$ and $\delta = 2$. The mapping function $f : V(B_n{}^2) \to \{1, 2, \dots, 2n\}$ is defined by, $f(u_i) = i+1$ for $1 \leq i \leq 2n$. Such that, the factori-difference labeling conditions are satisfied and that the conditions are every edge $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. So that, every vertices and edges labels are distinct. Hence, the function f is a factori-difference labeling for a square graph of a brush graph $B_n{}^2, n \geq 2$. Therefore, the square graph of a brush graph $B_n{}^2, n \geq 2$ is a factori-difference graph. The chromatic number $\chi(B_n{}^2)$ of a square graph of a brush graph $B_n{}^2, n \geq 2$ is the minimum k. i.e. $\chi(B_n{}^2) = 4$.

1.22. Example

The factori-difference labeling of a square graph of a brush graph $B_n{}^2, n \geq 2$ is shown figure 4.

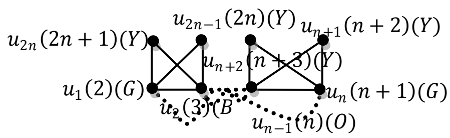

Figure 4. Factori-difference labeling for a graph $B_n{}^2$ and $\chi(B_n{}^2) = 4$.

1.23. Theorem

The square graph $T_n{}^2, n \geq 2$ of a friendship graph T_n is a factori-difference graph.

Proof

Let the friendship graph T_n be the n copies of cycle graph C_3 with $2n+1$ vertices and $3n$ edges. i.e., $u_1, u_2, \dots, u_n, u_{n+1}, \dots, u_{2n+1}$ be the successive vertices of T_n and u_1 be the apex vertex. Let $G = T_n{}^2, n \geq 2$ which means the square graph of a friendship graph with $2n+1$ vertices and $2n^2+n$ edges. Let $V(G) = V(T_n{}^2) = \{u_i / 1 \leq i \leq 2n+1\}$. Here we note that, $|V(T_n{}^2)| = 2n+1$ and $|E(T_n{}^2)| = 2n^2+n$. The maximum and minimum degree of a square graph of a friendship graph $T_n{}^2, n \geq 2$ are $\Delta = 2n = \delta$. The labeling function $f : V(T_n{}^2) \to \{1, 2, \dots, 2n+1\}$ is defined by, $f(u_i) = i+1$ for $1 \leq i \leq 2n+1$. Such that, the factori-difference labeling conditions are satisfied and that the conditions are every edge $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, every vertices and edges labels are distinct. Hence, the function f is a factori-difference labeling for a square graph of a friendship graph $T_n{}^2, n \geq 2$. Therefore, the square graph of a friendship graph $T_n{}^2, n \geq 2$ is a factori-difference graph. The chromatic number $\chi(T_n{}^2)$ of a square graph of a friendship graph $T_n{}^2, n \geq 2$ is the minimum k. i.e. $\chi(T_n{}^2) = 2n+1$.

1.24. Example

The factori-difference labeling of a square graph of a friendship graph $T_n{}^2, n \geq 2$ is shown figure 5.

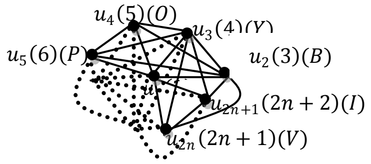

Figure 5. Factori-difference labeling for a graph $T_n{}^2$ and $\chi(T_n{}^2) = 2n + 1$.

1.25. Remark
The resultant graph of the square graph of a friendship graph $T_n{}^2, n \geq 2$ is a complete graph.

1.26. Theorem
The square graph $W_n{}^2, n \geq 4$ of a wheel graph $W_n, n \geq 3$ produces a factori-difference graph.

Proof

Let $W_n, n \geq 3$ be the wheel graph with successive vertices $u_1, u_2, \ldots, u_{n+1}$ and u_1 be the central vertex. i.e., $W_n, n \geq 3$ have $n+1$ vertices and $2n$ edges. Let $G = W_n{}^2, n \geq 4$ which means square graph of a wheel graph with $n+1$ vertices and $\frac{n(n+1)}{2}$ edges. Let $V(G) = V(W_n{}^2) = \{u_i / 1 \leq i \leq n+1\}$. Here, $|V(W_n{}^2)| = n + 1$ and $|E(W_n{}^2)| = \frac{n(n+1)}{2}$. The maximum and minimum degree of a square graph of a wheel graph $W_n{}^2, n \geq 4$ are $\Delta = n = \delta$. The bijection mapping function is defined $f : V(W_n{}^2) \to \{1, 2, \ldots, n+1\}$ as follows, $f(u_i) = i + 1$ for $1 \leq i \leq n+1$. Then the factori-difference labeling conditions are satisfied and that the conditions are every edge $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, every vertices and edges labels are distinct. Thus, the function f is a factori-difference labeling for a square graph of a wheel graph $W_n{}^2, n \geq 4$. Therefore, the square graph of a wheel graph $W_n{}^2, n \geq 4$ produces a factori-difference graph. The chromatic number $\chi(W_n{}^2)$ of a square graph of a wheel graph $W_n{}^2, n \geq 4$ is the minimum *k*. i.e. $\chi(W_n{}^2) = n + 1$.

1.27. Example
The factori-difference labeling of a square graph of a wheel graph $W_n{}^2, n \geq 4$ is shown figure 6.

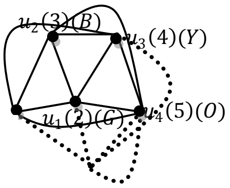

Figure 6. Factori-difference labeling for a graph $W_n{}^2$ and $\chi(W_n{}^2) = n + 1$.

1.28. Remark
The resultant graph of the square graph of a wheel graph $W_n{}^2, n \geq 4$ is a complete graph.

1.29. Theorem
The square graph $H_n{}^2, n \geq 3$ of a helm graph $H_n, n \geq 3$ acknowledges a factori-difference graph.

Proof

Let $H_n, n \geq 3$ be the helm graph with $2n+1$ vertices and $3n$ edges. Let $G = H_n{}^2, n \geq 3$ be the square graph of a helm graph with $2n+1$ vertices and $7n, n \geq 5$ edges. Let $V(G) = V(H_n{}^2) = \{u_i / 1 \leq i \leq 2n+1\}$ and $E(G) = E(H_n{}^2) = \{u_1 u_i \,/\, 2 \leq i \leq 2n+1\} \cup \{u_i u_{i+1} \,/\, 2 \leq i \leq n+1\} \cup \{u_{n+1} u_2\} \cup \{u_i u_{n+i} \,/\, 2 \leq i \leq$ *n+1∪uiui+2 / 2≤i≤n−1∪uiun+i−2 / 2≤i≤n−2∪u2u2n+1 ∪uiun+1+i / 2≤i≤n∪uiun−1+i / 3≤i≤n+1*. Here, *V(Hn2)=2n+1* and *E(Hn2)= 7n, n≥5* . The maximum and minimum degree of a square graph of a helm graph $H_n{}^2, n \geq 3$ are $\Delta = 2n$ and $\delta = 4$. The function $f : V(H_n{}^2) \to \{1, 2, \ldots, 2n+1\}$ is defined by, $f(u_i) = i + 1$ for $1 \leq i \leq 2n+1$. Then the factori-difference labeling conditions are satisfied and that the conditions are every edge $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, every vertices and edges labels are distinct. Hence, the function f is a factori-difference labeling for a square graph of a helm graph $H_n{}^2, n \geq 3$. Therefore, the square graph of a helm graph $H_n{}^2, n \geq 3$ acknowledges a factori-difference graph. The chromatic number $\chi(H_n{}^2)$ of a square graph of a helm graph $H_n{}^2, n \geq 3$ is the minimum *k*. i.e. $\chi(H_n{}^2) = \begin{cases} n+2, & n = 3 \\ n+1, & n \geq 4 \end{cases}$.

1.30. Example
The factori-difference labeling of a square graph of a helm graph $H_n{}^2, n \geq 3$ is shown figure 7.

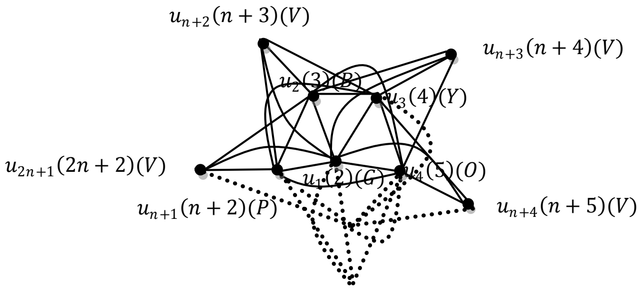

Figure 7. Factori-difference labeling for a graph $H_n{}^2$ and $\chi(H_n{}^2) = \begin{cases} n+2, & n=3 \\ n+1, & n \geq 4 \end{cases}$.

1.31. Theorem

The square graph $L_n{}^2, n \geq 2$ of a ladder graph $L_n, n \geq 2$ produces a factori-difference graph.

Proof

Let the product graph $P_2 \, X \, P_n$ is called the ladder graph $L_n, n \geq 2$ has $2n$ vertices and $3n-2$ edges. Let the resultant graph G is the square graph of a ladder graph $L_n{}^2, n \geq 2$ with $2n$ vertices and $7n-8$ edges. Let $V(G) = V(L_n{}^2) = \{u_i / 1 \leq i \leq 2n\}$ and $E(G) = E(L_n{}^2) = \{u_i u_{i+1} \, / \, 1 \leq i \leq 2n-1\} \cup \{u_i u_{2n-i+1} \, / \, 1 \leq i \leq n\} \cup \{u_i u_{i+2} \, / \, 1 \leq i \leq 2n-2\} \cup \{u_i u_{2n-i} \, / \, 1 \leq i \leq n-1\} \cup \{u_i u_{2n+1-i} \, / \, 2 \leq i \leq n\}$. Such that, $|V(L_n{}^2)| = 2n$ and $|E(L_n{}^2)| = 7n-8$. The maximum and minimum degree of a square graph of a ladder graph $L_n{}^2, n \geq 2$ are $\Delta = \begin{cases} n+1 & \text{if } n=2 \\ n+2 & \text{if } n=3,4 \\ 7 & \text{if } n \geq 5 \end{cases}$ and $\delta = \begin{cases} n+1 & \text{if } n=2 \\ 4 & \text{if } n \geq 3 \end{cases}$ respectively. The labeling function $f : V(L_n{}^2) \to \{1, 2, \ldots, 2n\}$ is defined by, $f(u_i) = i+1$ for $1 \leq i \leq 2n$. Then the factori-difference labeling conditions are satisfied and that the conditions are every edge $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, every vertices and edges labels are distinct. Thus, the function f is a factori-difference labeling for a square graph of a ladder graph $L_n{}^2, n \geq 2$. Therefore, the square graph of a helm graph $L_n{}^2, n \geq 2$ produces a factori-difference graph. The chromatic number $\chi(L_n{}^2)$ of a square graph of a ladder graph $L_n{}^2, n \geq 2$ is the minimum k. i.e. $\chi(L_n{}^2) = \begin{cases} 4, & n=2 \\ n+1, & n=3,4. \\ 6, & n \geq 5 \end{cases}$

1.32. Example

The factori-difference labeling of a square graph of a ladder graph $L_n{}^2, n \geq 2$ is shown figure 8.

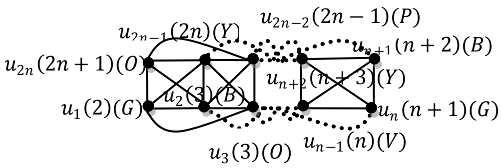

Figure 8. Factori-difference labeling for a graph $L_n{}^2$ and $\chi(L_n{}^2) = \begin{cases} 4, & n=2 \\ n+1, & n=3,4. \\ 6, & n \geq 5 \end{cases}$

1.33. Theorem

The square graph $S_n{}^2, n \geq 3$ of a sun let graph $S_n, n \geq 3$ is a factori-difference graph.

Proof

Let $S_n, n \geq 3$ be the sun let graph with both $2n$ vertices say $u_1, u_2, \ldots, u_{2n}$ and $2n$ edges. Let the resultant graph G be a square graph of a sun let graph and it is denoted by $S_n{}^2, n \geq 3$ with $2n$ vertices say $u_1, u_2, \ldots, u_{2n}$ and $\begin{cases} 4n, & n=3 \\ 4n+2, & n=4 \\ 5n, & n \geq 5 \end{cases}$ edges. Here, $V(G) = V(S_n{}^2) = \{u_i / 1 \leq i \leq 2n\}$ and $E(G) = E(S_n{}^2) = \{u_i u_{i+1} \, / \, 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_i u_{i+2} \, / \, 1 \leq i \leq n-2\} \cup \{u_{n-1} u_1\} \cup \{u_n u_2\} \cup \{u_i u_{n+i} \, / \, 1 \leq i \leq n\} \cup \{u_i u_{n-1+i} \, / \, 2 \leq i \leq$

$n\} \cup \{u_i u_{n+1+i} \;/\; 1 \le i \le n-1\} \cup \{u_n u_{n+1}\} \cup \{u_1 u_{2n}\}$. Also, $|V({S_n}^2)| = 2n$ and $|E({S_n}^2)| = \begin{cases} 4n, & n = 3 \\ 4n+2, & n = 4 \\ 5n, & n \ge 5 \end{cases}$.

The maximum and minimum degree of a square graph of a sun let graph ${S_n}^2, n \ge 3$ are $\Delta = \begin{cases} n+2, & n = 3,4 \\ 7, & n \ge 5 \end{cases}$ and $\delta = 3$ respectively. The labeling mapping function $f : V({S_n}^2) \to \{1, 2, \dots, 2n\}$ is defined by, $f(u_i) = i + 1$ for $1 \le i \le 2n$. Clearly, every vertices and edges labels are distinct. So that, the factori-difference labeling conditions are satisfied and that the conditions are every edge $e = uv = \frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$ and for any edge $f(e_i) \ne f(e_j), i \ne j$. Then, the function f is a factori-difference labeling for a square graph of a sun let graph ${S_n}^2, n \ge 3$. Thus, the square graph of a sun let graph ${S_n}^2, n \ge 3$ is a factori-difference graph. The chromatic number $\chi({S_n}^2)$ of a square graph of a sun let graph ${S_n}^2, n \ge 3$ is the minimum k. i.e. $\chi({S_n}^2) = \begin{cases} 4, & n \text{ is even} \\ 5, & n \text{ is odd} \end{cases}$.

1.34. Example

The factori-difference labeling of a square graph of a sun let graph ${S_n}^2, n \ge 3$ is shown figure 9.

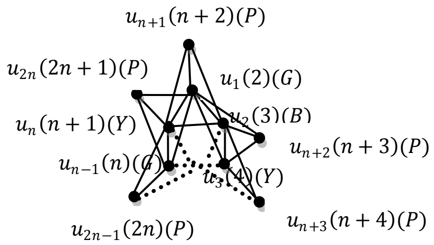

Figure 9. Factori-difference labeling for a graph ${S_n}^2$ and $\chi({S_n}^2) = \begin{cases} 4, & n \text{ is even} \\ 5, & n \text{ is odd} \end{cases}$.

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III. CONCLUSION

In this paper, we discussed the factori-difference labeling and that the factori-difference graph. The factori-difference labeling conditions are satisfied to some square graphs of a classes of graphs likely path, cycle, fan, brush, friendship, wheel, helm, ladder, sun let graphs and also the above graphs are produces the factori-difference graphs. Also, we found that the chromatic number of the square graph of some classes of graphs.

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