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Solution of Space Time Fractional Partial Differential Equations By Adomian Decomposition Method

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ABSTRACT
The purpose of this paper is to solve different types of initial value problems (IVPs) for space time fractional transport equations, space time fractional diffusion and wave equations as well as space time fractional Airy's equation using the Adomian decomposition method. The method is successfully applied and series solution of initial value problems are obtained, converging to a function known as solution function of the initial value problems.
KEYWORDS: Adomian decomposition method, Caputo fractional derivative, Mittag-Leffler functions, Riemann-Liouville integral, Space time fractional transport equation, Space time fractional diffusion-wave equations, Space time fractional Airy's equation.

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I. INTRODUCTION :

Many problems in mathematical, physical, chemical and biological sciences and technologies are governed by differential equations. In recent years, fractional differential equations have attracted many researchers due to their applications in the field ofvisco-elasticity, feedback amplifiers, electrical circuits, electro analytical chemistry, fractional multipoloes etc. Consider the general linear fractional partial differential equation

$D_t^{\alpha}u(x, t) = \sum_{j=1}^{n} a_j\, D_{xj}^{\delta j}\, u(x, t) + \sum_{j=1}^{n} bj\, D_{xj}^{\beta j} u(x,t) + \sum_{j=1}^{n} c_j D_{xj}^{\gamma j} u(x, t) + du(x, t) + f(x, t);$

$m - 1 < \alpha \leq m,\ 2 < \delta_j \leq 3,\ 1 < \beta_j \leq 2,\ 0 < \gamma_j \leq 1,\ m \in N$ where $x = (x_1, x_2, \ldots x_n) \in R^n$,

a_j , b_j , c_j, d are real constants, $0 \leq t \leq T$, $f(x, t)$ is known real as valued continuous function and $D^{\mu}_t\, u(x, t)$ is the μth-order Caputo partial fractional derivative of a function $u(x, t)$ with respect to ' t '.These equations appear in many interesting physical processes such as transportation, diffusion of heat, propagation of wave.

Fractional transport equations

$$D_t^{\alpha} u(x, t) = \sum_{j=1}^{n} cj\, D_{xj}^{\gamma j}\, u(x, t) + f(x , t); \quad 0 < \alpha \leq 1,\ 0 < \gamma_j \leq 1 \qquad (1.1)$$

represent transportation phenomena. Fractional diffusion-wave equations

$$D_t^{\alpha} u(x, t) = \sum_{j=1}^{n} bj\, D_{xj}^{\beta j}\, u(x, t) + du(x , t) + f(x , t); \quad m - 1 < \alpha \leq m,\ 1 < \beta_j \leq 2, \qquad (1.2)$$

$$D_t^{\alpha} u(x, t) = \sum_{j=1}^{n} a_j D_{xj}^{\delta j}\, u(x, t) + f(x, t); \quad m - 1 < \alpha \leq m,\ 2 < \delta_j \leq 3,\ m \in N \qquad (1.3)$$

represent relaxation phenomena in complex viscoelastic material, propagation of mechanical waves in visco-elastic media, non-Markovian diffusion process with memory, electromagnetic acoustic and mechanical responses, Roman and Alemany investigated a continuous time random walks on fractals. Fractional differential equations are solved by A domain decomposition method , Finite sine transform method, an iteration method, method of images and Fourier transform, Green's function method .

We define Caputo partial fractional derivative. It needs following Riemann-Liouville fractional integral.

Definition 1.1 :

The (left sided)Riemann-Liouville fractional integral of order μ, μ > 0 of a function u(x, t) ϵ C_α , α ≥ -1 is denoted by I_t^{μ} u(x, t) and defined as

$$D_t^{\mu} u(x, t) = \frac{1}{\Gamma(\mu)} \int_0^t (t - \tau)^{\mu-1} u(x, \tau) d\tau,\ \ t > 0$$

Definition 1.2

The (left sided) Caputo partial fractional derivative of a function

u(x, t)∈ C_l^m, w. r. t ' t ' is denoted by D_t^{μ} u(x, t) and is defined as

$$D_t^{\mu} u(x,t) = \begin{cases} \frac{\partial^m}{\partial t^m} u(x,t), \mu = m, \ m \in N \\ I_t^{m-\mu} \frac{\partial^m}{\partial t^m} u(x,t), m-1 < \mu < m \end{cases}$$

where $I_t^{\mu} u(x, t)$ is Riemann-Liouville fractional integral of order μ, μ > 0

Note that, $I_t^{\mu} D_t^{\mu} u(x, t) = u(x, t) - \sum_{k=0}^{m-1} \frac{\partial^k u}{\partial t^k} u(x, 0) \frac{t^k}{k!}$, $m-1 < \mu < m$, $m \in N$

and $I_t^{\mu} t^{\upsilon} = \frac{\Gamma(\upsilon+1)}{\Gamma(\mu+\upsilon+1)} t^{(\upsilon+\mu)}$

Mittag-LefflerFunction : In 1902, Mittag-Leffler introduced the one parameter function commonly known as Mittag-Leffler function which is denoted by $E_{\alpha}(z)$ and defined as

$E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)}$, $(\alpha > 0)$ (1.4)

Example:(i) If we put $\alpha = 1$ then equation (1.4)becomes,

$E_1(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k+1)} = \sum_{k=0}^{\infty} \frac{z^k}{k!} = e^z$

Example:(ii) If we put $\alpha = 2$ then equation (1.4) becomes,

$E_2(z) = \sum_{k=0}^{\infty} \frac{z^{2k}}{\Gamma(2k+1)} = \sum_{k=0}^{\infty} \frac{z^{2k}}{(2k!)} = coshz$

The rest of this paper is divided into the following sections. In section 2, Adomian decomposition method is discussed. Iterative solution of initial value problem for general fractional diffusion-wave equation is obtained as an application of Adomian decomposition method. In section 3, some illustrative examples for fractional transport, fractional diffusion, fractional Airy's equation as well as fractional wave equation are discussed.

II. ANALYSIS OFADOMIAN METHOD FOR FRACTIONAL INITIAL VALUE PROBLEMS:

Consider the general fractional partial differential equation

$D_t^{\alpha} u(x, t) = \sum_{j=1}^{n} a_j D_{xj}^{\delta j} u(x, t) + \sum_{j=1}^{n} b_j D_{xj}^{\beta j} u(x, t) + \sum_{j=1}^{n} c_j D_{xj}^{\gamma j} u(x, t) + du(x, t) + f(x, t)$; (2.1)

With the initial condition $\frac{\partial^k u(x,0)}{\partial t^k} = h_k(x)$, $0 \le k \le m-1$ (2.2)

Fractional partial differential equation (2.1) together With the initial condition(2.2) is called initial value problem.

We are interested for series solution of such type of initial value problem (IVP).

$u(x, t) = \sum_{i=0}^{\infty} u_i(x, t)$ (2.3)

Applying I_t^{α}, to the equation (2.1) on both sides

$I_t^{\alpha} D_t^{\alpha} u(x,t) = I_t^{\alpha}(\sum_{j=1}^{n} a_j D_{xj}^{\delta j} u(x, t) + \sum_{j=1}^{n} b_j D_{xj}^{\beta j} u(x, t) + \sum_{j=1}^{n} c_j D_{xj}^{\gamma j} u(x, t) + du(x, t) + f(x,t))$

And using initial conditions (2.2), we get

$u(x,t) = \sum_{k=0}^{m-1} h_k(x) \frac{t^k}{k!} + I_t^{\alpha}(\sum_{j=1}^{n} a_j D_{xj}^{\delta j} u(x, t) + \sum_{j=1}^{n} b_j D_{xj}^{\beta j} u(x, t) + \sum_{j=1}^{n} c_j D_{xj}^{\gamma j} u(x, t) + du(x, t) + f(x,t))$

$\sum_{i=0}^{\infty} u_i(x,t) = \sum_{k=0}^{m-1} h_k(x) \frac{t^k}{k!} + I_t^{\alpha}(\sum_{j=1}^{n} a_j D_{xj}^{\delta j} \sum_{i=0}^{\infty} u_i(x,t) + \sum_{j=1}^{n} b_j D_{xj}^{\beta j} \sum_{i=0}^{\infty} u_i(x,t)$
$+ \sum_{j=1}^{n} c_j D_{xj}^{\gamma j} \sum_{i=0}^{\infty} u_i(x,t) + d\sum_{i=0}^{\infty} u_i(x,t) + f(x,t))$

$\sum_{i=0}^{\infty} u_i(x,t) = \sum_{k=0}^{m-1} h_k(x) \frac{t^k}{k!} + I_t^{\alpha}(\sum_{j=1}^{n} a_j D_{xj}^{\delta j} \sum_{i=0}^{\infty} u_i(x,t)) + I_t^{\alpha}(\sum_{j=1}^{n} b_j D_{xj}^{\beta j} \sum_{i=0}^{\infty} u_i(x,t))$
$+ I_t^{\alpha}(\sum_{j=1}^{n} c_j D_{xj}^{\gamma j} \sum_{i=0}^{\infty} u_i(x,t)) + I_t^{\alpha}(d\sum_{i=0}^{\infty} u_i(x,t)) + I_t^{\alpha}(f(x,t))$ (2.4)

Now we define the recursive scheme

$u_0(x, t) = \sum_{k=0}^{m-1} h_k(x) \frac{t^k}{k!} + I_t^{\alpha}(f(x, t)$

$u_1(x, t) = I_t^{\alpha}(\sum_{j=1}^{n} a_j D_{xj}^{\delta j} u_0(x, t)) + I_t^{\alpha}(\sum_{j=1}^{n} b_j D_{xj}^{\beta j} u_0(x, t)) + I_t^{\alpha}(\sum_{j=1}^{n} c_j D_{xj}^{\gamma j} u_0(x, t)) + I_t^{\alpha}(du_0(x, t))$

$u_2(x, t) = I_t^{\alpha}(\sum_{j=1}^{n} a_j D_{xj}^{\delta j} u_1(x, t)) + I_t^{\alpha}(\sum_{j=1}^{n} b_j D_{xj}^{\beta j} u_1(x, t)) + I_t^{\alpha}(\sum_{j=1}^{n} c_j D_{xj}^{\gamma j} u_1(x, t)) + I_t^{\alpha}(du_1(x, t))$.

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$u_n(x, t) = I_t^{\alpha}(\sum_{j=1}^{n} a_j D_{xj}^{\delta j} u_{n-1}(x, t)) + I_t^{\alpha}(\sum_{j=1}^{n} b_j D_{xj}^{\beta j} u_{n-1}(x, t)) + I_t^{\alpha}(\sum_{j=1}^{n} c_j D_{xj}^{\gamma j} u_{n-1}(x, t)) + I_t^{\alpha}(du_{n-1}(x, t))$

It is noteworthy that the recursive scheme is constructed on the basis that the zeroth component $u_0(x, t)$ defined by a term that arises from the initial condition and the source term $f(x, t)$, both are known. Hence $u_0(x, t)$ is known. The remaining components $u_n(x, t)$, $n \geq 1$ can be completely determined; each component is computed by using the previous component. As a result, the components u_0, u_1, u_2, ... are calculated and the series solution is determined. Based on the Adomian Decomposition method, we considered the solution $u(x, t)$ as

$u(x, t) = \lim_{n\to\infty} \varphi_n$ (2.5) where the (n+1) term approximation of the solution is defined in the following form

$\phi_n = \sum_{k=0}^{n} u_n(x\ ,\ t)$ (2.6)

We apply this method to some examples, and the solutions are obtained in closed form. In linear problems, the practical solution ϕ_n ,the n-term approximation is converging. However, in many cases, it may not be possible to obtain the exact solution in a closed form. The question of convergence is established.

III. ILLUSTRATIVE EXAMPLES :

In this section, we discuss some illustrative examples for space time fractional transport equations, space time fractional diffusion equations, space time fractional wave and Airy'sequations one by one.

I) Space Time Fractional Transport Equation:

These equations appear in the mathematical description of many phenomena in classical and statistical physics. Now we consider some examples of space time fractional transport equations with suitable initial condition

Example 3.1 *Consider the space time fractional transport equation*

$\frac{\partial^\alpha}{\partial t^\alpha} + \frac{\partial^\beta}{\partial t^\beta} = 0, \quad 0< \alpha \leq 1,\ \ 0 < \beta \leq 1,\ \gamma = 1,\ t > 0,\ x_1 \in R$ (3.1)

with the initial condition

$u(x_1 , 0) = x_1^\beta$ (3.2)

The initial value problem (3.1)-(3.2) *is a special case of IVP* (2.1)-(2.2).

Solution:*We know that*

$\frac{\partial^\alpha}{\partial t^\alpha} = D_t^\alpha \ , \frac{\partial^\beta}{\partial x_1^\beta} = D_{x1}^\beta$

are the Caputo time fractional derivative of order α and the space fractional derivative of order β respectively. We look for series solution

$u(x_1,t) = \sum_{i=0}^{\infty} u_i(x_1 , t)$

Multiply by the inverse operator I_t^α to equation (3.1),*we get,*

$D_t^\alpha u(x_1 , t) = - D_{x1}^\beta u(x_1 , t)$

$I_t^\alpha D_t^\alpha u(x_1 , t) = - I_t^\alpha D_{x1}^\beta u(x_1 , t)$

$u(x_1 , t) = u(x_1 , 0) - I_t^\alpha [D_{x1}^\beta u(x_1 , t)]$

$\sum_{i=0}^{\infty} u_i(x_1 , t) = u(x_1 , 0) - I_t^\alpha [D_{x1}^\beta \sum_{i=0}^{\infty} u_i(x_1 , t)]$

$u_1 + u_2 + u_3 \ldots\ldots = u(x_1 , 0) - I_t^\alpha [D_{x1}^\beta u_0(x_1 , t)] - I_t^\alpha [D_{x1}^\beta u_1(x_1 , t)] - I_t^\alpha [D_{x1}^\beta u_2(x_1 , t)] - I_t^\alpha [D_{x1}^\beta u_3(x_1 , t)] - \ldots\ldots\ldots\ldots\ldots$

From recursive relations, we get,

$u_0(x_1 , t) = u(x_1 , 0) = x_1^\beta$

$u_1(x_1 , t) = - I_t^\alpha [D_{x1}^\beta u_0(x_1 , t)] = - I_t^\alpha [D_{x1}^\beta x_1^\beta] = \frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^\alpha$

$u_2(x_1 , t) = - I_t^\alpha [D_{x1}^\beta u_1(x_1 , t)] = - I_t^\alpha [D_{x1}^\beta (\frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^\alpha)] =$

Substituting u_0, u_1, $u_2$in series (3.3), *we have, the solution of IVP* (3.1)-(3.2)

$u(x_1 , t) = x_1^\beta - \frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^\alpha$

II. Fractional Diffusion Equation:

Now we consider an example of space time fractional diffusion equation with suitable initial condition

Example 3.2 *Consider the Cauchy problem for space and time-fractional diffusion equation,*

$\frac{\partial^\alpha u(x1,\ x2,\ t)}{\partial t^\alpha} = K \left(\frac{\partial^\beta u(x1 ,x2 ,t)}{\partial x1} + \frac{\partial^\beta u(x1 ,x2 ,t)}{\partial x2}\right)$ (3.4)

$0<\alpha\leq 1,\ 1 < \beta \leq 2, \quad (x_1 , x_2) \in R^2$

with the initial condition, $u(x_1 , x_2 , 0) = \frac{1}{2} (x_1^\beta + x_2^\beta)$(3.5)

The initial value problem (3.4)-(3.5) is a special case of IVP (2.1)-(2.2).

Solution: We know that

$\frac{\partial^\alpha}{\partial t^\alpha} = D_t^\alpha, \frac{\partial^\beta}{\partial x1} = D_{x1}^\beta, \frac{\partial^\beta}{\partial x2} = D_{x2}^\beta,$

are the Caputo time fractional derivative of order α and the space fractional derivatives of order β respectively. We look for the series solution

$u(x_1, x_2, t) = \sum_{i=0}^{\infty} u_i(x_1, x_2, t)$ (3.6)

Multiplying by I_t^α to both the sides of equation (3.4), *we get,*

$I_t^\alpha D_t^\alpha u(x_1 , x_2 , t) = K\ I_t^\alpha [D_{x1}^\beta u(x_1 , x_2 , t) + D_{x2}^\beta u(x_1 , x_2 , t)]$

$u(x_1 , x_2 , t) - u(x_1 , x_2 , 0) = K I_t^\alpha [D_{x1}^\alpha u(x_1 , x_2 , t) + D_{x2}^\alpha u(x_1 , x_2 , t)]$

$u(x_1, x_2, t) = u(x_1, x_2, 0) + K I_t^{\alpha}[D_{x1}^{\beta} u(x_1, x_2, t) + D_{x2}^{\beta} u(x_1, x_2, t)]$
$\sum_{i=0}^{\infty} u_i(x_1, x_2, t) = u(x_1, x_2, 0) + K I_t^{\alpha}[D_{x1}^{\beta} \sum_{i=0}^{\infty} u_i(x_1, x_2, t) + D_{x2}^{\beta} \sum_{i=0}^{\infty} u_i(x_1, x_2, t)]$
$u_1 + u_2 + u_3 \ldots\ldots = u(x_1, x_2, 0) + K I_t^{\alpha}[D_{x1}^{\beta} u_0(x_1, x_2, t) + D_{x2}^{\beta} u_0(x_1, x_2, t)] + K I_t^{\alpha}[D_{x1}^{\beta} u_1(x_1, x_2, t)$
$+ D_{x2}^{\beta} u_1(x_1, x_2, t)] + K I_t^{\alpha}[D_{x1}^{\beta} u_2(x_1, x_2, t) + D_{x2}^{\beta} u_2(x_1, x_2, t)] + \ldots\ldots$
From recursive scheme, we get,
$u_0(x_1, x_2, t) = u(x_1, x_2, 0) = \frac{1}{2}(x_1^{\beta} + x_2^{\beta})$
$u_1(x_1, x_2, t) = K I_t^{\alpha}[D_{x1}^{\beta} u_0(x_1, x_2, t)] + K I_t^{\alpha}[D_{x2}^{\beta} u_0(x_1, x_2, t)]$
$= K I_t^{\alpha}[D_{x1}^{\beta} \frac{1}{2}(x_1^{\beta} + x_2^{\beta})] + K I_t^{\alpha}[D_{x2}^{\beta} \frac{1}{2}(x_1^{\beta} + x_2^{\beta})]$
$= K\frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^{\alpha}$
$u_2(x_1, x_2, t) = K I_t^{\alpha}[D_{x1}^{\beta} u_1(x_1, x_2, t)] + K I_t^{\alpha}[D_{x2}^{\beta} u_1(x_1, x_2, t)]$
$= K I_t^{\alpha}[D_{x1}^{\beta} \frac{1}{2}(x_1^{\beta} + x_2^{\beta})] + K I_t^{\alpha}[D_{x2}^{\beta} \frac{1}{2}(x_1^{\beta} + x_2^{\beta})]$
$= I_t^{\alpha}[D_{x1}^{\beta} [K \frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^{\alpha}] + I_t^{\alpha}[D_{x2}^{\beta} [K \frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^{\alpha}]$
$= 0$
Substituting u_0, u_1, u_2 in series solution(3.6), *we have the solution of IVP* (3.4)-(3.5)
$u(x_1, x_2, t) = \frac{1}{2}(x_1^{\beta} + x_2^{\beta}) + K\frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^{\alpha}$

III.Fractional Airy's Equation:
Now we consider an examples of space time fractional Airy's equation with suitable initial condition
Example 3.3 *Consider the space time-fractional Airy's equation*
$\frac{\partial^{\alpha} u(x1, t)}{\partial t^{\alpha}} = \frac{\partial^{\beta} u(x1, t)}{\partial x1}$ (3.7)
$0 < \alpha \leq 1, \quad 2 < \beta \leq 3, \quad x_1 \in R$
with the initial condition
$u(x_1, 0) = \frac{1}{6} x_1^{\beta}$ (3.8)
The initial value problem (3.7)-(3.8)for space time fractional Airy's equation is a special case of IVP (2.1)-(2.2).
Solution: We know that
$\frac{\partial^{\alpha}}{\partial t^{\alpha}} = D_t^{\alpha}, \quad \frac{\partial^{\beta}}{\partial x1} = D_{x1}^{\beta}$
are the Caputo time fractional derivative of order α and the space fractional derivative of order β respectively. We look for the series solution
$u(x_1, t) = \sum_{i=0}^{\infty} u_i(x_1, t)$ (3.9)
Multiplying by I_t^{α} to both the sides of equation (3.7), we get,
$I_t^{\alpha} D_t^{\alpha} u(x_1, t) = I_t^{\alpha}(D_{x1}^{\beta} u(x_1, t)$
$u(x_1, t) - u(x_1, 0) = I_t^{\alpha}[D_{x1}^{\alpha} u(x_1, t)$
$u(x_1, t) = u(x_1, 0) + I_t^{\alpha}[D_{x1}^{\beta} u(x_1, t)$
$\sum_{i=0}^{\infty} u_i(x_1, t) = u(x_1, 0) + I_t^{\alpha}[D_{x1}^{\beta} \sum_{i=0}^{\infty} u_i(x_1, t)]$
$u_1 + u_2 + u_3 \ldots\ldots = u(x_1, 0) + I_t^{\alpha}[D_{x1}^{\beta} u_0(x_1, t)] + I_t^{\alpha}[D_{x1}^{\beta} u_1(x_1, t)] + \ldots$
From recursive scheme, we get,
$u_0(x_1, t) = u(x_1, 0) = \frac{1}{6}(x_1^{\beta})$
$u_1(x_1, t) = I_t^{\alpha}[D_{x1}^{\beta} u_0(x_1, t)]$
$= I_t^{\alpha}[D_{x1}^{\beta} \frac{1}{6}(x_1^{\beta})]$
$= \frac{1}{6}\frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^{\alpha}$
$u_2(x_1, t) = I_t^{\alpha}[D_{x1}^{\beta} u_1(x_1, t)]$
$= I_t^{\alpha}[D_{x1}^{\beta} \frac{1}{6}[\frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^{\alpha}]$
$= 0$
Substituting u_0, u_1, u_2 in series (3.9), we have the solution of IVP (3.7)-(3.8),
$u(x_1, t) = \frac{1}{6}[(x_1^{\beta}) + \frac{\Gamma(\beta+1)}{\Gamma(\alpha+1)} t^{\alpha}]$

IV. FRACTIONAL WAVE EQUATION:

Now we consider an examples of space time fractional wave equation with suitable initial conditions

Example 3.4 Consider the space time fractional wave equation

$D_t^{\alpha}u(x_1, t) = D_{x1}^{\beta}u$ (3.10)

$0<\alpha\leq 1,\ 2<\beta\leq 3,\quad x_1\in R$

with the initial condition

$u(x_1, 0) = \cos x_1, u_t(x_1, 0) = 0$ (3.11)

The initial value problem (3.10)-(3.11)for space time fractional wave equation is a special case of IVP (2.1)-(2.2).

Solution: We know that

$\frac{\partial^{\alpha}}{\partial t^{\alpha}} = D_t^{\alpha}$, $\quad \frac{\partial^{\beta}}{\partial x1} = D_t^{\beta}$

are the Caputo time fractional derivative of order α and the space fractional derivative of order β respectively. We look for the series solution

$u(x_1, t) = \sum_{i=0}^{\infty} u_i(x_1, t)$ (3.12)

Multiplying by $I_t^{\alpha}$to both the sides of equation (3.10), we get,

$I_t^{\alpha} D_t^{\alpha}u(x_1, t) = I_t^{\alpha}[D_{x1}^{\beta}\, u(x_1, t)]$

$u(x_1, t) = u(x_1, 0) + u_t(x_1, 0) + I_t^{\alpha}[D_{x1}^{\alpha}\, u(x_1, t)]$

$\sum_{i=0}^{\infty} u_i(x_1, t) = u(x_1, 0) + I_t^{\alpha}[D_{x1}^{\beta} \sum_{i=0}^{\infty} u_i(x_1, t)]$

$u_1 + u_2 + u_3 \ldots\ldots = u(x_1, 0) + I_t^{\alpha}[D_{x1}^{\beta}\, u_0(x_1, t)] + I_t^{\alpha}[D_{x1}^{\beta}\, u_1(x_1, t)] + \ldots\ldots\ldots$

From recursive scheme, we get,

$u_0(x_1, t) = u(x_1, 0) = \cos x_1$

$u_1(x_1, t) = I_t^{\alpha}[D_{x1}^{\beta}\, u_0(x_1, t)]$

$= I_t^{\alpha}[D_{x1}^{\beta} \cos x_1]$

$= \cos(x_1 + \frac{\pi}{2}\beta)\frac{1}{\Gamma(\alpha+1)} t^{\alpha}$

$u_2(x_1, t) = I_t^{\alpha}[D_{x1}^{\beta}\, u_1(x_1, t)]$

$= I_t^{\alpha}[D_{x1}^{\beta} \cos(x_1 + \frac{\pi}{2}\beta)\frac{1}{\Gamma(\alpha+1)} t^{\alpha}]$

$= [\cos(x_1 + \frac{2\pi}{2}\beta)\frac{1}{\Gamma(2\alpha+1)} t^{2\alpha}]$

$u_3(x_1, t) = I_t^{\alpha}[D_{x1}^{\beta}\, u_2(x_1, t)]$

$= I_t^{\alpha}[D_{x1}^{\beta} \cos(x_1 + \frac{2\pi}{2}\beta)\frac{1}{\Gamma(2\alpha+1)} t^{2\alpha}]$

$= [\cos(x_1 + \frac{3\pi}{2}\beta)\frac{1}{\Gamma(3\alpha+1)} t^{3\alpha}]$

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$u_i(x_1, t) = I_t^{\alpha}[D_{x1}^{\beta}\, u_{i-1}(x_1, t)]$

$= [\cos(x_1 + \frac{i\pi}{2}\beta)\frac{1}{\Gamma(i\alpha+1)} t^{i\alpha}]$

Substituting u_0, u_1, u_2 in series (3.12), we have the solution of IVP (3.10)-(3.11),

$u(x_1, t) = \cos x_1 + \cos(x_1 + \frac{\pi}{2}\beta)\frac{1}{\Gamma(\alpha+1)} t^{\alpha}$

$+ \cos(x_1 + \frac{2\pi}{2}\beta)\frac{1}{\Gamma(2\alpha+1)} t^{2\alpha} + \ldots$

$u(x_1, t) = \sum_{i=0}^{\infty} \cos\left(x1 + \frac{i\pi}{2}\beta\right)\frac{1}{\Gamma(i\alpha+1)}(t^{\alpha})^i$ (3.13)

Remark 3.1 If we put α = 2, β = 2 in IVP (3.10)-(3.11) for space time fractionalWave equation and in its solution (3.13), we have

$\frac{\partial^2 u(x1, t)}{\partial t^2} = \frac{\partial^2 u(x1, t)}{\partial x_1^2}$ (3.14)

$u(x_1, 0) = \cos x_1 u_t(x_1, 0) = 0$ (3.15)

$u(x_1, t) = \sum_{i=0}^{\infty} \cos(x1 + i\pi\beta)\frac{1}{\Gamma(2\alpha+1)}(t^2)^i$

$= \cos x_1 \cos t$ (3.16)

Now we observe that (3.16) is solution of IVP (3.14)-(3.15).Therefore we conclude that our results obtained for IVP of time space fractional wave equation agree with IVP for classical wave equation.

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