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Ionization Effect of Soliton Propagation in Non-uniform Dusty Plasmas

Dr. L. B. Gogoi
Associate Professor, Department of Mathematics, Duliajan College, Duliajan, Dibrugarh, Assam, India
Corresponding Author: Dr. L. B. Gogoi

ABSTRACT

Propagation of solitons in spatially non-uniform (inhomogeneous) dusty plasma in presence of weak ionization is studied. For this study, fluid model of the plasma is considered. The related fluid equations are treated by reductive perturbation analysis with a suitable space–time stretched coordinate. The propagation characteristics are described by modified Korteweg–de–Vries (mKdV) equation. The soliton solutions are found to be affected by weak ionization and plasma inhomogeneity. The effective conditions like peak amplitude and width for soliton propagation in this inhomogeneous dusty plasma model are analyzed.

Keywords: Inhomogeneous dusty plasma, Ion–acoustic solitons, ionization, mKdV equation, plasma inhomogeneity, sine-cosine method, solitary wave.

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I. INTRODUCTION

The properties of propagation of solitons in plasma dynamics have received a special attention of the researchers in last decades. The concept of study of such properties was first augmented by Washimi and Tanuity[1] through a nonlinear wave equation known as Korteweg- de-Vries(KdV) equation[2]. Majority of the theoretical results of various aspects of solitary waves were limited to uniform (homogeneous) plasmas. As homogeneity is a special case of inhomogeneity due to which we invariably encounter inhomogeneous plasmas both in space and laboratory. Non-uniformity (inhomogeneity) may be due to density gradient, temperature gradient or magnetic field etc. In uniform plasma, solitary wave travels without change in amplitude, shape and speed however in non-uniform plasma; the soliton is altered as it propagates. Also, dust grains are quite common throughout the universe. Dusty plasma normally contains nanometer or micrometer sized dust particles together with ordinary plasma particles such as electrons, ions and neutrals. A considerable amount of analytical as well as experimental investigations have been done so far to investigate extensively about various aspects of the propagation of soliton in inhomogeneous medium [3-16]. Dust grains in inhomogeneous medium [17] were also considered in some previous studies. In this present work, we have considered weakly ionized inhomogeneous dusty plasma. The reductive perturbation analyses of fluid equations are carried out by employing a set of 'stretched coordinates' appropriate for spatially inhomogeneous plasma. From the fluid equations we have derived a modified KdV (mKdV) equation which describes the propagation of solitons.

II. BASIC EQUATIONS

We have considered an unmagnetised weakly inhomogeneous with weak ionization dusty plasma. The Boltzmann distribution for electrons, constant electron temperature and zero ion temperature are assumed. The continuity and momentum equation for this plasma model with Poisson's equation and electron Boltzmann distribution can be written as follows:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}\left(\mathrm{n}_d \mathrm{u}_d\right) = \nu n_e \tag{1}$$

$$\frac{\partial u_d}{\partial t} + \mathrm{u}\,\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial x} = -\left(\nu n_e / n_d\right) u_d \tag{2}$$

$$n_e = n_e^{(0)} \exp\phi \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2} - n_e - n_d + n_i = 0 \tag{4}$$

Where n_i, n_d and n_e are respectively, the ion, dust and electron densities, u_d is the dust fluid velocity, ϕ is the electrostatic potential, ν is the ionization frequency and x, t are space and time variables respectively.

Normalizing n_i, n_d and n_e by the zero-order ion density at $x=0$, the quantity u_d is normalized by the ion-acoustic speed and ϕ by $K_B T_e / e$ where K_B, T_e and e are Boltzmann's constant, electron temperature and ion charge respectively. The time t and spatial coordinate x are normalized respectively by the reciprocal of the ion plasma frequency at $x=0$ and the Debye length at $x=0$. The ionization frequency ν is normalized by ion-plasma frequency at x = 0. It has been assumed further that the quasineutrality condition under which the zero-order ion and electron densities are equal at $x=0$. Also from equation (1) we assume that $\nu n_e \approx \nu n$.

The normalized form the above equations (1) – (4) becomes

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}\left(n_d u_d\right) = \nu n_d \tag{5}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \frac{\partial \phi}{\partial x} = -\nu \tag{6}$$

$$\frac{\partial^2\phi}{\partial x^2} - n_e - n_d + n_i = 0 \tag{7}$$

III. DERIVATION AND SOLUTION OF THE MODIFIED KDV EQUATION

The usual form of spatial stretched coordinates [14], appropriate for especially inhomogeneous plasma is as follows:

$$\xi = \varepsilon^{\frac{1}{2}}\left(\frac{x}{\lambda_0} - t\right), \quad \tau = \varepsilon^{\frac{3}{2}} x \tag{8}$$

where ε is an expansion parameter and λ_0 is the phase velocity of the dust-acoustic wave.

Further, because of weak ionization we ordered as

$$\nu = \varepsilon^{\frac{3}{2}}\nu_0, \quad \nu_0 = o(1) \tag{9}$$

Using equations (8) and (9), equations (5) – (7) becomes

$$-\frac{\partial n_d}{\partial \xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial \xi}\left(n_d u_d\right) + \varepsilon\frac{\partial}{\partial \tau}\left(n_d u_d\right) = \varepsilon \nu_0 n_d \tag{10}$$

$$-\frac{\partial u_d}{\partial \xi} + \frac{u_d}{\lambda_0}\frac{\partial u_d}{\partial \xi} + \varepsilon u_d \frac{\partial u_d}{\partial \tau} + \frac{1}{\lambda_0}\frac{\partial \phi}{\partial \xi} + \varepsilon\frac{\partial \phi}{\partial \tau} = -\varepsilon \nu_0 u_d \tag{11}$$

and
$$\frac{\varepsilon}{\lambda_0^{\;2}}\frac{\partial^2\phi}{\partial \xi^2} + \frac{2\varepsilon^2}{\lambda_0}\frac{\partial^2\phi}{\partial\xi\,\partial\tau} + \varepsilon^3\frac{\partial^2\phi}{\partial\tau^2} - \frac{\varepsilon^2}{\lambda_0^{\;2}}\frac{\partial\lambda_0}{\partial\tau}\frac{\partial\phi}{\partial\xi} - n_e^{(0)}e^{\phi} + n_d + n_i^{(0)}e^{-\phi} = 0 \tag{12}$$

To employ the reductive perturbation technique [1], the plasma parameters n, u and ϕ are expressed as power series in ε as

$$\left.\begin{aligned} n_d &= n_d^{(0)} + \varepsilon n_d^1 + \varepsilon^2 n_d^2 + \varepsilon^3 n_d^3 + \dots \\ u &= u_d^{(0)} + \varepsilon u_d^{(1)} + \varepsilon^2 u_d^{(2)} + \varepsilon^3 u_d^{(3)} + \dots \\ \phi &= \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \varepsilon^3\phi^{(3)} + \dots \end{aligned}\right\} \tag{13}$$

where $n_0^{(d)}$ are $u_d^{(0)}$ are the plasma parameters in unperturbed state.

Since $n_d^{(0)}$ and λ_0 are independent of ξ , we have

$$\frac{\partial n_d^{(0)}}{\partial \xi} = \frac{\partial \lambda_0}{\partial \xi} = 0 \qquad (14)$$

From equations (10) – (12), the zeroth-order of ε gives

$$\frac{\partial u_d^{(0)}}{\partial \xi} = 0 \qquad (15)$$

from which we get

$$\frac{\partial}{\partial \tau}\left(n_d^{(0)} u_d^{(0)}\right) = \nu_0\, n_d^{(0)} \quad \text{and} \quad u_d^{(0)} \frac{\partial u_d^{(0)}}{\partial \tau} = \nu_0\, u_d^{(0)} \;\Rightarrow \frac{\partial u_d^{(0)}}{\partial \tau} = \nu_0 \qquad (16)$$

Using Eq. (16) into Eqs. (10) – (12), the lowest order of ε together with the boundary conditions $u_d^{(0)} \to 0,\; u_d^{(1)} = \phi^{(1)}$ and $n_d^{(0)}, \lambda_0 \to 1$ as $|\xi| \to \propto$ we get

$$\left.\begin{aligned} u_d^{(1)} &= P n_d^{(1)} \\ \phi^{(1)} &= n_d^{(0)} u_d^{(0)} P \end{aligned}\right\} \quad \text{where} \quad P = \frac{\lambda_0 - u_d^{(0)}}{n_d^{(0)}}$$

and hence we get $\left(\lambda_0 - u_d^{(0)}\right)^2 = 1$ (17)

Considering second order relations of ε from Eqs. (10) – (12) and then eliminating all the second order quantities, we get the following equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + \alpha \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \beta \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + \gamma \phi^{(1)} = 0 \qquad (18)$$

Where $\alpha = \dfrac{1}{\lambda_0^{\;2}}, \quad \beta = \dfrac{1}{2 n_d^{(0)} \lambda_0^{\;4}}, \; \gamma = \dfrac{1}{\lambda_0}\dfrac{\partial \lambda_0}{\partial \tau} + \dfrac{1}{2 n_d^{(0)}}\dfrac{\partial n_d^{(0)}}{\partial \tau}$ (19)

The Eq. (18) is a modified form of Korteweg-de-Vries (mKdV) equation. This equation contains an additional term with coefficient γ. This coefficient γ depend entirely on the inhomogeneity parameters which would have vanished in case of homogeneous plasma. Also the Eq. (18) contains a term with variable coefficient. To solve the eq.(18), we use the sine-cosine method which was first applied by Yan [15] to solve the KdV equation derived in a homogeneous plasma and find the exact soliton propagation in plasmas. Later, *Yan et al* extended this method further for inhomogeneous plasmas. To use sine-cosine method in eq.(18), we use the following variable transformations:

$$\zeta = \kappa\left(\xi - U\tau\right), \; \psi\left(\xi, \tau\right) = \Phi\left(\zeta\right) \qquad (20)$$

where κ^{-1} the width of the solitary wave and U is the shift in the velocity when the wave evolves as a soliton, then the equation (18) becomes

$$-\kappa U \frac{d\Phi}{d\zeta} + \alpha \kappa \Phi \frac{d\Phi}{d\zeta} + \beta\, \kappa^3 \frac{d^3\Phi}{d\zeta^3} + \gamma\, \Phi = 0 \qquad (21)$$

Using Sine-Cosine method, the solution of eq.(21) can be written as

$$\Phi\left(\omega\right) = A_0 + \sum_{i=1}^{p}\left(B_i \sin\omega + A_i \cos\omega\right)\cos^{i-1}\omega \qquad (22)$$

where $\dfrac{d\omega}{d\zeta} = \sin\omega$ and A_i, B_i are functions of ζ and ω but they will not appear explicitly as functions of $\sin\omega$ and $\cos\omega$. Also, p is determined by the balance of the leading order of nonlinear to linear terms. Due to lower order nonlinearity, we take p=2 in our present case. With these considerations, the solution in the form of variable ω can be written as

$$\Phi\left(\omega\right) = A_0 + A_1 \cos\omega + B_1 \sin\omega + A_2 \cos^2\omega + B_2 \cos\omega \sin\omega \qquad (23)$$

To determine the coefficients A_i, B_i, U and k , putting the values of $\Phi\left(\omega\right)$ from equation (23) in equation (22) and then the coefficients of the various trigonometric identities are put equal to zero. The odd functions $\sin\omega$, $\cos\omega\sin\omega$ etc. do not play any rule in solution, so $B_1 = B_2 = 0$. Finally as by *Das et al* [16], we get

$$A_0 = -A_2 = \frac{2\beta\kappa^2}{\alpha},\ A_1 = -\frac{40\beta\kappa^5}{\gamma\alpha},\ \ k^4 = \frac{\gamma^2}{2\beta U} \ \text{ and } \ U = \frac{\alpha}{6A_2}\left(3A_1^{\ 2} + 6A_0A_2 - 4A_2^{\ 2}\right) - \frac{\gamma A_1}{2kA_2} \qquad (24)$$

So, from eq.(22) the solution of mKdV equation becomes

$$\Phi(\omega) = \frac{2\beta\kappa^2}{\alpha}\sin^2\omega - \frac{40\beta\kappa^5}{\gamma\alpha}\cos\omega \qquad (25)$$

In terms of ϕ_1, this solution becomes

$$\phi_1(\xi,\tau) = \frac{2\beta\kappa^2}{\alpha}\sec h^2\left[\kappa(\xi - U\eta)\right] \pm \frac{40\beta\kappa^5}{\gamma\alpha}\tanh\left[\kappa(\xi - U\eta)\right] \qquad (26)$$

IV. CONCLUSION

We have derived the modified KdV(mKdV) equation in the weakly ionized non-uniform dusty plasma. To study the characteristics of propagation of solitons, we solved the mKdV equation using Sine-Cosine method. The solution which have two terms represents two forms of soliton structures, the first term containing $\mathrm{sech}^2[\kappa(\xi - U\eta)]$ could be observed for simple KdV equation derived in case of uniform plasma where as the second part $\tanh[\kappa(\xi - U\eta)]$ due to the presence of plasma inhomogeneity. The second term containing $\tanh[\kappa(\xi - U\eta)]$ represents a soliton like tailing structure which follows the main soliton. The behavior depends completely on the variation of inhomogeneity term γ. The amplitude of the main soliton decreases while the tailing soliton grows faster for increasing unperturbed dust density $n_d^{(0)}$. These happen as the energy generates continuously from the main soliton to the tailing soliton due to presence of ionization when it propagates in the inhomogeneous medium. The width of both the solitons (main and tailing like) decreases for increasing $n_d^{(0)}$ with slightly higher rate for greater values of dust fluid velocity $u_d^{(0)}$.

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