ISSN (e): 2250 – 3005 || Volume, 08 || Issue, 02|| February – 2018 ||
International Journal of Computational Engineering Research (IJCER)

Over View of Cosmological Model

ParshuramSahoo[1],J.P. Tripathy[2] ,A.K. Mishra[3], Ashok Panigrahi[4]

[1,3,4]Assistant Professor, Department of Basic Science and Humanities, Gandhi Institute For Technology (GIFT), Bhubaneswar

[2]Assistant Professor, Department ofBasic Science and Humanities, Gandhi Engineering College, Bhubaneswar

I. INTRODUCTION

A cosmological model is a mathematical description of the universe, which tries to explain the reasons of its current aspect, and to describe its evolution during time.

Of course, it must account for the observations, and be able to make predictions that later observations will be able to check .

The current models are based on general relativity, because this theory is what produces at present the best agreement for large-scale behavior.

From his theory of general relativity, which is a theory of gravitation, Einstein writes the equations which govern an universe filled with matter. But he thought that the universe had to be static. So, he introduced a term, called the cosmological constant, into his equations, in order to obtain this result.

Afterward, in view of the results of Hubble, he will return on this idea, and will admit that universe can effectively be expanding.

Soon after him, the Dutch De Sitter, the Russian Friedmann and the Belgian Lemaître introduce non-static universes as solutions for the Einstein equations of relativity.

If the universe of De Sitter corresponds to an empty universe, the Friedmann model is dependent on the density of matter inside the universe. This model is always the basis of current cosmological models.

The basic hypothesis

The first hypothesis of a cosmological model rests on the *cosmological principle* : the Earth has no reason for standing at the center of the Universe, or in any privileged area. According to this hypothesis, the universe is considered as :

- Homogeneous, i.e. it presents the same properties everywhere on a cosmological scale - of course, on a smaller scale there are different situations, if we look at the solar system or outside the Galaxy, for example.
- Isotropic, i.e. it always has the same properties, in every direction where we could look. Especially, it is not "flattened" in one direction.

The second necessary hypothesis is the universality of the laws of physics. These laws are the same, in any place and any time.

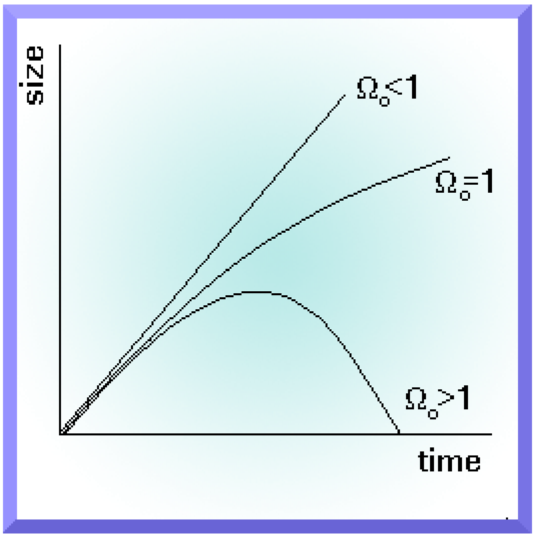
Considering the content of the universe as a perfect fluid is another hypothesis. The characteristic dimensions of its constituents are negligible in front of the distances which separate them .

The parameters of the model

In order to completely describe a model, in accordance with these previous hypothesis, the Friedmann-Lemaître models use three parameters which totally characterize the evolution of the universe:

- The Hubble constant, which characterizes the rate of expansion of the universe,
- The mass density parameter, noted Ω, which measures the ratio between the density ρ of the studied universe and a particular density, called the critical density ρ_c, linked with the Hubble constant. The current value of this parameter is noted Ωo.
- The cosmological constant, noted Λ , which represents a force opposing to the gravitation.

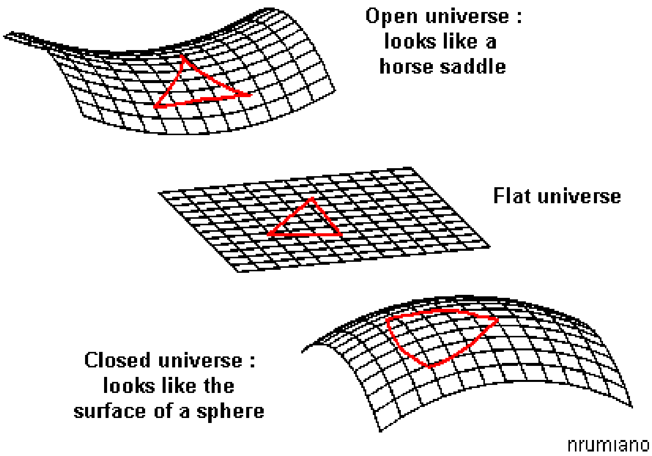
The density of matter within the universe is the key parameter for the foreseeing of its evolution : if it is very dense ($\Omega o> 1$), gravity will be able to win against expansion, and the universe will go back to its initial state. In the opposite case, the expansion will continue forever.



Intuitively, we can realize the evolution of the universe, according to the amount of matter inside: A great quantity of matter will result in a closed universe. This one will end in its initial state. A low amount of matter will result in an open universe, with an infinite expansion.
The value $\Omega o = 1$ leeds to to the particular case of a flat universe.
The value of the critical density ρ_c is about 6 10^{-27} kg/m^3, that is two atoms of hydrogen per cubic meter. This very low figure explains why the models which suppose an empty universe are not so bad !

Closed or open universe

The density of matter inside the universe determines its geometry : for a high density we will obtain a closed universe with a positive curvature, but with a density lower than the critical density, we will obtain an open universe.
Let us note that a closed universe is necessarily of finished size, whereas a flat or open universe can be finite or infinite.



In an open universe, the sum of the angles of a triangle is lower than 180°. In a closed universe (like the surface of the Earth), this sum is always greater than 180°.
All the measurements until now did not allow to put in evidence a curvature of the universe.
Measurements of the fossil radiation by the Boomerang baloon tend once again to accredit the hypothesis of a flat universe.
The universe would be flat. But this fact puts us two questions:

- If it is flat, it means that the matter density is equal to the critical density $\Omega o = 1$. But, at the very most, the visible matter in the universe accounts for only 5% of this density. Where is the rest ? In the same way as the birth of the galaxies, we once again have to appeal to the dark matter.
- Why is it flat ? This is a very special situation. We shall try to find an answer to this question a bit later.

1.3.4 ***The age of the universe***
We can show that the age of the universe is proportional to the inverse of the Hubble constant. So, an accurate determination of this constant is a crucial problem for the cosmology. Recent measurements indicate a value included between 50 and 100 km/s/MPc, i.e. the universe would be between 7 and 20 billions years old.
But the universe must necessarily be older than its oldest stars. These are estimated between 13 and 16 billions years old.
The last results supplied by the Hubble and Spitzer satellites conclude with a value of H=67.15 ±1.2 km/sec./Mpc, hence the universe would be 13,8billions years old.

Physical cosmology is the study of the largest-scale structures and dynamics of the Universe and is concerned with fundamental questions about its formation, evolution, and ultimate fate.For most of human history, it was a branch of metaphysics and religion. Cosmology as a science originated with the Copernican principle, which implies that celestial bodies obey identical physical laws to those on Earth, and Newtonian mechanics, which first allowed us to understand those laws.
Physical cosmology, as it is now understood, began with the 20th century development of Albert Einstein's general theory of relativity, and better astronomical observations of extremely distant objects. These advances made it possible to speculate about the origin of the Universe, and allowed the establishment of the Big Bang Theory, by Fr. Georges Lemaitre, as the leading cosmological model. Some researchers still advocate a handful of alternative cosmologies; however, most cosmologists agree that the Big Bang theory best explains observations.
Cosmology draws heavily on the work of many disparate areas of research in theoretical and applied physics. Areas relevant to cosmology include particle physics experiments and theory, theoretical and observational astrophysics, general relativity, quantum mechanics, and plasma physics.

History of study
Modern cosmology developed along tandem tracks of theory and observation. In 1916, Albert Einstein published his theory of general relativity, which provided a unified description of gravity as a geometric property of space and time. At the time, Einstein believed in a static universe, but found that his original formulation of the theory did not permit it. This is because masses distributed throughout the Universe gravitationally attract, and move toward each other over time. However, he realized that his equations permitted the introduction of a constant term which could counteract the attractive force of gravity on the cosmic scale. Einstein published his first paper on relativistic cosmology in 1917, in which he added this *cosmological constant* to his field equations in order to force them to model a static universe. However, this so-called Einstein model is unstable to small perturbations—it will eventually start to expand or contract. The Einstein model describes a static universe; space is finite and unbounded (analogous to the surface of a sphere, which has a finite area but no edges). It was later realized that Einstein's model was just one of a larger set of possibilities, all of which were consistent with general relativity and the cosmological principle. The cosmological solutions of general relativity were found by Alexander Friedmann in the early 1920s. His equations describe the Friedmann-Lemaître-Robertson-Walker universe, which may expand or contract, and whose geometry may be open, flat, or closed.
In the 1910s, VestoSlipher (and later Carl Wilhelm Wirtz) interpreted the red shift of spiral nebulae as a Doppler shift that indicated they were receding from Earth. However, it is difficult to determine the distance to astronomical objects. One way is to compare the physical size of an object to its angular size, but a physical size must be assumed to do this. Another method is to measure the brightness of an object and assume an intrinsic luminosity, from which the distance may be determined using the inverse square law. Due to the difficulty of using these methods, they did not realize that the nebulae were actually galaxies outside our own Milky Way, nor did they speculate about the cosmological implications. In 1927, the BelgianRomanCatholicpriestGeorgesLemaître independently derived the Friedmann-Lemaître-Robertson-Walker equations and proposed, on the basis of the recession of spiral nebulae, that the Universe began with the "explosion" of a "primeval atom"—which was later called the Big Bang. In 1929, Edwin Hubble provided an observational basis for Lemaître's theory. Hubble showed that the spiral nebulae were galaxies by determining their distances using measurements of the brightness of Cepheid variable stars. He discovered a relationship between the redshift of a galaxy and its distance. He interpreted this as evidence that the galaxies are receding from Earth in every direction at speeds proportional to their distance. This fact is now known as Hubble's law, though the numerical factor Hubble found relating recessional velocity and distance was off by a factor of ten, due to not knowing about the types of Cepheid variables.
Given the cosmological principle, Hubble's law suggested that the Universe was expanding. Two primary explanations were proposed for the expansion. One was Lemaître's Big Bang theory, advocated and

developed by George Gamow. The other explanation was Fred Hoyle's steady state model in which new matter is created as the galaxies move away from each other. In this model, the Universe is roughly the same at any point in time.

For a number of years, support for these theories was evenly divided. However, the observational evidence began to support the idea that the Universe evolved from a hot dense state. The discovery of the cosmic microwave background in 1965 lent strong support to the Big Bang model, and since the precise measurements of the cosmic microwave background by the Cosmic Background Explorer in the early 1990s, few cosmologists have seriously proposed other theories of the origin and evolution of the cosmos. One consequence of this is that in standard general relativity, the Universe began with a singularity, as demonstrated by Roger Penrose and Stephen Hawking in the 1960s.

Cosmologists cannot explain all cosmic phenomena exactly, such as those related to the accelerating expansion of the Universe, using conventional forms of energy. Instead, cosmologists propose a new form of energy called dark energy that permeates all space. One hypothesis is that dark energy is the energy of virtual particles, which are believed to exist in a vacuum due to the uncertainty principle.

There is no clear way to define the total energy in the Universe using the most widely accepted theory of gravity, general relativity. Therefore, it remains controversial whether the total energy is conserved in an expanding universe. For instance, each photon that travels through intergalactic space loses energy due to the redshift effect. This energy is not obviously transferred to any other system, so seems to be permanently lost. On the other hand, some cosmologists insist that energy is conserved in some sense; this follows the law of conservation of energy.

Thermodynamics of the universe is a field of study that explores which form of energy dominates the cosmos - relativistic particles which are referred to as radiation, or non-relativistic particles referred to as matter. Relativistic particles are particles whose rest mass is zero or negligible compared to their kinetic energy, and so move at the speed of light or very close to it; non-relativistic particles have much higher rest mass than their energy and so move much slower than the speed of light.

As the Universe expands, both matter and radiation in it become diluted. However, the Universe also cools down, meaning that the average energy per particle gets smaller. Therefore radiation becomes weaker, and dilutes fasterthan matter. Thus with the expansion of the Universe, radiation becomes less dominant than matter. The very early Universe is said to have been 'radiation dominated' and radiation controlled the deceleration of expansion. Later, as the average energy per photon becomes roughly 10 eV and lower, matter dictates the rate of deceleration and the Universe is said to be 'matter dominated'. The intermediate case is not treated well analytically. As the expansion of the universe continues, matter dilutes even further and the cosmological constant becomes dominant, leading to an acceleration in the universe's expansion.

History of the universe

The history of the Universe is a central issue in cosmology. The history of the Universe is divided into different periods called epochs, according to the dominant forces and processes in each period. The standard cosmological model is known as the Lambda-CDM model.

Equations of motion

The equations of motion governing the Universe as a whole are derived from general relativity with a small, positive cosmological constant. The solution is an expanding universe; due to this expansion, the radiation and matter in the Universe cool down and become diluted. At first, the expansion is slowed down by gravitation attracting the radiation and matter in the Universe. However, as these become diluted, the cosmological constant becomes more dominant and the expansion of the Universe starts to accelerate rather than decelerate. In our universe this happened billions of years ago.

Particle physics in cosmology

Particle physics is important to the behavior of the early Universe, because the early Universe was so hot that the average energy density was very high. Because of this, scattering processes and decay of unstable particles are important in cosmology.

As a rule of thumb, a scattering or a decay process is cosmologically important in a certain cosmological epoch if the time scale describing that process is smaller than, or comparable to, the time scale of the expansion of the Universe. The time scale that describes the expansion of the Universe is $1/H$ with H being the Hubble constant, which itself actually varies with time. The expansion timescale $1/H$ is roughly equal to the age of the Universe at that time.

Timeline of the Big Bang

Observations suggest that the Universe began around 13.8 billion years agoSince then, the evolution of the Universe has passed through three phases. The very early Universe, which is still poorly understood, was the split second in which the Universe was so hot that particles had energies higher than those currently accessible in particle accelerators on Earth. Therefore, while the basic features of this epoch have been worked out in the Big Bang theory, the details are largely based on educated guesses. Following this, in the early Universe, the evolution of the Universe proceeded according to known high energy physics. This is when the first protons, electrons and neutrons formed, then nuclei and finally atoms. With the formation of neutral hydrogen, the cosmic microwave background was emitted. Finally, the epoch of structure formation began, when matter started to aggregate into the first stars and quasars, and ultimately galaxies, clusters of galaxies and superclusters formed. The future of the Universe is not yet firmly known, but according to the ΛCDM model it will continue expanding forever.

Areas of study

Below, some of the most active areas of inquiry in cosmology are described, in roughly chronological order. This does not include all of the Big Bang cosmology, which is presented in Timeline of the Big Bang.

Very early Universe

The early, hot Universe appears to be well explained by the Big Bang from roughly 10^{-33} seconds onwards. But there are several problems. One is that there is no compelling reason, using current particle physics, for the Universe to be flat, homogeneous, and isotropic (see the cosmological principle). Moreover, grand unified theories of particle physics suggest that there should be magnetic monopoles in the Universe, which have not been found. These problems are resolved by a brief period of cosmic inflation, which drives the Universe to flatness, smooths out anisotropies and inhomogeneities to the observed level, and exponentially dilutes the monopoles. The physical model behind cosmic inflation is extremely simple, but it has not yet been confirmed by particle physics, and there are difficult problems reconciling inflation and quantum field theory. Some cosmologists think that string theory and brane cosmology will provide an alternative to inflation.

Another major problem in cosmology is what caused the Universe to contain more particles than antiparticles. Cosmologists can observationally deduce that the Universe is not split into regions of matter and antimatter. If it were, there would be X-rays and gamma rays produced as a result of annihilation, but this is not observed. This problem is called the baryon asymmetry, and the theory to describe the resolution is called baryogenesis. The theory of baryogenesis was worked out by Andrei Sakharov in 1967, and requires a violation of the particle physics symmetry, called

CP-symmetry, between matter and antimatter. However, particle accelerators measure too small a violation of CP-symmetry to account for the baryon asymmetry. Cosmologists and particle physicists look for additional violations of the CP-symmetry in the early Universe that might account for the baryon asymmetry.

Both the problems of baryogenesis and cosmic inflation are very closely related to particle physics, and their resolution might come from high energy theory and experiment, rather than through observations of the Universe.

Big bang nucleosynthesis

Big Bang nucleosynthesis is the theory of the formation of the elements in the early Universe. It finished when the Universe was about three minutes old and its temperature dropped below that at which nuclear fusion could occur. Big Bang nucleosynthesis had a brief period during which it could operate, so only the very lightest elements were produced. Starting from hydrogenions (protons), it principally produced deuterium, helium-4, and lithium. Other elements were produced in only trace abundances. The basic theory of nucleosynthesis was developed in 1948 by George Gamow, Ralph Asher Alpher, and Robert Herman. It was used for many years as a probe of physics at the time of the Big Bang, as the theory of Big Bang nucleosynthesis connects the abundances of primordial light elements with the features of the early Universe. Specifically, it can be used to test the equivalence principle, to probe dark matter, and test neutrino physics. Some cosmologists have proposed that Big Bang nucleosynthesis suggests there is a fourth "sterile" species of neutrino.

Cosmic microwave background

The cosmic microwave background is radiation left over from decoupling after the epoch of recombination when neutral atoms first formed. At this point, radiation produced in the Big Bang stopped Thomson scattering from charged ions. The radiation, first observed in 1965 by Arno Penzias and Robert Woodrow Wilson, has a perfect thermal black-body spectrum. It has a temperature of 2.7 kelvins today and is isotropic to one part in 10^5. Cosmological perturbation theory, which describes the evolution of slight inhomogeneities in the early Universe, has allowed cosmologists to precisely calculate the angular power spectrum of the radiation, and it has been measured by the recent satellite experiments (COBE and WMAP) and many ground and balloon-based experiments (such as Degree Angular Scale Interferometer, Cosmic Background Imager, and Boomerang). One of the goals of these efforts is to measure the basic parameters of the Lambda-CDM model with increasing accuracy, as well as to test the predictions of the Big Bang model and look for new physics. The recent measurements made by WMAP, for example, have placed limits on the neutrino masses.

Newer experiments, such as QUIET and the Atacama Cosmology Telescope, are trying to measure the polarization of the cosmic microwave background. These measurements are expected to provide further confirmation of the theory as well as information about cosmic inflation, and the so-called secondary anisotropies, such as the Sunyaev-Zel'dovich effect and Sachs-Wolfe effect, which are caused by interaction between galaxies and clusters with the cosmic microwave background.

Formation and evolution of large-scale structure

Understanding the formation and evolution of the largest and earliest structures (i.e., quasars, galaxies, clusters and superclusters) is one of the largest efforts in cosmology. Cosmologists study a model of **hierarchical structure formation** in which structures form from the bottom up, with smaller objects forming first, while the largest objects, such as superclusters, are still assembling. One way to study structure in the Universe is to survey the visible galaxies, in order to construct a three-dimensional picture of the galaxies in the Universe and measure the matter power spectrum. This is the approach of the Sloan Digital Sky Survey and the 2dF Galaxy Redshift Survey.

Another tool for understanding structure formation is simulations, which cosmologists use to study the gravitational aggregation of matter in the Universe, as it clusters into filaments, superclusters and voids. Most simulations contain only non-baryonic cold dark matter, which should suffice to understand the Universe on the largest scales, as there is much more dark matter in the Universe than visible, baryonic matter. More advanced simulations are starting to include baryons and study the formation of individual galaxies. Cosmologists study these simulations to see if they agree with the galaxy surveys, and to understand any discrepancy.

Other, complementary observations to measure the distribution of matter in the distant universe and to probe reionization include:

- The Lyman-alpha forest, which allows cosmologists to measure the distribution of neutral atomic hydrogen gas in the early Universe, by measuring the absorption of light from distant quasars by the gas.
- The 21 centimeter absorption line of neutral atomic hydrogen also provides a sensitive test of cosmology
- Weak lensing, the distortion of a distant image by gravitational lensing due to dark matter.

These will help cosmologists settle the question of when and how structure formed in the Universe.

Dark matter

Evidence from Big Bang nucleosynthesis, the cosmic microwave background and structure formation suggests that about 23% of the mass of the Universe consists of non-baryonic dark matter, whereas only 4% consists of visible, baryonic matter. The gravitational effects of dark matter are well understood, as it behaves like a cold, non-radiative fluid that forms haloes around galaxies. Dark matter has never been detected in the laboratory, and the particle physics nature of dark matter remains completely unknown. Without observational constraints, there are a number of candidates, such as a stable supersymmetric particle, a weakly interacting massive particle, an axion, and a massive compact halo object. Alternatives to the dark matter hypothesis include a modification of gravity at small accelerations (MOND) or an effect from brane cosmology.

Dark energy

If the Universe is flat, there must be an additional component making up 73% (in addition to the 23% dark matter and 4% baryons) of the energy density of the Universe. This is called dark energy. In order not to interfere with Big Bang nucleosynthesis and the cosmic microwave background, it must not cluster in haloes like baryons and dark matter. There is strong observational evidence for dark energy, as the total energy density of the Universe is known through constraints on the flatness of the Universe, but the amount of clustering matter is tightly measured, and is much less than this. The case for dark energy was strengthened in 1999, when measurements demonstrated that the expansion of the Universe has begun to gradually accelerate.

Apart from its density and its clustering properties, nothing is known about dark energy. Quantum field theory predicts a cosmological constant (CC) much like dark energy, but 120 orders of magnitude larger than that observed. Steven Weinberg and a number of string theorists (see string landscape) have invoked the 'weak anthropic principle': i.e. the reason that physicists observe a universe with such a small cosmological constant is that no physicists (or any life) could exist in a universe with a larger cosmological constant. Many cosmologists find this an unsatisfying explanation: perhaps because while the weak anthropic principle is self-evident (given that living observers exist, there must be at least one universe with a cosmological constant which allows for life to exist) it does not attempt to explain the context of that universe. For example, the weak anthropic principle alone does not distinguish between:

- Only one universe will ever exist and there is some underlying principle that constrains the CC to the value we observe.
- Only one universe will ever exist and although there is no underlying principle fixing the CC, we got lucky.
- Lots of universes exist (simultaneously or serially) with a range of CC values, and of course ours is one of the life-supporting ones.

Other possible explanations for dark energy include quintessence or a modification of gravity on the largest scales. The effect on cosmology of the dark energy that these models describe is given by the dark energy's equation of state, which varies depending upon the theory. The nature of dark energy is one of the most challenging problems in cosmology.

A better understanding of dark energy is likely to solve the problem of the ultimate fate of the Universe. In the current cosmological epoch, the accelerated expansion due to dark energy is preventing structures larger than superclusters from forming. It is not known whether the acceleration will continue indefinitely, perhaps even increasing until a big rip, or whether it will eventually reverse.

Consequences of our investigation

In the preceding sections of this chapter we have made a brief survey on cosmology.now we have presented consequences of our investigations which are under taken and recorded in the succeeding chapters.

In chapter-II we have studies the geometrical aspects of the space time described by metric[2.1] which regards to the space of constant curvature,symmetric space and recurrence space. The solutions corresponding to Einstein vacuum field equations are derived. It is shown that the space time [2.1] does not represent a space of constant curvature and is not an Einstein.also the space time becomes asymmetric space only when $A=B=C=e^t$.

In this case the space time is flat at initial epoch and admit a singularity at infinite future. How ever the flat space time given by [2.1] is neither a symmetric space nor a recurrent space.

In chapter-III we have taken metric[2.1] to obtained the cosmological model with time varying G andΛ.we obtain an expansion scalar or themetricbearing a constant ratio to the anisotropy in the direction of space –like unit vector λ^i.

The anisotropy in the model does not die out asymptotically. We also obtain that the model satisfies the condition for a machine cosmologyical solution that is$G\rho \sim H^2$.Also the metric with condition $A = B^m C^n$,

In the case of perfect fluid distribution represents a stiff matter if and only if$\Lambda=0$.also the second model reduces to a static universe with repulsive gravity.

REFERENCES

[1]. Radojevic, D.: One example of Einstein space, Teor. Prim. Meh. 13, (1988), 77-81.
[2]. Godel, K: An example of a new type of cosmological solutions to Einstein's field equation of gravity, Rev. Mod. Phys. 21, (1949), 447-450.
[3]. Mohanty, G and Pattanaik, R.R: Some geometrical aspects of Bertotti-Robinson like space-time, Teor. Prim. Meh. 16, 75-78. (1990).
[4]. Willmore, T.J: An Introduction to Dierential Geometry, Oxford University Press, Oxford, 236-38, 1978.
[5]. Venkateswarlu, R and Reddy, D.R.K: Bianchi type-I models in self-creation theory of gravitation, Astrophys.& Space Sc. 168, (1990),193-199.
[6]. A G Riess*et al*, *Astron. J.* 116, 1099 (1998)
[7]. S Perlmutter*et al*, *Astrophys. J.* 517, 565 (1999)
[8]. S Perlmutter*et al*, *Nature (London)* 391, 51 (1998)
[9]. J P Ostriker and P J Steinhardt, *Nature (London)* 377, 600 (1995)
[10]. S Weinberg, *Rev. Mod. Phys.* 61, 1 (1989)
[11]. S Weinberg, astro-ph/0005265
[12]. M Gasperini, *Class Quantum Gravit.* 5, 521 (1988)
[13]. M Ozer and M O Taha, *Nucl. Phys.* B287, 776 (1987)
[14]. K Freese*et al*, *Nucl. Phys.* B287, 797 (1987)
[15]. J AFrieman*et al*, *Phys. Rev. Lett.* 75, 2077 (1995)
[16]. K Coble, S Dodelson and J A Frieman, *Phys. Rev.* D55, 1851 (1997)
[17]. P A M Dirac, *Proc. R. Soc. London* A165, 199 (1938)
[18]. A-M M Abdel-Rahman, *Nuovo Cimento*B102, 225 (1988)
[19]. A-M M Abdel-Rahman, *Phys. Rev.* D45, 3497 (1992)
[20]. J D Barrow, *Phys. Lett.* B180, 335 (1986)
[21]. J D Barrow and P Parsons, *Phys. Rev.* D55, 1906 (1997)
[22]. A Beesham, *Gen. Relativ. Gravit.* 26, 159 (1994)
[23]. M S Berman, *Phys. Rev.* D43, 1075 (1991)
[24]. J C Carvalho, J A S Lima and I Waga, *Phys. Rev.* D46, 2404 (1992)
[25]. W Chen and Y S Wu, *Phys. Rev.* D41, 695 (1990)
[26]. D Kalligas, P S Wesson and C W F Everitt, *Gen. Relativ. Gravit.* 24, 351 (1992)