ISSN (e): 2250 – 3005 || Volume, 08 || Issue, 4|| April – 2018 ||
International Journal of Computational Engineering Research (IJCER)

Some New Classes of Unicyclic Graphsof Strong Edge Coloring

Narayan Poddar[*1], Basudeb Mondal[2]
[*1]Assistant Teacher, Department of Mathematics, Bakshirhat High School, Bakshirhat, Coochbehar, West Bengal-736131, India.
[2]Assistant Teacher, Bajitpur Primary School, Bajitpur, Purapara, Murshidabad, West Bengal, 742201, India.
Corresponding author: Narayan Poddar

ABSTRACT
Let G be an undirected simple graph. A strong edge coloring of a graph G is a function $f : E \rightarrow \{1,2,3,\ldots,k\}$ such that $f(e_1) \neq f(e_2)$ whenever e_1 and e_2 lie within distance 2 from each other. Again, no two edges lie on a path of length 3 receive same colors. The smallest number of colors necessary for strong edge coloring of a graph G is entitled as strong chromatic index and is represented by $\chi_S'(G)$. In this paper, we investigate strong chromatic index of some new classes of unicyclic graphs.
KEYWORDS: Strong edge coloring, strong chromatic index, unicyclic graph.

Date of Submission: 21-03-2018 Date of acceptance: 05-04-2018

I. INTRODUCTION

The concept of strong edge coloring was first introduced by Fouquet and Jolivet [4] for cubic planar graphs. Strong edgecoloring of graphs discovers its application in the area of frequency assignment. The problem of frequency assignment ariseswhen different radio transmitters function in the same geographical region interfere with one another when assigned thesame or closely related frequency channels. This situation is common in a wide variety of real world applications relatedto mobile or general wireless networks and can be modelled as a problem in strong edge coloring [2] [6]. The problem of finding$\chi_S'(G)$of a graph is NP -complete [5]. One of the open problems proposed by Erdos and Nesetril [3] is as follows. If G is asimple graph with maximum degree Δ, then $\chi_S'(G) \leq \frac{5\Delta^2-2\Delta+1}{4}$ if Δ is odd and $\chi_S'(G) \leq \frac{5\Delta}{4}$ if Δ is even.Thisconjecture is true for $\Delta \leq 3$. They asked if there is any $\epsilon > 0$such that, for every such G, $\chi_S'(G) \leq (2-\epsilon)\Delta^2$. Chung etal [1] studied extensively the upper bound of strong chromatic index. They showed that the upper bounds are exactly thenumbers of edges in $2K_2$ free graphs. Recently strong edge coloring was studied by several authors as well.

II. CHARACTERIZATION OF UNICYCLIC GRAPHS

Definition 2.1. A unicyclic graph is a connected graph that contains exactly one cycle.

Lemma 2.2. For any unicyclic graph$G$ with cycleC_3, $\chi_S'(G) = |E(G)|$ if and only if every vertex not in C_3 is a leaf.

Proof. Since G contains the cycle C_3, the three edges of C_3 should be assigned with three distinct colors for strong edgecoloring. Assume that $\chi_S'(G) = |E(G)| \Leftrightarrow$ The minimum number of colors required for strong edge coloring of G is equalto the number of edges of the graph $G \Leftrightarrow$ The distance $d(e_i, e_j) < 2$ for any two edges $e_i$ande_j, $(i \neq j)$ of G , Since $G$is unicyclic and the distance $d(e_i, e_j) < 2$for any two edges $e_i$ande_j, $(i \neq j)$ of G, implying that the edges incident atthe cycle vertices other than the cycle edges must be the pendent edges , Vertices which are not in C_3 must be the leafvertices. Hence the proof.

Lemma 2.3. Let G be a unicyclic graph with cycle C_4 and $\Delta = 3$. Then $5 \leq \chi_S'(G) \leq 6$.

Proof. Assume the contrary that $\chi_S'(G) < 5$. Let $C_4 = (u_1, u_2, u_3, u_4, u_1)$.

For strong edge coloring of C_4, the edges ofC_4 should be colored as $c(u_1u_2) = c_1$, $c(u_2u_3) = c_2$,$c(u_3u_4) = c_3$, and $c(u_4u_1) = c_4$.

It is given that $\Delta = 3$. Let e_1 and $e_2$be the edges incident at u_1 and u_2 respectively as shown in Figure 1. The edges $e_1$and e_2 lie within distance two from eachother implying that they should be assigned with two new colors c_5 and c_6. Similarly, if $e_1$and e_2 are the edges incident at$u_1$and $u_3$respectively.

Then the edges $e_1$and e_2 can be assigned with a new color c_5 as the distance between $e_1$and e_2 isexactly 2.

This is a contradiction to our assumption that $\chi_S'(G) < 5$. It is obvious that $\chi_S'(G)$cannot be greater than 6 as$\Delta = 3$. Thus $5 \leq \chi_S'(G) \leq 6$.

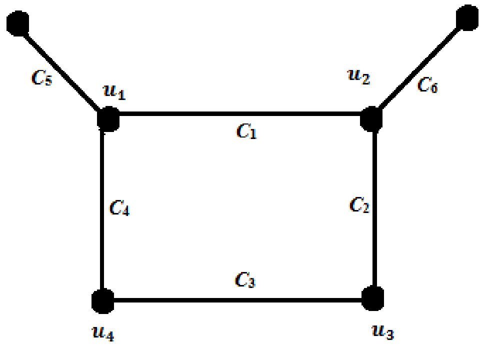

Figure 1. Unicyclic graph with $\chi_S'(G) = 6$

Lemma 2.4. Let G be a unicyclic graph with cycle C_5 and $\Delta = 3$. Then $\chi_S'(G) = 5$.

Proof. Let $V(C_5) = \{ v_i | 1 \leq i \leq 5\}$ and $E(C_5) = \{v_i v_{i+1} | 1 \leq i \leq 4\} \cup \{v_5 v_1\}$.The cycle$C_5$requires 5 colors for strongedge coloring. Attaching a pendent edge at v_i, $1 \leq i \leq 5$, will not increase the number of colors as the pendent edge incidentat the cycle vertex acquire color from the cycle edge that is two distance away from it. Thus $\chi_S'(G) = 5$.

Theorem 2.5. Let$G$ be a unicyclic graph with cycle C_n, $n \geq 6$ and $\Delta = 3$. Then $\chi_S'(G) \leq 6$.

Proof. Let $V(C_n) = \{ v_i | 1 \leq i \leq n\}$ and $E(C_n) = \{e_i = v_i v_{i+1} | 1 \leq i \leq n-1\} \cup \{e_n = v_n v_1\}$.The following cases arise.

Case (1):$n \equiv 0 (mod\ 3)$. In this case $\chi_S'(C_n) = 3$. Also, n can be even or odd. If n is even, then attaching edges to thecycle vertices will increase the number of colors by at most 2. Thus $\chi_S'(G) \leq 5$. If n is odd, then attaching edges tothe cycle vertices will increase the number of colors by at most 3. Consequently, $\chi_S'(G) \leq 6$.

Case (2):$n \equiv 1 (mod\ 3)$. In this case $\chi_S'(C_n) = 4$. Also, n can be even or odd. In both the cases, attaching edges to thecycle vertices will increase the number of colors by at most 2. Thus $\chi_S'(G) \leq 6$.

Case (3):$n \equiv 2 (mod\ 3)$. In this case $\chi_S'(C_n) = 4$. Also, n can be even or odd. In both the cases, attaching edges to thecycle vertices will increase the number of colors by at most 2. Thus $\chi_S'(G) \leq 6$.

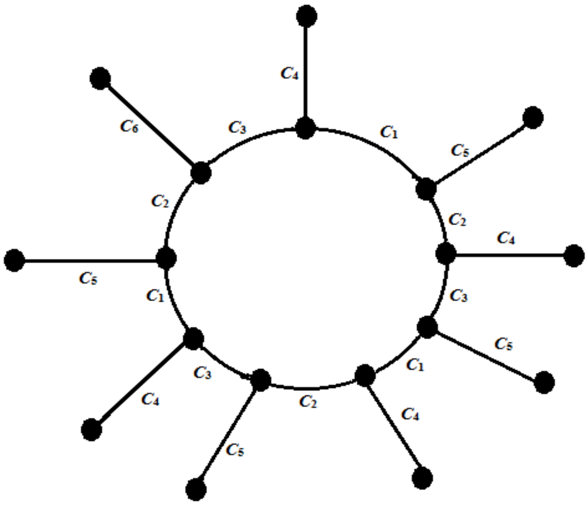

Figure 2. Strong edge coloring of unicyclic graph with C_9 and $\Delta = 3$

From the above cases, it is clear that $\chi_S'(G) \leq 6$. See Figure 2. Let $U_{n,r}$be a unicyclic graph of order n obtained from thetwo vertex disjoint graphs $C_r$and$P_{n-r}$by adding an edge that joins a vertex of C_r to an end vertex of P_{n-r}.

Theorem 2.6. Let$G$ be a unicyclic graph $U_{n,r}$For $n > r > 5$, $4 \leq \chi_S'(G) \leq 5$.

Proof. Consider the following cases.

Case (1):$r \equiv 0 (mod\ 3)$. In this case,$\chi_S'(C_r) = 3$. Further, the edge joining $C_r$and P_{n-r}should be assigned with a newcolor and the edges of the pathP_{n-r}acquire the colors that are already assigned to the cycle edges. Thus $\chi_S'(G) = 4$.

Case (2): $r \equiv 1(mod\ 3)$. Then the cycle C_r requires 4 colors for strong edge coloring. The edges of the path P_{n-r} shouldacquire colors from the edges of the cycle C_r. The edge joining$C_r$andP_{n-r}can be colored according as $r \equiv 0(mod\ 4)$ or $r \not\equiv 0(mod\ 4)$. If $r \equiv 0(mod\ 4)$, then the edge joining C_r andP_{n-r}should be assigned with a new color. Thus$\chi_S^{'}(G) = 5$. Otherwise, the edge joining $C_r$and P_{n-r}as well as the edges of the path P_{n-r} can acquire colors fromthe colors that are assigned to the edges of the cycle. In this case $\chi_S^{'}(G) = 4$.
Case (3): $r \equiv 2(mod\ 3)$. In this case, the cycle C_r requires 4 colors for strong edge coloring. The edge joining C_r andP_{n-r} can be colored according as $r \equiv 0(mod\ 4)$ or $r \not\equiv 0(mod\ 4)$. If $r \equiv 0(mod\ 4)$, then the edge joining $C_r$and P_{n-r}should be assigned with a new color. The edges of the path P_{n-r}should acquire colors from the colors alreadyassigned to the edges of the cycle C_r. Thus $\chi_S^{'}(G) = 5$. If $r \not\equiv 0(mod\ 4)$, then the edge joining $C_r$and P_{n-r}as well as the edges ofP_{n-r} can acquire colors from the colors that are already assigned to the edges of the cycle as given inFigure 3 In this case $\chi_S^{'}(G) = 4$.
So, from the above discussion $4 \leq \chi_S^{'}(G) \leq 5$.

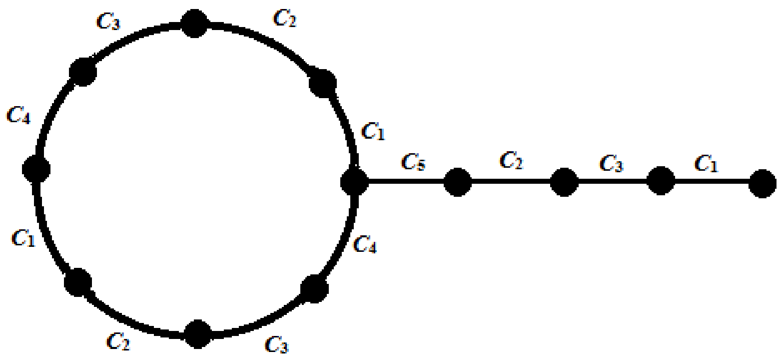

Figure 3. Strong edge coloring of $U_{12,8}$

III. CONCLUSION

In this paper, we characterized unicyclic graphs. It would be interesting to study the strong chromatic index for cycle relatedgraphs and interconnection networks.

REFERENCES

[1]. Chung, F.R. K., Gyarfas, A., Tuza, Z.,&Trotter, W.T.(1990). "The maximum number of edges in $2K_2$ Free graphs of bounded degree", Discrete Mathematics, 81, pp.129-135.
[2]. Deo, N. (2014). "Graph Theory with Applications to Engineering and Computer Science", Prentice Hall of India, Delhi, India.
[3]. [3].Erdos, P.,Nesetril, J. (1989)."Problem, in: Halasz. G, Sos.V.T. T (Eds.), Irregularities of Partitions", Springer, Berlin, pp.162-163.
[4]. Fouquet, J.L., Jolivet, J.L. (1984)."Strong edge-coloring of cubic planar graphs, in: Progress in graph theory (Waterloo, Ont.,1982)", Academic Press, Toronto, pp. 247-264.
[5]. Mahdian, M. (2002). "On the Computational Complexity of Strong Edge Coloring", Discrete Applied Mathematics, 118(3).
[6]. Mondal, B., De, K. (2017). "Overview Applications of Graph Theory in Real Field". International Journal of Scientific Research in Computer Science, Engineering and Information Technology, 2(5), pp. 751-759.

Narayan Poddar ."Some New Classes of Unicyclic Graphsof Strong Edge Coloring." International Journal of Computational Engineering Research (IJCER), vol. 08, no. 04, 2018, pp. 01-03.