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# A Review of Soil Constitutive Models and Their Use in Geotechnical Engineering

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**ABSTRACT:**
Various constitutive models for describing the stress strain behaviour of soils have been created, and these models have been applied to finite element modelling for use in geotechnical engineering and for the investigation of soil structure problems under various loading circumstances. On the basis of the mechanical principle (hooks law of linear elasticity and others), simple and complicated models have been developed.Howeversoilsarenotentirelylinearlyelasticandperfectlyplasticfortheentirerangeofloading.Infact,actualbehaviorofsoilsisverycomplicated to understand and it shows range of behaviorsunder different conditions. Hence, different models have beenproposed to describe its response. Moreover, no model cancompletely describe the complex behavior of soils. This paperpresentsbriefintroductionofvarioussoilmodelsandparticularlythecamclay Modelandalsotheirapplication.

**Keywords:Finiteelement,Cam clayModel,Application**

## INTRODUCTION:

Soilsbeingcomplexmaterialsconsistofsolidgrainsincontactwitheachotherandvoidspresentinbetween may be filled either by water or air. The solidgrainstransmitnormalandshearforcesandthissolidSkeltonbehavesinacomplexfashiondependinguponfactorslike permeability,voidratio etc.

Soilsexhibitacomplicatedbehaviorwheneversubjectedtostresses.Soilsmostlyshownonlinear,anisotropic,timedependentresponseunderdifferentloading conditions. They undergo plastic deformationandis inconsistent in dilatancy. They also undergo small strainstiffness at small strain levels and when subjected to stressreversal. Different aspects of soil behavior have to be takeninto consideration (Brinkgreve2005).

1. Influenceofwateronsoilfromeffectivestressandporepressures.
2. Factorsinfluencingsoilstiffnesslikestresspath,stresslevel,soildensity,strainlevels.
3. Factorinfluencingsoilstrength likeageandsoildensity,consolidationratio,undrained behavior,loadingspeed.
4. Compaction,dilatancyetc.

Moreover, the failure of soil under three dimensional stateof stress is extremely complicated. Various criteria havebeen proposed to explain failure condition under this state.Withtheadvancementinnumericalmethodslikedevelopment of finite element method, it has been possibleto analyze and predict the complex behavior of soils andsoil-structure interaction problems. Such analysis dependson relation between stress and strain of variousmaterials.In numerical methods, this relation of stress and strain of agiven material is represented by **constitutive Model** whichmodels the behavior of soil in a single element. The mainaimoftheseconstitutivemodelsistosimulatesoilbehaviorwithsufficientaccuracyunderallloadingconditions.

Constitutive models have developed over a periodof time, from being simple to more complex in order tocapturethebehaviorofsoilundercomplexloadingconditions. These models have been formulated based onthe principles of continuum mechanics and also numericalevaluation of the models with respect to the facility whichcan be implemented in computer calculations (Chen 1985).Differenttypesofmodels forsimulation of soilbehaviorare Hookes model, Mohr-Coulumb Model, (modified) camclayModel,HyperelasticModel,HypoelasticModel,PlaxishardeningsoilModel etc.

HOOKE'SMODEL:

It is a linear elastic model which is based uponHooke's law of linear elasticity. This model consists of twobasic parameters viz modulus of elasticity (E) and poisonsratioμ.Since the soil behavior is highly non linear andirreversible,thismodelisinsufficienttocapturetheessentialfeaturesofsoilhowever,thismodelcanbeusedtomodelthestiffvolumesinsoilliketheconcretewalls.

MOHR-COULUMBMODEL:

It is a linear elastic perfectly plastic model andrepresents first order approximation of soil behavior .ThegraphofMohrcolumbmodelisshowninFig. 1

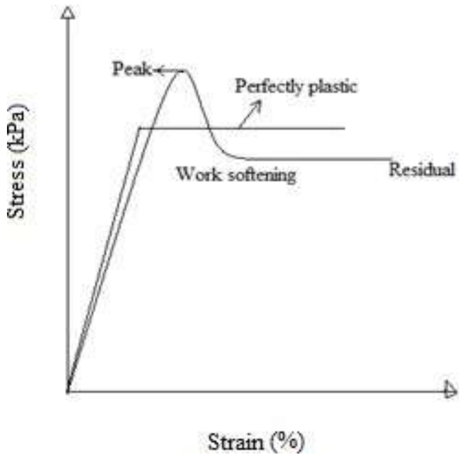


Fig.1Elasticperfectyplasticassumption ofMohrColumbModel

FromFig.1,itisclearthatmaterialbehaveslinearly in the elastic range, defined by two parameters likeYoung's modulus (E) and poisons ratio μ. For defining thefailure criteria, parameters are friction angle ø, cohesionintercept C. Also a parameter to describe the flow rule isdilatancy angle Ψ which is used to model the irreversiblevolumechange duetotheshearing.

Inplastictheory,flowruleisusedforplasticstrain rates.Inorder to evaluate whether or not the plasticityoccurs, a yield function f is introduced which is a functionof stress and strain. The condition f =0 is related to theplasticyielding. In principalstressspace,conditionf=0canberepresentedasasurfaceasshowninFig.2.MohrCoulumbmodelissimpleandapplicabletothreedimensionalstressspacemodel to describe plastic behavior. This model finds itsapplicationinstabilityanalysisofdams,slopes,embankmentsandshallowfoundations.

Inthismodel,thefailurebehaviorofsoiliscaptured under drained condition, but the effective stresspath followed in undrained materials shows considerablevariation from observations. Thus in undrained analysis, itispreferred touseundrainedshearparametersø=0◦.

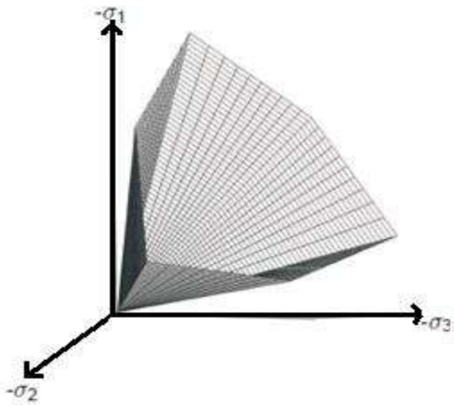


Fig.2MohrColumbyieldsurfaceinprincipalstressspace

HYPERBOLICMODEL(DUNCAN-CHANGMODEL):
Since the mathematical behavior of soils is highlynonlinearandalsoshowsstressdependencyintheirstiffness. Duncan-Chang Model also known as hyperbolicModel(Duncan-Chang1970)isbasedonstressstrain curve in drainedtriaxial compression tests of clays andsands and can be approximated by a hyperbolic function asshown inFig.3.

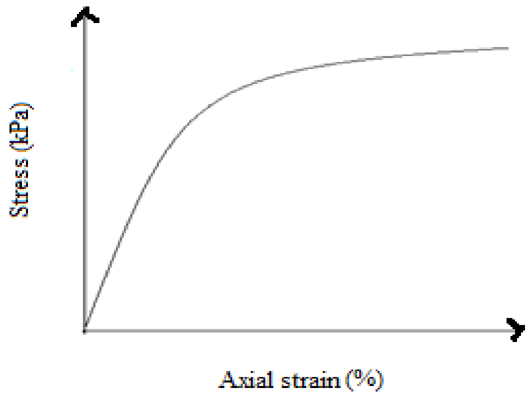


Fig.3Hyperbolicstressstraincurve

Itisanincrementalnonlinearstressdependentmodelhavingbothloadingandunloadingmodulus.Thefailure criterion is based on Mohr columb Model and also itisformulatedusingpower lawfunctions(Ohde 1939).
Undertheloadingcondition($\sigma 1/\sigma 3>0$),plasticdeformation continues till stress point is on yield surface ,otherwise, the stress state must drop below yield value, andin this case all deformations are elastic which occurs underunloading($\sigma 1/\sigma 3<0$).
primarydeviatoricloadingandcompressionhardeningisusedtomodelirreversibleplasticstrainsduetoprimarycompressioninoedometerloadingandisotropicloading.Hardening is assumed to beisotropic depending on bothplastic shear and volumetric strain. Due to the involvementof two types of hardening, it is used for problems whichinvolve reduction of mean effective stress and mobilizationof shearstrength likeexcavation andtunnelconstructionprojects.

HARDENINGSOILMODEL:
The hardening soil Model (Brinkgreve and Vermer1997) is an advanced model for simulation of soil behavior.It is a true second order model for soils in general for anytype of application (Brinkgreve 2005).The background ofthis model is the hyperbolic relationship between verticalstrain and deviatoric stress in primary loading. It uses theoryofplasticityand includessoildilatancyandayield cap.
The model involves two types of hardening namelyshearhardeningandcompressionhardening.Shearhardeningisusedtomodelirreversiblestrainsdueto
In contrast to MohrColumbModel, yield surfaceof hardening soil model is not fixed in principal stress space,but it can expand due to plastic strain as shown in Fig. 4.This model accounts for stress dependent stiffness modulii.e, stiffness increases with pressure. It also uses power lawfor formulation of stress dependent stiffness. Here, the soilstiffness is described accurately by usingthree differentstiffnessnamely,

1. TriaxialloadingstiffnessE$_{50}$
2. TriaxialunloadingstiffnessE$_{ur}$.
3. OedometerloadingstiffnessE$_{oed}$.

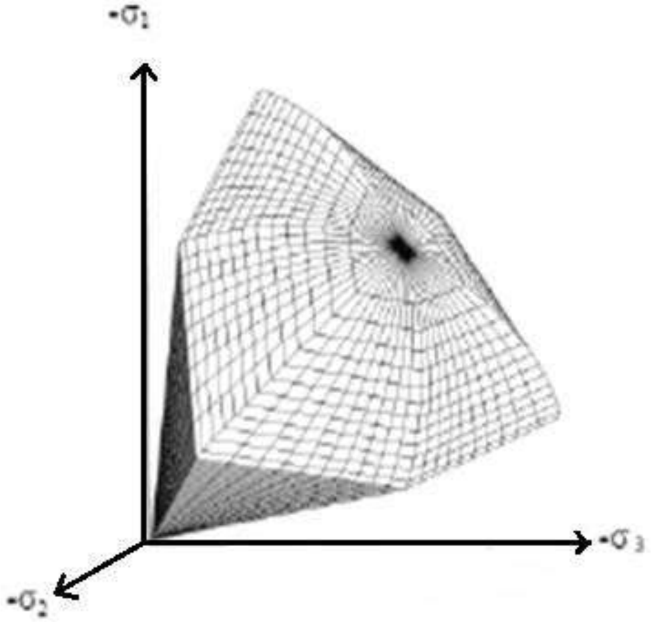


Fig.4YieldsurfaceofhardeningsoilModel

HYPERELASTICMODEL:

Hyperelastic or green elastic Model is a type of constitutivemodelforideally elasticmaterials,wherethestress isafunction of current strain and not a function of history ofstrain i.e, it depends on current state of deformation. TheHyperelasticmaterialisalsocalledasCauchy-elasticmaterial. A Cauchy-elastic material is one in which stress ateach point is determinedby current state of deformationwith respecttoan arbitrary referenceconfiguration.Thus,thestresscomputationinaCauchyelasticmaterialisindependentofhistory,pathofdeformationandalsothetimetakentoachievedeformation.AlsoCauchyelasticmaterialexhibitsenergydissipationviolatingenergydissipationpropertiesofelasticmaterial.

Whenanexternalforceisappliedtoanelasticmaterial in its natural state, the body undergoes deformationand reaches a different energy state. When the external forceis removed, body regains its original state and there is noenergy dissipation. Materials for which work performed isindependent of load path are Hyperelastic materials.

ThusHyperelasticmaterialsareelasticmaterialwhichderivesstressstrainrelation(asshowninFig.5)fromastrainenergy densityfunction.Fordeductionofthisenergyfunction, it is assumed material is isotropic, constant volumeandincompressible.ThemostcommonfunctionsofdeformationenergyaretheMooney-RivlinModel,Neo-Hookean Model, Odgen Model (Gent1996).

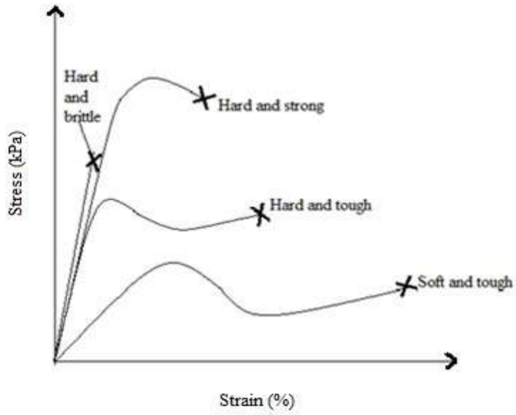


Fig.5Stressstraincurvesfordifferentmaterialsusinghyperelasticity

Truesdell(1955)proposedaratetheoryonthe

HYPOELASTICMODEL:
Hypoelasticityisusedtomodelmaterialsthatexhibitnonlinear, but reversible stress strain behavior even at smallstrain.Herealso,thestraininmaterialdependsonlyonstress applied rather than history of loading and the rate ofloading. In this model, the stress is a non linear function ofstrain,eventhoughstrainsaresmallasshowninFig.6.Becausestrainsaresmallthisistruewhateverstressmeasurewe adopt(Cauchystress) and is truewhateverstrain measure we adopt (Langrage strain or infinitesimalstrain).
basis of Cauchy formulation for such materials. From thistheory, incremental stress strain laws can be developed. Thestress strain curve for such materials is shown in Fig. 6 (A FBowler).This curve is developed based on assumption thatstrainsandrotationsareassumedtobesmall.Thus,deformationischaracterizedusinginfinitesimalstraintensor.

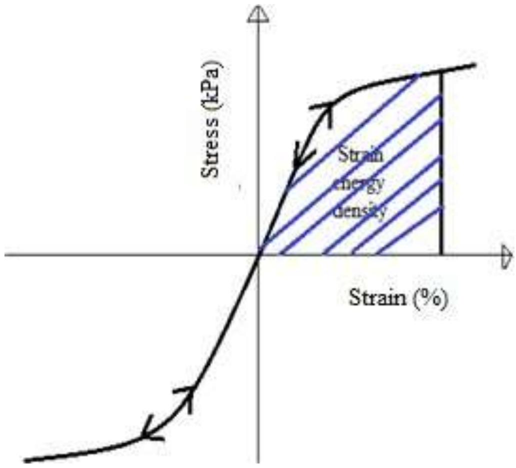


Fig.6Stressstraincurveofhypoelasticmaterials

Chen (1985) discussedthatthepath-independentbehavior implied in the previous secant type of stress-strainformulation can be improved by the hypoelastic formulationinwhichtheincrementalstressandstraintensorsarelinearly related through variable tangent material responsemoduli that are functions of the stress or strain state. In thesimplest case of hypoelastic Models, the incremental stress-strain relations are formulated directly as simple extensionsoftheisotropiclinearelasticmodelwiththeelasticconstants replaced by variable tangential moduli which aretakento befunctionsofthestressand/orstraininvariants.

(MODIFIED)CAMCLAY MODEL:
The cam clay Model was developed by researchersat Cambridge University for past thirty years (Roscoe 1970).Roscoe et al. (1963) utilized the strain hardening theory todevelopstressstrainmodelfornormallyconsolidatedorover consolidated soils in triaxial test known as cam clayModel (Schofield and wroth 1968). Also cam clay Modelwasbasedonassumptionthatsoilisisotropic,elasto-plastic,deformsascontinuum,anditisnotaffectedbycreep. Burland (1968) was responsible for the modificationoforiginalcamclayModelwheretheyieldsurfaceis
described by an ellipse i.e, three dimensional stress states(Roscoe and Burland 1968) and hence modified cam clayModel.BothcamclayandmodifiedcamclayModeldescribethree importantaspectsofsoilbehavior.

1. Strength
2. Compression or dilatancy (the volume change that occurswithshearing).
3. Criticalstateatwhichsoilelementscanexperienceunlimiteddistortionwithoutanychangesinstressorvolume.

The modified cam clay Model is an elastic plasticstrainhardeningmodelwherethenon linearbehaviorismodeled by means of hardening plasticity. Formulation ofmodified cam clay Model is based on plastic theory whichmakes it possible to predict volume changes due to differentloadings using an associated flow rule. This model is basedoncriticalstate theory.
In critical state theory, state of soil is characterized by threeparameters:

1. Effective meanstressp’.
2. Deviatoricstressq
3. Specificvolumev.

Where, $p' = \frac{(\sigma'_1+\sigma'_2+\sigma'_3)}{3}$

$q = \frac{1}{\sqrt{2}}\sqrt{(\sigma'_1-\sigma'_2)^2+(\sigma'_2-\sigma'_3)^2+(\sigma'_3-\sigma'_1)^2}$

Thespecificvolumeisdefinedasv=1+ewheree=void ratioofsoil.

Followingarethecomponentsofcriticalstatetheory:

*1.* VIRGINCOMPRESSIONLINEANDSWELLINGLINES:

The models assume that under isotropic stress conditions, when a soft soil sample is slowly compressed under perfectlydrained conditions, the relationship between specific volume v and logarithm of mean effective stress $p'$ consists of a straight lineknown as normal compression line or virgin compression line and upon reloading and unloading the sample, we get swelling linesasshowninFig. 7.

Thevirginconsolidationlineisgivenas:

Whiletheequationforswellingline hastheformas:

$v = N - \lambda \ln p'$

$v = v_S - \kappa \ln p'$

Thevalues $\lambda$,$\kappa$,Narethecharacteristicproperties of aparticularsoil,$\lambda$is theslopeofNCL on $v - \ln p'$ plane,while$\kappa$istheslopeofswellingline Nisthespecificvolume ofNCLat unitpressure.

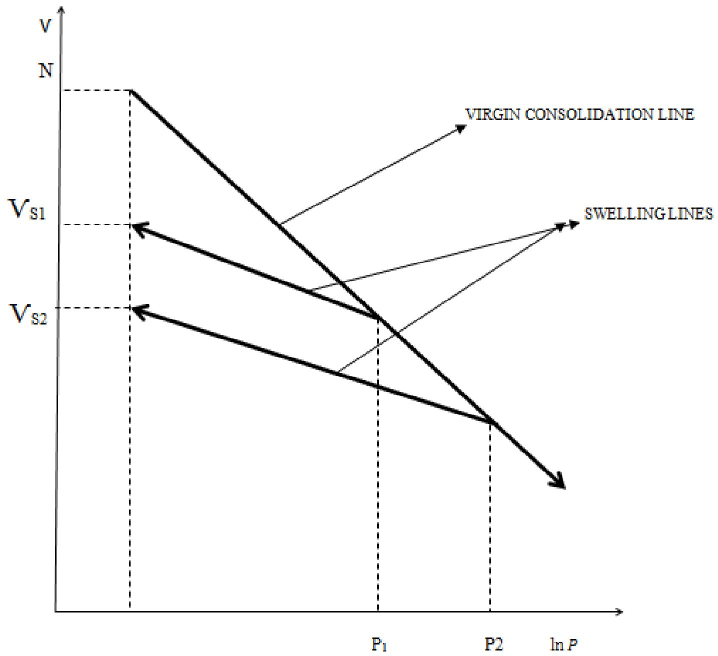


Fig.7Behaviorofsoilsampleunderisotropiccompression

*2.* CRITICALSTATELINE:

When the soil sample is continuously sheared, itwill eventually reach a state where further deformations canoccurwithoutanychangeinstressorvolumei.e,soildistorts at constant stress without any change in volume andthis state is known as critical state which is characterized bycritical state line (CSL).Critical state line is always paralleltoNCLin $v-\ln p'$ spaceasshowninFig.8:

Atcriticalstate, $q=Mp'$ whereMistheslopeofstraightlinepassingthroughoriginin $P'-q$ plane.

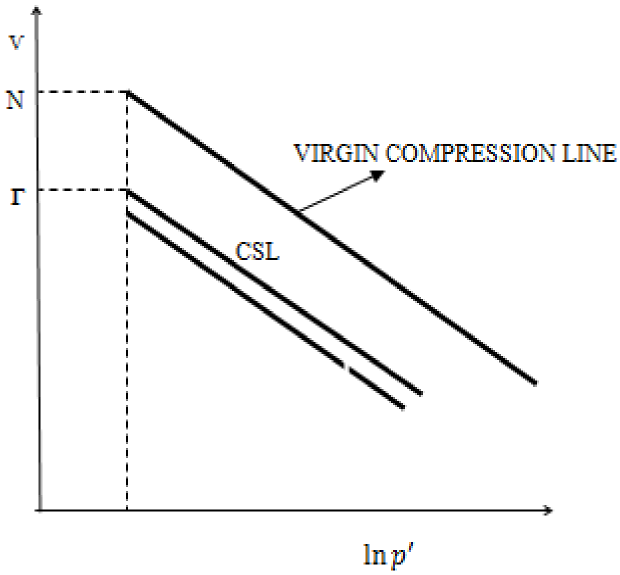


Fig.8LocationofCSLwithrespecttovirgincompressionline

Further,atcriticalstate,

$v=\Gamma-\lambda\ln p'$

Where, Γ is the specific volume of swelling line at unit pressure.ForcamclayModel, equationofCSLisas:

$\Gamma=N-(\lambda-\kappa)$

Andformodified camclayModel,relationship betweentheseparametersisas:

$\Gamma=N-(\lambda-\kappa)\ln 2$

*3.* YIELDFUNCTIONS:

Theyieldfunctionofcamclayandmodifiedcamclaymodelsaredetermined fromfollowingequations:Forcamclaymodel:

Formodifiedcamclaymodel:

$q+Mp'\ln\frac{p'}{p'_o}=0$

$\frac{q^2}{p'^2}+M^2\left(1-\frac{p'_0}{p'}\right)=0$

In the$p^{'} - q$space, the CC yield surface is a logarithmic curve while for MCC , the yield surface plots as an ellipticalcurve as shown in Fig. 9.The parameter $p'_0$(known as yield stress or preconsolidation pressure) controls the size of yield surface.The parameter M is the slope of CSL in$p^{'} - q$plane. A characteristic of CSL is that it intersects the yield surface at a point atwhichthemaximumvalue ofqisobtained.
In three dimensional space defined by$v - p^{'} - q$, the yield surface for CC and MCC formulation is known as state boundarysurfacewhichisshowninFig.10.

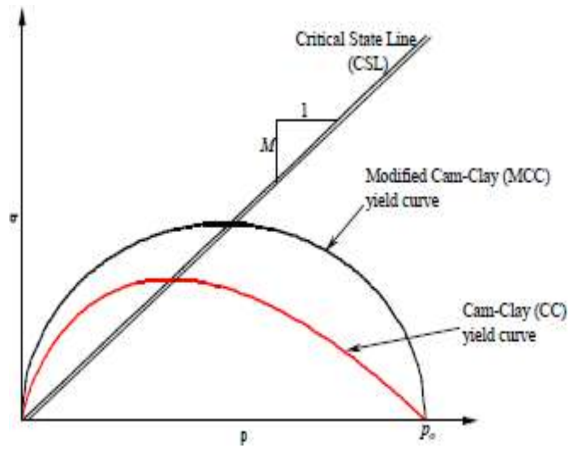


Fig.9Yield surfaceofCamclayandModifiedCamclayModel

This model is more suitable to describe deformationsthan failure especially for normally consolidated soils. Thesemodelsfindapplicationsinembankmentsandfoundations.
Despite some success in modifying the standard cam clay in1980'sYu(1995,1998)found somelimitationsofthismodel:
1. Yieldsurfacesadoptedinmanycriticalstatemodelssignificantlyoverestimatefailurestressesondryside.
2.
3. Sincethemodelisbasedonassociatedflow rule,itisunabletopredictthepeakindeviatorstressbeforecriticalstateisapproachedinundrainedtests.
4. Due to inability to predict observed softening and dilatancyof dense sands, this model does not work well for modelinggranularmaterials.

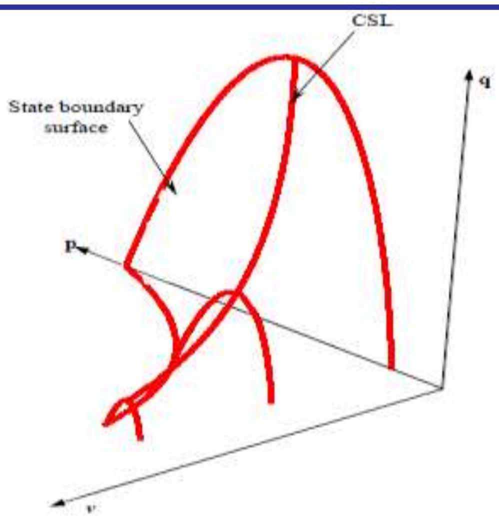


Fig. 10Stateboundarysurfaceofmodifiedcamclay

APPLICATIONSOFCONSTITUTIVEMODELS:
Various types of geotechnical problems can be analyzed usingthe constitutive models. The following types of geotechnicalapplicationshave been found(Duncan1994).

- Soilstructureinteraction
- Soilreinforcementandanchorage.
- Tunnels
- Dams
- Embankments
- Settlementsduetofluidextraction.
- Naturalandunbracedcutslopes

Thus it has been possible to analyze and predict thebehaviorofanytypeofcomplexsoilstructureandsoilstructureinteractionproblems.Thesemodelsfindgreatadvantageinplaceswherethereisaninteractionbetweenstresses and soilvolume changes plays a dominant role asshown inFig.11.

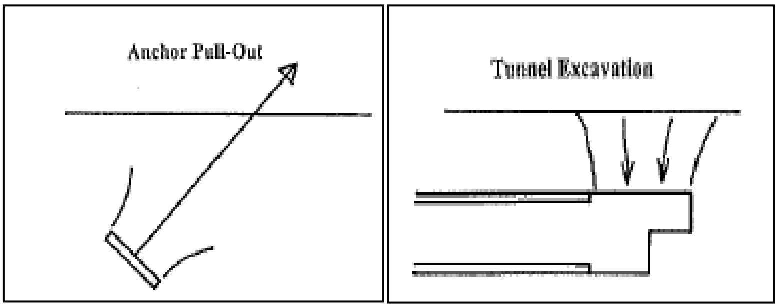


Fig.11Structuresinwhichsoilbehaviorplaysanimportantrole

Another type of problem depicting the importanceof soil behavior is shown in Fig. 12.Due to filling of waterreservoir,theupstreamshellofrockfilldammaycollapse

duetowetting.Thisproducesdisplacementsofcrestwhichcanbebetteranalyzedbymodelingofsoilbehavior.

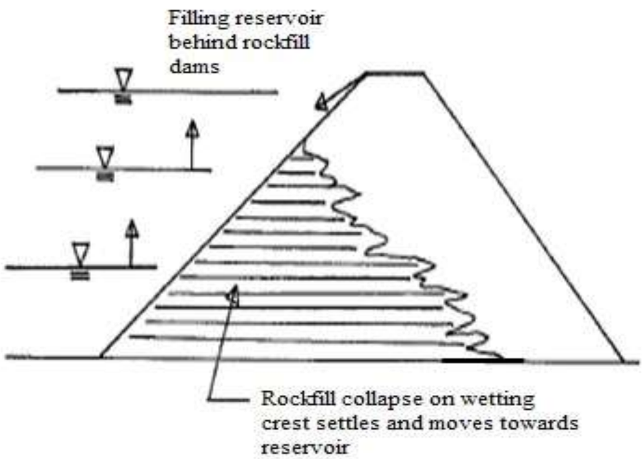


Fig.12Fillingofwaterreservoirbehindarockfilldamwiththeresultingcrestdisplacementduetowettingandcollapseofrockfillmaterialinupstreamshell.

Also, since most of the embankments have beenbuiltonsoftsoils,theconstitutivelawsemployedtodescribe their behavior are: linear and non linear elasticity,elastoplasticitywithoutstrainhardening;elastoplasticitywithstrainhardeningandelastoplasticity.Whenthe embankmentisdescribedbyfiniteelements,themostwidespreadconstitutivelawisisotropiclinearelasticity(55%), followed by perfect elastoplasticity (36%) and non-linearelasticity(9%).
The constitutive models used for describe soil behavior incase of tunnels includes the following types of laws: linearandnon-linearelasticity,elastoplasticitywithoutstrainhardening; elastoplasticity with strain hardening and

elasto-viscoplasticity.Generallyspeaking,themostwidespreadconstitutivelawistheMohr-CoulombperfectelastoplasticityModelwithisotropiclinearelasticity.Amongtheelastoplasticlawswithstrainhardening,theCam-clayModelsremainsthemostwidelyused.

## REFRENCES:


[1] Adachi, T and Okano, M. (1974), A constitutive equation fornormallyconsolidatedclay,soilAndFoundations,volume14,No.4,pp 55-73

[2] Brinkgreve, R.B.J, and Veemar, P.A. (1997), PLAXIS finiteelement code for soil and rock Analysis –version 7, Balkema,Rotterdam.

[3] Burland,JB.(1965),Theyieldinganddilationofclaycorrespondence, Geotechnique, Volume 15, No. 2, pp 211-219.

[4] Chen, CF. (1985), Mechanics of Geomaterials , InZ. Bazant,editorJohnWiley&Sons Ltd.Pg65-86.

[5] Duncan, JM and Chang, CY. (1970), Nonlinear analysis ofstress and strain in soils, ASCE Journal of the Soil MechanicsAnd FoundationDivisionVolume 96,pp1629-1653.

[6] Gens,A.,Potts,D.M.(1988),Criticalstatemodelsincomputationalgeomechanics,EngineeringComputations.Volume5, pp178 – 197.

[7] Mestat, PH, Bourgeois, E and Riou, Y. (2004), NumericalmodellingofembankmentsandUndergroundworks,ComputersandGeotechnics,Volume 31,pp227-236.

[8] Mitchell, JK. (1976),Fundamentals of Soils Behavior, JohnWiley&Sons.

[9] Mroz,Z,(1967),Onthedescriptionofworkhardening,Journal of Mechanics and physics of solids, volume 15, pp163-175

[10] Ogden, RW. (1972), Large deformation isotropic elasticity onthecorrelationoftheoryand

[11] experiment for incompressible rubberlike solids, ProceedingsoftheRoyal Societyof

[12] LondonA.