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Equilibrium Points In The Elliptical Magnetic Binary Problem

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ABSTRACT
The present paper deals with the existence and linear stability of equilibrium points in the magnetic binary problem when the primaries are moving in the elliptical orbit including the effect of the gravitational force of the bigger primary. We have observed that there exists three collinear and two non-collinear equilibrium points we have also observed that all the collinear and non-collinear equilibrium points are unstable.
KEY WORDS: equilibrium points, elliptical magnetic binary problem, stability.

I INTRODUCTION

The elliptical restricted three-body problem describes the dynamical system more accurately on account that the realistic assumptions of the, motion of the primaries are subjected to move along the elliptical orbit. The different aspects of the elliptical restricted three-body problem have been investigated by several mathematician. The stability of the triangular points in elliptic restricted problem of three bodies have been discussed by Danby [3]. L. Floria [4] has studied an analytical solution in the planar elliptic restricted three body problem In (2004). A. Narayan and C. Ramesh [1,2] have investigated the stability of triangular equilibrium points in elliptical restricted three body problem under the effects of oblateness of the primaries. The effect of solar radiation pressure on the Lagrangian points in the elliptical restricted three body problem has benn discussed by M.K. Ammer [5].

A. Mavragnais [6-9] and Mohd Arif [10-11] have studied the motion of a charge particle which is moving in the field of two rotating magnetic dipoles in the circular motion around their centre of mass. Being motivated by the above discussion in this article we have discussed the motion of a charged particle when the dipoles move on elliptic orbits including the effect of the gravitational force of the bigger dipole. The dimensionless variables are introduced by using the distance r between dipoles given by

$$r = \frac{a(1-e^2)}{1+e\, cos\gamma} \qquad (1.1)$$

Here a and e are the semi-major axis and the eccentricity of the elliptical orbit of the either dipole around other and γ is the true anomaly of one of the dipole of mass m_1. We have introduced a coordinate system which rotates with the variable angular velocity ω with

$$\frac{d\omega}{dt^*} = \frac{k\,(m_1+m_2)^{\frac{1}{2}}}{a^{3/2}\,(1-e^2)^{3/2}}\,(1+e\, cos\gamma)^2 \qquad (1.2)$$

Where t^* is the dimensional time and $k = k_1 + k_2$ where k_1 and k_2 are the product of the universal gravitational constants with the mass of dipoles. Equation (1.2) follows from the principle of the conservation of the angular momentum

II EQUATION OF MOTION

Two dipoles (the primaries), with magnetic fields move in the elliptical orbits and a charged particle P of charge q and mass m moves in the vicinity of these dipoles. The question of the elliptical magnetic-binaries problem is to describe the motion of this particle. The equation of motion in the rotating coordinate system including the effect of the gravitational force of the bigger primary written as:

$$\xi'' - \eta' f = U_\xi \qquad (2.1)$$
$$\eta'' + \xi' f = U_\eta \qquad (2.2)$$

Where

$f = 2 - 2\zeta\left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right)$, $U_\xi = \frac{\partial U}{\partial \xi}$ and $U_\eta = \frac{\partial U}{\partial \eta}$

$$U = (\xi^2 + \eta^2)\left\{\frac{m}{2(1+e\cos\gamma)} + \zeta\left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right)\right\} + \xi\,\zeta\left\{\frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} + \frac{(1-\mu)}{r_1(1+e\cos\gamma)} + \frac{2\,q\,\xi\,\eta\,e\sin\gamma}{m\,c\,a^{3/2}\,(1-e^2)^{3/2}} \tag{2.3}$$

$\zeta = \frac{q\sqrt{a(1-e^2)}}{m\,c\,(1+e\cos\gamma)}$, $r_1^2 = (\xi-\mu)^2 + \eta^2$, $r_2^2 = (\xi+1-\mu)^2 + \eta^2$, $\lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion)
$m_2 = \mu$, $m_1 = (1-\mu)$, c = velocity of light
Let us define the averaged potential function of the problem with respect to true anomaly as:

$$U^* = \frac{1}{2\pi}\int_0^{2\pi} U\,d\alpha, \tag{2.4}$$

Hence

$$U^* = (\xi^2 + \eta^2)\left\{\frac{m}{2\sqrt{(1-e^2)}} + a^{\frac{1}{2}}\left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right)\right\} + \xi\,a^{\frac{1}{2}}\left\{\frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} + + \frac{(1-\mu)}{r_1\sqrt{(1-e^2)}}$$

is the modified potential function where e is the eccentricity and a is the semi-major axis of the orbit, μ is the mass parameter, for numerical calculation we have taken a particular case $q = mc$.

III INVESTIGATION OF EQUILIBRIUM POINTS

The equilibrium points of the system are determined by the equation:

$$\frac{\partial U^*}{\partial \xi} = 0 \text{ and } U_\eta = \frac{\partial U^*}{\partial \eta} = 0 \tag{2.5}$$

We group the solution of equation (2.5) into two kinds; those $\eta = 0$ the collinear equilibrium points and those with $\eta \neq 0$ non-collinear equilibrium points.
We investigate the existence and location of the collinear equilibrium points into the following three intervals.
$C_1 = \{\xi : \xi > \mu\}$, $C_2 = \{\xi : -(\mu-1) < \xi \le \mu\}$ and $C_3 = \{\xi : \xi \le -(\mu-1)\}$.
If $\xi \in C_1$ the substitution $r_1 = \xi - \mu = \tau$ and $r_2 = \xi + 1 - \mu = \tau + 1$ in (2.5) we have 11th degree equation

$$(\tau+1)^5\tau^5(\tau+\mu)\,m - a^{\frac{1}{2}}\sqrt{1-e^2}\,[3\,(\tau+1)^5\tau^2\,(\tau+\mu)^2 + (\tau+1)^2\tau^5\,(\tau+\mu)^2\lambda - -(\tau+1)^5\tau^2(\tau+\mu) - (\tau+1)^2\tau^5(\tau+\mu)\,\lambda - (\tau+1)^5\tau^3 + \lambda\,(\tau+1)^3\tau^5] - (\tau+1)^5\tau^3(1-\mu) = 0. \tag{2.6}$$

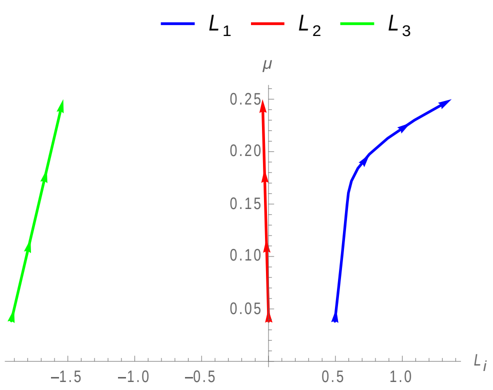
If $\xi \in C_2$ the substitution $r_1 = \xi - \mu = 1 - \tau$ and $r_2 = \xi + 1 - \mu = \tau$ in (2.5) again given 11th degree equation

$(1-\tau)^5\tau^5(1-\tau+\mu)\,m - a^{\frac{1}{2}}\sqrt{1-e^2}\,[3(1-\tau)^2\tau^5(1-\tau+\mu) + (1-\tau)^5\tau^2\,(1-\tau+\mu)\lambda - (1-$
τ2τ51−τ+μ−1−τ5τ21−τ+μ λ−1−τ3τ5+λ 1−τ5τ3]−1−τ3τ51−μ=0.
(2.7)

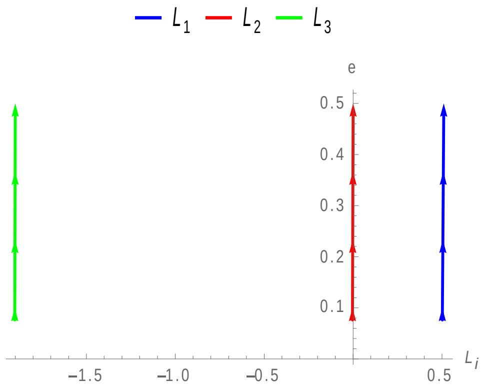
And when $\xi \in C_3$ the substitution $r_1 = \mu - \xi = 1 + \tau$ and $r_2 = -(\xi + 1 - \mu) = \tau$ in (2.5) we have again 11th degree equation

$$(1+\tau)^5\tau^5(-1-\tau+\mu)\,m - a^{\frac{1}{2}}\sqrt{1-e^2}\,[3\,(1+\tau)^2\tau^5(-1-\tau+\mu) + (1+\tau)^5\tau^2\,(-1-\tau+\mu)\lambda - (1+\tau)^2\tau^5(-1-\tau+\mu) - (1+\tau)^5\tau^2(-1-\tau+\mu)\,\lambda - (1+\tau)^3\tau^5 + \lambda\,(1+\tau)^5\tau^3] - (1+\tau)^3\tau^5(1-\mu) = 0 \tag{2.8}$$

To find the location of the collinear-equilibrium points we solve the equations (2.6), (2.7) and (2.8) numerically by using Mathmatica-11 and we have observed that the each equation have one real root for various values of mass parameter μ and eccentricity e and these roots are denoted by L_1, L_2 and L_3 respectively. The variation in the values of L_i $(i = 1,2,3)$ for various values of μ and e are shown in the figures (1) and (2) respectively and these figures shows that the collinear-equilibrium points L_i $(i = 1,2,3)$ moves away from the centre of mass as μ and e increases.



Fig(1)



Fig(2)

The non-collinear equilibrium points denoted by L_4 and L_5 are the solution of the equation (2.5) when $\eta \neq 0$. In table (4) and (5) we give the position of the points L_4 and L_5 for various values of μ and e respectivaly. We observed that both L_4 and L_5 shifted towards the centre of mass as μ increases and moves away as e increases.

IV STABILITY OF EQUILIBRIUM POINTS

Let (ξ_0 , η_0) be the coordinate of any one of the equilibrium point and let α, β denote small displacement from the equilibrium point. Therefore we have

$\alpha = \xi - \xi_0$,

$\beta = \eta - \eta_0$,

Put this value of ξ and η in equation (2.1) and (2.2), we have the variation equation as:

$$\alpha'' - \beta' f_0 = \alpha\left(U^*_{\xi\xi}\right)^0 + \beta\left(U^*_{\xi\eta}\right)^0 \tag{3.1}$$

$$\beta'' + \alpha' f_0 = \alpha\left(U^*_{\xi\eta}\right)^0 + \beta\left(U^*_{\eta\eta}\right)^0 \tag{3.2}$$

Retaining only linear terms in α and β. Here superscript indicates that these partial derivative of U^* are to be evaluated at the equilibrium point (ξ_0 , η_0) . So the characteristic equation at the equilibrium points is

$$\lambda_1^{\,4} + \lambda_1^{\,2}\left\{f_0^{\,2} - \left(U^*_{\xi\xi}\right)^0 - \left(U^*_{\xi\xi}\right)^0\right\} + \left(U^*_{\xi\xi}\right)^0\left(U^*_{\eta\eta}\right)^0 - \left(U^*_{\xi\eta}\right)^{0\,2} = 0 \tag{3.3}$$

The equilibrium point (ξ_0 , η_0) is said to be stable if all the four roots of equation (3.3) are either negative real numbers or pure imaginary.

From tables 1, 2, 3, 4 and 5 it is clear that all the equilibrium point in these tables are unstable .

Table (1)

μ	L_1	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	0.506189	± 1.2693	$\pm 18.876\, i$
.1	.549337	± 1.2747	$\pm 19.991\, i$
.15	.589419	± 1.2911	$\pm 21.681\, i$
.20	.622841	± 1.3308	$\pm 24.796\, i$
.25	1.36918	± 1.2031	$\pm 1.4329\, i$
e	L_1	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.1	.502181	± 1.4724	$\pm 19.569\, i$
.2	.502994	± 1.4317	$\pm 19.426\, i$
.3	.504420	± 1.3598	$\pm 19.178\, i$
.4	.506585	± 1.2487	$\pm 0.9407\, i$
.5	.509714	± 1.0813	$\pm 18.290\, i$

Table (2)

μ	L_2	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	-0.00162	± 7.3434	$\pm 15973.6\,i$
.1	-0.0063	± 5.2509	$\pm 1847.33\,i$
.12	-0.01457	± 4.286	$\pm 508.008\,i$
.15	-0.02700	± 3.6624	$\pm 200.74\,i$
.6	0.44260	± 3.1766	$\pm 98.167\,i$
e	L_2	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.1	-0.00350	± 6.9478	$\pm 14351.3i$
.2	-0.00312	± 7.0282	$\pm 14660.6i$
.3	-0.002452	± 7.1690	$\pm 15227.0i$
.4	-0.00144	± 7.3810	$\pm 16141.9i$
.5	$-.00002$	± 7.6834	$\pm 17552.8i$

Table (3)

μ	L_3	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	-1.90153	$\pm .20373$	$\pm 6.47009\,i$
.1	-1.80566	$\pm .22178$	$\pm 7.0943\,i$
.15	-1.71251	$\pm .21987$	$\pm 7.7904\,i$
.20	-1.62215	$\pm .20127$	$\pm 8.5602\,i$
.25	-1.53465	$\pm .16890$	$\pm 9.4030\,i$
e	L_3	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.1	-1.90307	$\pm .16669$	$\pm 6.4536i$
.2	-1.90273	$\pm .17497$	$\pm 6.4572i$
.3	-1.90222	$\pm .18866$	$\pm 6.4626i$
.4	-1.90136	$\pm .20685$	$\pm 6.4719\,i$
.5	-1.90003	$\pm .22932$	$\pm 6.4867\,i$

Table (4)

μ	$(L_{4,5})_\xi$	$(L_{4,5})_\eta$	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	-0.59574	± 0.621255	± 3.1921	$\pm 8.7699i$
.1	-0.53877	± 0.608585	± 3.2199	$\pm 8.9802i$
.15	-0.48071	± 0.595206	± 3.2424	$\pm 9.205\, i$
.20	-0.42128	± 0.581075	± 3.2573	$\pm 9.4456i$
.25	-0.36009	± 0.566185	± 3.2492	$\pm 9.9609\, i$
.30	-0.29655	± 0.55062	± 3.2157	$\pm 10.223i$
.35	-0.22981	± 0.53470	± 3.1574	$\pm 10.468i$
.40	-0.15865	± 0.51936	± 3.2464	$\pm 9.6938i$

Table (5)

e	$(L_{4,5})_\xi$	$(L_{4,5})_\eta$	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.1	-0.474647	± 0.601144	± 3.218	$\pm 8.9954i$
.2	-0.475884	± 0.59994	± 3.2237	$\pm 9.0372i$
.3	-0.478046	± 0.59783	± 3.2322	$\pm 9.0884i$
.4	-0.48131	± 0.594617	± 3.2447	$\pm 9.2269i$
. 5	-0.48598	± 0.58996	± 3.2626	$\pm 9.3988i$

V. CONCLUSION

In this article we have seen that there exist five equilibrium points, three collinear and two non-collinear. We observed that the collinear-equilibrium points L_i $(i = 1,2,3)$ moves away from the centre of mass as both μ and e increases. We also observed that the points L_4 and L_5 shifted towards the centre of mass as μ increases and moves away as e increases. We have observed that all points given in tables 1,2,3 4 and 5 are unstable.

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