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Double-Framed Soft Version of Chain Over Distributive Lattice

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ABSTRACT
In this study, Using the double-framed soft de nitions, we de ne some new concept such as the double-framed soft lattice, distributive double-framed soft lattice, double-framed soft chain then we study the relationship and observe common properties.
KEYWORDS: Double-framed soft sets, Double-framed soft lattice, Double-framed soft chain, Modular double framed soft lattice.

I. INTRODUCTION

Most of the problems in engineering, medical science, economics and social science etc. have vagueness and various uncertainties. To overcome these uncertainties, some kinds of theories were given which we can use as mathematical tools for dealing with uncertain-ties. However, these theories have their own di culties. In 1999, Molodtsov [1] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. From then on, works on the soft set theory are progressing rapidly. After Molodtsovs work, same di erent applications of soft sets were studied in [2,3]. Furthermore Maji, Biswas and Roy worked on soft set theory in [4,5]. Roy et al. presented some applications of this notion to decision making problems in [6]. The algebraic structures of soft sets have been studied by some authors [714]. Birkho s work in 1930 started the general development of lattice theory [15]. The lattice theory has been applied to many kinds of elds. Recently, the work introducing the soft set theory to the lattice theory and the fuzzy set theory have been initiated. Fu [16] and a gman et al. [17] presented the nation of the soft lattice and derived the properties of the soft lattice and discussed the relationship between the soft lattices. Karaaslan et al. [18] introduced the fuzzy soft lattice theory, some related properties on it. Saibaba, [20] initi-ated the study of L-fuzzy lattice ordered groups Specialty journal of Engineering and Applied Science, 2017, Vol, 3 (3): 10-21 and introduced the notion L fuzzy sub l-groups [22] replaced the val-uation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy

sets. K.V. Thomas and Latha. S. Nair studied Rough intuition-istic fuzzy sets in a lattice [21]. Hayat [25] de ned applications of double-framed soft ideals in BEalgebra. Jun et. al, [23] intro-duced the notion of double-framed soft sets (brie y, DFSsets), and applied it to BCK/BCI- algebras. They discussed double-framed soft algebras (brie y, DFS-algebras) and investigated related prop-erties. Hadipour, [24] de ned double-framed soft BF-algebras and Yongukcho et al. [26] studied on double-framed soft Near-rings.

II. PRELIMINARIES

De nition 2.1. (Double-framed set)(DF-set)
De nition 2.2. Let U be an initial universe set and E be a set of parameters. Consider E. Let P (U) denote the set of all double-framed sets of U. The collection (F; A) is termed to be the soft double-framed set (DFS-set) over U, where F is a mapping given by F : A ! P (U).
De nition 2.3. Let P be a non-empty ordered set.

(i) If x _ y and x ^ y exist for all x; y 2 P; then P is called a lattice.

(ii) If _S and ^S exist for all S P; then P is called a complete lattice.

De nition 2.4. An algebra (L; _; ^) is called a lattice if L is a non-empty set, _ and ^ are binary operations on L, both _ and ^ are idempotent, commutative and associative and they satisfy the two absorbtion identities. that is , for all a; b; c 2 L

(i) a ^ a = a; a _ a = a

(ii) a ^ b = b ^ a; a _ b = b _ a

(iii) (a ^ b) ^ c = a ^ (b ^ c); (a _ b) _ c = a _ (b _ c)

(iv) a ^ (a _ b) = a; a _ (a ^ b) = a:

De nition 2.5. Let f_A and g_B be two double-framed soft sets over U. f_A is said to be a double-framed soft subset of g_B if A B,

and $f_{(e)}$ (x) $g_{(e)}$ (x); $f_{(e)}$ (x) $g_{(e)}$ (x)8x 2 U and e 2 A: we denote it by f_A g_B.

De nition 2.6. The complement of a double-framed soft set f_A dented by $f_A{}^c$ where f^c : A ! P (U) is a mapping given by $f_A{}^c$( ) = double-framed soft complement with $_f$c (x) = $_f$ (x) and $_f$c (x) = $_f$ (x).

De nition 2.7. Let h_A and g_B be two DFS-sets over U. then the union of h_A and g_B is denoted by h_A [g_B and is de ned by h_A [g_B = k_C, where C = A [B and its membership functions of k_C are as follows;

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$_{ke}$ (m) = > $_{he}$ (m); if e 2 A B
< $_{ge}$ (m); if e 2 B A
maxf $_{he}$ (m); $_{ge}$ (m)g; if e 2 A \ B
> $_{he}$ (m); if e A B
:

$_{ke}$ (m) = 8 $_{ge}$ (m); if e 2 B A
> 2
< min h (m); g (m) ; if e A B
> f e e g 2 \

De nition 2.8. Let
:

Let h_A and g_B be two DFS-sets over U. then the intersection of h_A and g_B is denoted by h_A \ g_B and is de ned by h_A \ g_B = k_C, where C = A\B and its membership functions of k_C are as follows;

$k_{(e)}$ (m) = minf $h_{(e)}$ (m); $g_{(e)}$ (m)g

$k_{(e)}$ (m) = maxf $h_{(e)}$ (m); $g_{(e)}$ (m)g; if e 2 A \ B

III. LATTICE STRUCTURES OF DOUBLE-FRAMED SOFT SETS

In this section, the notion of double-framed soft lattice is de ned and several related properties are investigated.

De nition 3.1. Let DF^L be a double-framed soft set over U: $_L$ and ^ be two binary operation on DF^L. If elements of DF^L are equipped with two commutative and a associative binary operations _ and ^ which are connected by the absorption law, then algebraic structure (DF^L; _; ^) is called a double-framed soft lattice.

Example 3.2. Let U = fu$_1$; u$_2$; u$_3$; u$_4$g be a universe set and DF^L = ff$_A$; f$_B$; f$_C$; f$_D$g DFS(U):

Suppose that

f_A = e$_1$; u1 / 0:5; 0:7 ; e$_2$; u2 / 0:6; 0:4 ; u3 / 0:4; 0:7 ; e$_3$; u4 / 0:6; 0:8

f_B = e$_1$; u1 / 0:4; 0:5 ; u2 / 0:8; 0:6 ; e$_2$; u3 / 0:5; 0:3

f_C = e$_1$; u1 / 0:4; 0:5 ; u2 / 0:5; 0:6 ; e$_2$; u2 / 0:5; 0:6 ; u3 / 0:4; 0:7

f_D = e$_1$; u1 ; u2 ; u3

0:5; 0:4 0:6; 0:2 0:7; 0:4

Then (DF ^{L}; _; ^) is a double-framed soft lattice. Here binary op-erations are double-framed union and double-framed intersection.

Theorem 3.3. (DF ^{L}; _; ^) be a double-framed soft lattice and f_A; f_B 2 DFS(U). then f_A ^ f_B = f_A , f_A _ f_B = f_B.
Proof.

f_A ^ f_B = f_A ^ (f_A _ f_B)

= (f_A ^ f_A) _ (f_A ^ f_B)

= f_A _ f_A = f_A

Conversely,

f_A _ f_B = (f_A ^ f_B) _ f_B

= (f_A _ f_B) ^ (f_B _ f_B)

= f_B ^ f_B = f_B

□

Theorem 3.4. (DF ^{L}; _; ^) be a double-framed soft lattice and f_A; f_B 2 DFS(U). then the relation which is de ned by f_A f_B , f_A ^ f_B = f_A or f_A _ f_B = F_B is an ordering relation on
DFS(U).

Proof. For all f_A; f_B; f_C 2 DF ^{L},

(i) For all f_A 2 DF ^{L}, is re exive, f_A f_A , f_A ^ f_A = f_A:

(ii) For all f_A; f_B 2 DF ^{L}, is antisymmetric, Let f_A f_B and f_B f_A. then

f_A = f_A ^ f_B = f_B ^ f_A = f_B:

(iii) For all f_A; f_B; f_C 2 DF ^{L}, is transitive,

If f_A f_B and f_B f_C) f_A f_C. Indeed

f_A ^ f_C = (f_A ^ f_B) ^ f_C
= f_A ^ (f_B ^ f_C) = f_A ^ f_B = f_A:

□

Theorem 3.5. (DF ^{L}; _; ^) be a double-framed soft lattice and f_A; f_B 2 DFS(U). then f_A _ f_B and f_A ^ f_B are the least upper and greatest lower bound of f_A and f_B respectively.

Proof. Suppose that f_A ^ f_B is not the greatest lower bound of f_A and f_B.

Then there exists f_C 2 DFS(U) such that f_A ^ f_B f_C f_A and f_A ^ f_B f_C f_B.
Hence f_C ^ f_C f_A ^ f_B. Thus f_C f_A ^ f_B:

Therefore f_C = f_A ^ f_B: But this is a contradiction. f_A _ f_B being the least upper bound of f_A and f_B can be shown similarly. □

Theorem 3.6. Let DF ^{L} 2 DFS(U): then double-framed soft lat-tice inclusion relation that is de ned by f_A f_B , f_A [f_B = f_B or f_A \ f_B = f_A is an ordered relation on DF ^{L}.

Proof. For all f_A; f_B; f_C 2 DF ^{L},

(i) f_A 2 DF ^{L}; is re exive, f_A f_A , f_A \ f_A = f_A:

(ii) f_A; f_B 2 DF ^{L}; is antisymmetric, Let f_A f_B and f_B f_A , f_A = f_B:

(iii) f_A; f_B; f_C 2 DF ^{L} is transitive,

If f_A f_B and f_B f_C) f_A f_C:

□

Corollary 3.7. (DF ^{L}; [; \;) is a double-framed soft lattice.

De nition 3.8. Let (DF ^{L}; _; ^;) be a double-framed soft lattice and let f_A 2 DF ^{L}: If f_B f_A or f_A f_B for all f_A; f_B 2 DF ^{L}, then DF ^{L} is called a double-framed soft chain.

Example 3.9. Consider the double-framed soft lattice in example 3.2. A double-framed soft subset DF ^{S} = ff_A; f_B; f_Cg DFS(U) of DF ^{L} is a double-framed soft chain. But (DF ^{L}; [; \;) is not a double-framed soft chain. since f_B and f_C can not be comparable.

De nition 3.10. Let (DF ^{L}; _; ^;) be a double-framed soft lat-tice. If every subset of DF ^{L} have both a greatest lower bound and the least upper bound. then DF ^{L} is called a complete double-framed soft lattice.

Example 3.11. Let U = fu_1; u_2; u_3; u_4; u_5g be a universe set and
DF ^{L} = ff_A; f_B; f_C; f_Dg DFS(U):

$$f_A = e_1; \frac{u_1}{0{:}5;\ 0{:}7}; \frac{u_5}{0{:}8;\ 0{:}4}$$

$$f_B = e_1; \frac{u_1}{0{:}4;\ 0{:}5}; \frac{u_4}{0{:}8;\ 0{:}6}; \frac{u_5}{0{:}5;\ 0{:}3}; e_2; \frac{u_3}{0{:}5;\ 0{:}7}; \frac{u_4}{0{:}8;\ 0{:}3}$$

$$f_C = e_1; \frac{u_1}{0{:}4;\ 0{:}5}; \frac{u_2}{0{:}5;\ 0{:}6}; \frac{u_4}{0{:}7;\ 0{:}3}; \frac{u_5}{0{:}6;\ 0{:}4}; e_2; \frac{u_1}{0{:}4;\ 0{:}5}; \frac{u_2}{0{:}5;\ 0{:}6}; \frac{u_4}{0{:}7;\ 0{:}3}$$

f_D = $f_;$:

then (DF ^{L}; [; \;) is a complete double-framed soft lattice.

De nition 3.12. Let (DF ^{L}; _; ^;) be a double-framed soft lattice and DF ^{M} DF ^{L}. If f_A _ f_B 2 DF ^{M} and f_A ^ f_B 2 DF ^{M} for all f_A; f_B 2 DF ^{M} , then DF ^{M} is a double-framed soft sublattice.

Example 3.13. Let U = fu_1; u_2; u_3; u_4; u_5g be a universe set and

DF ^{L} = ff_A; f_B; f_C; f_Dg DFS(U):

$$f_A = e_1; \frac{u_1}{0{:}3;\ 0{:}8}; \frac{u_5}{0{:}2;\ 0{:}5}$$

$$f_B = e_1; \frac{u_1}{0{:}4;\ 0{:}5}; \frac{u_4}{0{:}9;\ 0{:}5}; \frac{u_5}{0{:}5;\ 0{:}2}; e_2; \frac{u_3}{0{:}5;\ 0{:}7}; \frac{u_4}{0{:}9;\ 0{:}5}$$

$$f_C = e_1; \frac{u_1}{0{:}4;\ 0{:}6}; \frac{u_2}{0{:}5;\ 0{:}6}; \frac{u_4}{0{:}7;\ 0{:}3}; \frac{u_5}{0{:}6;\ 0{:}5}$$

f_D = $f_;$

Then, if DF ^{M} = ff_A; f_B; f_Dg DFS(U), then DF ^{M} is a double-framed soft sublattice.

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De nition 3.14. Let $(DF^L; \_; \wedge;\ )$ be a double-framed soft lattice and $f_A; f_B$ and f_C 2 DF^L. If $(f_A \wedge f_B) \_ (f_A \wedge f_c)\ f_A \wedge (f_B \_ f_C)$ or $f_A \wedge (f_B \_ f_C)\ (f_A \wedge f_B) \_ (f_A \wedge f_C)$ then DF^L is called a one-sided distributive double-framed soft lattice.

Theorem 3.15. Every double-framed soft lattice is a one-sided dis-tributive double-framed soft lattice.

Proof. Let $f_A; f_B$ and f_C 2 DF^L.

Since $f_A \wedge f_B\ f_A$ and $f_A \wedge f_B\ f_B\ f_B \_ f_C$. $f_A \wedge f_B\ f_A$ and $f_A \wedge f_B\ f_B \wedge f_C$.
Therefore,

$$f_A \wedge f_B = (f_A \wedge f_B) \wedge (f_A \wedge f_B)\quad f_A \wedge (f_B \_ f_C) \tag{1}$$

and also we have $f_A \wedge f_C\quad f_A$ and $f_A \wedge f_C\quad f_C\quad f_B \_ f_C$.

Since $f_A \wedge f_C\quad f_A$ and $f_A \wedge f_C\quad f_B \_ f_C$; then

$$f_A \wedge f_C = (f_A \wedge f_C) \wedge (f_A \wedge f_C)\quad f_A \wedge (f_B \_ f_C) \tag{2}$$

□

from (1) and (2), we get the desired result, $(f_A \wedge f_B) \_ (f_A \wedge f_C)\ f_A \wedge (f_B \_ f_C)$:

De nition 3.16. Let $(DF^L; \_; \wedge;\ )$ be a double-framed soft lat-tice. If DF^L satis es the following axioms, it is called a distributive double-framed soft lattice.

(i) $f_A \_ (f_B \wedge f_C) = (f_A \_ f_B) \wedge (f_A \_ f_C)$

(ii) $f_A \wedge (f_B \_ f_C) = (f_A \wedge f_B) \_ (f_A \wedge f_C)$ for all $f_A; f_B$ and f_C 2 DF^L.

Example 3.17. Let U = fu$_1$; u$_2$; u$_3$; u$_4$; u$_5$g be a universe set and DF^L = ff$_;$; f_A; f_B; f_C; f_D; f_E g DFS(U): then DF^L DFS(U) is a double-framed soft lattice with the operations [and \. Suppose that

$$f_A = e_1; \frac{u_5}{0{:}4; 0{:}6}; e_2; \frac{u_1}{0{:}5; 0{:}7}; \frac{u_2}{0{:}5; 0{:}6}$$

$$f_B = e_1; \frac{u_1}{0{:}4; 0{:}5}; \frac{u_3}{0{:}6; 0{:}4}; \frac{u_5}{0{:}3; 0{:}2};$$
$$e_2; \frac{u_2}{0{:}4; 0{:}3}; \frac{u_4}{0{:}6; 0{:}3}; e_3; \frac{u_3}{0{:}7; 0{:}3}; \frac{u_4}{0{:}5; 0{:}3}$$

$$f_C = e_1; \frac{u_3}{0{:}4; 0{:}6}; \frac{u_4}{0{:}7; 0{:}4}; \frac{u_5}{0{:}5; 0{:}4};$$
$$e_2; \frac{u_1}{0{:}4; 0{:}6}; \frac{u_2}{0{:}5; 0{:}6}; \frac{u_4}{0{:}7; 0{:}3};$$
$$e_3; \frac{u_3}{0{:}7; 0{:}6}; \frac{u_4}{0{:}5; 0{:}3}$$

$$f_D = e_1; \frac{u_5}{0{:}6; 0{:}9}; e_2; \frac{u_1}{0{:}4; 0{:}5}; \frac{u_2}{0{:}5; 0{:}6}; \frac{u_3}{0{:}7; 0{:}2};$$
$$e_3; \frac{u_3}{0{:}7; 0{:}2}; \frac{u_4}{0{:}5; 0{:}3}$$

$$f_E = e_1; \frac{u_1}{0{:}4; 0{:}6}; \frac{u_2}{0{:}5; 0{:}7}; \frac{u_3}{0{:}7; 0{:}3}; \frac{u_5}{0{:}6; 0{:}4};$$
$$e_2; \frac{u_1}{0{:}4; 0{:}5}; \frac{u_2}{0{:}5; 0{:}6}; \frac{u_3}{0{:}7; 0{:}3};$$
$$e_3; \frac{u_3}{0{:}7; 0{:}3}; \frac{u_4}{0{:}5; 0{:}4}; \frac{u_5}{0{:}5; 0{:}3}$$

$f_; \ = \ ;$

$(DF^L; [; \backslash;\ )$ is a distributive double-framed soft lattice.

De nition 3.18. $(DF^L; \_; \wedge;\ )$ be a double-framed soft lattice. then DF^L is called a double-framed soft modular lattice, if it satis-es the following property

$f_C\quad f_A\ )\ f_A \wedge (f_B \_ f_C) = (f_A \wedge f_B) \_ f_C$

for all f_A; f_B and f_C 2 DF ^{L}:

Example 3.19. Let U = fu$_1$; u$_2$; u$_3$; u$_4$; u$_5$u$_6$g be a universe set and
DF ^{L} = ff$_;$; f_A; f_B; f_C; f_Dg DFS(U): then DF ^{L} DFS(U) is

a double-framed soft lattice with the operations [and \. Suppose that

$$f_A = e_5; \frac{u_5}{0{:}5;\,0{:}7}$$

$$f_B = e_1; \frac{u_1}{0{:}4;\,0{:}5}; \frac{u_2}{0{:}7;\,0{:}6}; e_2; \frac{u_4}{0{:}7;\,0{:}3}$$

$$f_C = e_3; \frac{u_2}{0{:}6;\,0{:}3}$$

$$f_D = e_1; \frac{u_1}{0{:}4;\,0{:}5}; \frac{u_2}{0{:}7;\,0{:}6}; \frac{u_3}{0{:}6;\,0{:}4}; e_2; \frac{u_2}{0{:}5;\,0{:}6}; \frac{u_4}{0{:}7;\,0{:}3}; e_3; \frac{u_2}{0{:}6;\,0{:}3}; \frac{u_5}{0{:}5;\,0{:}4}; e_5; \frac{u_1}{0{:}7;\,0{:}3}; \frac{u_2}{0{:}5;\,0{:}4}$$

$f_;$ = ;

(DF ^{L}; [; \;) is a double-framed soft modular lattice.

IV. CONCLUSION

In this paper, we introduce the concept of double-framed soft lat-tice as an algebraic structure and showed that these de nitions are equivalent. we then investigated some related properties and some characterization theorems.

Future Work : To extend this work one can obtain the properties of double-framed soft set in other algebraic structures and elds. In addition based on these results, we can further prove the applications of double-framed soft lattice.

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