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Square Difference Prime Labeling for Some Cycle Related Graphs

Sunoj B S[1], Mathew Varkey T K[2]
[1] Department of Mathematics, Government Polytechnic College, Attingal, Kerala, India
[2] Department of Mathematics, T K M College of Engineering, Kollam, Kerala, India

ABSTRACT

Abstract:
Square difference prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the squares of the labels of the incident vertices .The greatest common incidence number of a vertex (**gcin**) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is one, then the graph admits square difference prime labeling. Here we investigate; some cycle related graphs for square difference prime labeling.
Keywords: Graph labeling, square difference, prime labeling, prime graphs, cycle related graphs .

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I. INTRODUCTION

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.
A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4] . In [5] , we introduced the concept, square difference prime labeling and proved that some snake graphs admit this kind of labeling. In [6], [7] and [8] we extended our study and proved that the result is true for some path related graphs, some planar graphs, fan graph, helm graph, umbrella graph, gear graph, friendship and wheel graph. In this paper we investigated square difference prime labeling of some cycle related graphs.
Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection
f : V(G) → {0,1,2,------------------, p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{sdp}^* : E(G) → set of natural numbers N by $f_{sdp}^*(uv) = |f(u)^2 - f(v)^2|$. The induced function f_{sdp}^* is said to be a square difference prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.
Definition 2.2 A graph which admits square difference prime labeling is called a square difference prime graph.
Definition 2.3 Let V(G) and X(G) denote the vertex set and the edge set of G, respectively. The middle graph M(G) of G whose vertex set is V(G) union X(G) where two vertices are adjacent if and only if
(i) They are adjacent edges of G or
(ii) One is a vertex and other is an edge incident with it.
Definition 2.4 The total graph T(G) of G is the graph whose vertex set is V(G) union X(G) where two vertices are adjacent if and only if
(i) They are adjacent edges of G or
(ii) One is a vertex and other is an edge incident with it.
(iii) They are adjacent vertices of G
Definition 2.5 Let G = (V,E) be a simple graph and G' = (V',E') be another copy of graph G. Join each vertex v of G to the corresponding vertex v' of G' by an edge. The new graph thus obtained is the 2- tuple graph of G.

2-tuple graph of G is denoted by $T^2(G)$. Further if G = (p; q) then $V\{T^2(G)\}$ = 2p and $E\{T^2(G)\}$ = 2q+p.

Definition 2.6 The shadow graph D_2 (G) of a connected graph G is constructed by taking two copies of G say G_1 and G_2 join each vertex v in G_1 to the neighbors of the corresponding vertex u in G_2.

Definition 2.7 In a pair of cycles C_n, i^{th} vertex of a cycle C' is joined with $(i+1)^{th}$ vertex of a cycle C'' and first vertex of C' is joined with the nth vertex of C'' ,the resulting graph is denoted by [Z – (C_n)].

Theorem 2.1 Cycle C_n admits square difference prime labeling.

Proof: Let G = C_n and let $v_1,v_2,$----------------,v_n are the vertices of G.

Here |V(G)| = n and |E(G)| = n

Define a function f : V → {0,1,2,-----------------,n-1 } by

$f(v_i) = i-1$, i = 1,2,------,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$f^*_{sdp}(v_i\ v_{i+1})$ = 2i-1, i = 1,2,------------,n-1

$f^*_{sdp}(v_1\ v_n)$ = $(n-1)^2$.

Clearly f^*_{sdp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{sdp}(v_1\ v_2), f^*_{sdp}(v_1\ v_n)$

= gcd of $\{1, (n-1)^2\}$= 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{sdp}(v_i\ v_{i+1}), f^*_{sdp}(v_{i+1}\ v_{i+2})$

= gcd of { 2i-1, 2i+1)}

= gcd of {2, 2i-1} = 1, i = 1,2,------------,n-2

gcin of (v_n) = gcd of $\{f^*_{sdp}(v_{n-1}\ v_n), f^*_{sdp}(v_1\ v_n)$

= gcd of $\{2n-3, (n-1)^2\}$

= gcd of { n-1, 2n-3}

= gcd of {n-2, n-1} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence C_n , admits square difference prime labeling.

Theorem 2.2 2-tuple graph of cycle C_n admits square difference prime labeling, when n is odd.

Proof: Let G = $T^2(C_n)$ and let $v_1,v_2,$----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and | E(G) | = 3n

Define a function f : V → {0,1,2,-----------------,2n-1 } by

$f(v_i) = i-1$, i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$f^*_{sdp}(v_i\ v_{i+1})$ = 2i-1 i = 1,2,------------,2n-1

$f^*_{sdp}(v_i\ v_{2n-i+1})$ = (2n-1)(2n-2i+1) i = 1,2,------------,n-1

$f^*_{sdp}(v_1\ v_n)$ = $(n-1)^2$.

$f^*_{vsndp}(v_{n+1}\ v_{2n})$ = (3n-1)(n-1).

Clearly f^*_{sdp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{sdp}(v_1\ v_2), f^*_{sdp}(v_1\ v_{2n})$

= gcd of $\{1, (2n-1)^2\}$= 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{sdp}(v_i\ v_{i+1}), f^*_{sdp}(v_{i+1}\ v_{i+2})$

= gcd of { 2i-1, 2i+1)}

= gcd of {2, 2i-1} = 1, i = 1,2,------------,2n-2

gcin of (v_{2n}) = gcd of $\{f^*_{sdp}(v_{2n-1}\ v_{2n}), f^*_{sdp}(v_1\ v_{2n})$

= gcd of $\{4n-3, (2n-1)^2\}$

= gcd of { 2n-1, 4n-3}

= gcd of {2n-2, 2n-1} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $T^2(C_n)$, admits square difference prime labeling.

Theorem 2.3 The middle graph of cycle C_n admits square difference prime labeling.

Proof: Let G = M(C_n) and let $v_1,v_2,$----------------,v_{2n} are the vertices of G.

Here | V(G) | = 2n and |E(G)| = 3n

Define a function f : V → {0,1,2,-----------------,2n-1 } by

$f(v_i) = i-1$, i = 1,2,------,2n

Clearly f is a bijection

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$f^*_{sdp}(v_i\ v_{i+1})$ = 2i-1, i = 1,2,-----------,2n-1
$f^*_{sdp}(v_{2i}\ v_{2i+2})$ = $8i$, i = 1,2,-----------,n-1
$f^*_{sdp}(v_2\ v_{2n})$ = 4n(n-1).
$f^*_{sdp}(v_1\ v_{2n})$ = $(2n-1)^2$.
Clearly f^*_{sdp} is an injection.
gcin of (v_1) = gcd of {$f^*_{sdp}(v_1\ v_2)$, $f^*_{sdp}(v_1\ v_{2n})$ = 1.
gcin of (v_{i+1}) = gcd of {$f^*_{sdp}(v_i\ v_{i+1})$, $f^*_{sdp}(v_{i+1}\ v_{i+2})$
= 1, i = 1,2,------------,2n-2
gcin of (v_{2n}) = gcd of {$f^*_{sdp}(v_{2n-1}\ v_{2n})$, $f^*_{sdp}(v_1\ v_{2n})$
= 1.
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence M(C_n) , admits square difference prime labeling.

Theorem 2.4 The total graph of cycle C_n admits square difference prime labeling.
Proof: Let G = T(C_n) and let v_1,v_2,----------------,v_{2n} are the vertices of G.
Here |V(G) | = 2n and |E(G)| = 4n
Define a function f : V → {0,1,2,-----------------,2n-1 } by
f(v_i) = i-1 , i = 1,2,------,2n
Clearly f is a bijection
For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows
$f^*_{sdp}(v_i\ v_{i+1})$ = 2i-1, i = 1,2,-----------,2n-1
$f^*_{sdp}(v_{2i}\ v_{2i+2})$ = $8i$, i = 1,2,-----------,n-1
$f^*_{sdp}(v_{2i-1}\ v_{2i+1})$ = $8i-4$, i = 1,2,-----------,n-1
$f^*_{sdp}(v_1\ v_{2n})$ = $(2n-1)^2$.
$f^*_{sdp}(v_2\ v_{2n})$ = 4n(n-1).
$f^*_{sdp}(v_1\ v_{2n-1})$ = $(2n-2)^2$.
Clearly f^*_{sdp} is an injection.
gcin of (v_1) = gcd of {$f^*_{sdp}(v_1\ v_2)$, $f^*_{sdp}(v_1\ v_{2n})$ = 1.
gcin of (v_{i+1}) = gcd of {$f^*_{sdp}(v_i\ v_{i+1})$, $f^*_{sdp}(v_{i+1}\ v_{i+2})$
= 1, i = 1,2,------------,2n-2
gcin of (v_{2n}) = gcd of {$f^*_{sdp}(v_{2n-1}\ v_{2n})$, $f^*_{sdp}(v_1\ v_{2n})$
= 1.
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence T(C_n) , admits square difference prime labeling.

Theorem 2.5 The shadow graph of cycle C_n admits square difference prime labeling, when n is greater than 3.
Proof: Let G = $D_2(C_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 4n
Define a function f : V → {0,1,2,-----------------,2n-1 } by
f(v_i) = i-1 , i = 1,2,------,2n
Clearly f is a bijection
For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows
$f^*_{sdp}(v_i\ v_{i+1})$ = 2i-1, i = 1,2,-----------,2n-1
$f^*_{sdp}(v_i\ v_{n+i+1})$ = (n+1)(n+2i-1) i = 1,2,-----------,n-1
$f^*_{sdp}(v_{i+1}\ v_{n+i})$ = (n-1)(n+2i-1) i = 1,2,-----------,n-1
$f^*_{sdp}(v_1\ v_n)$ = $(n-1)^2$.
$f^*_{sdp}(v_1\ v_{2n})$ = $(2n-1)^2$.
$f^*_{sdp}(v_{n+1}\ v_{2n})$ = (3n-1)(n-1).
Clearly f^*_{sdp} is an injection.
gcin of (v_1) = gcd of {$f^*_{sdp}(v_1\ v_2)$, $f^*_{sdp}(v_1\ v_{2n})$ = 1.
gcin of (v_{i+1}) = gcd of {$f^*_{sdp}(v_i\ v_{i+1})$, $f^*_{sdp}(v_{i+1}\ v_{i+2})$
= 1, i = 1,2,------------,2n-2
gcin of (v_{2n}) = gcd of {$f^*_{sdp}(v_{2n-1}\ v_{2n})$, $f^*_{sdp}(v_1\ v_{2n})$
= 1.
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $D_2(C_n)$, admits square difference prime labeling.

Theorem 2.6 The graph [Z- (C_n)] admits square difference prime labeling when n is greater than 3.

Proof: Let G = [Z- (C_n)] and let v_1,v_2,----------------,v_{2n} are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 3n
Define a function f : V → {0,1,2,-----------------,2n-1 } by
f(v_i) = i-1 , i = 1,2,------,2n
Clearly f is a bijection
For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(v_i\, v_{i+1})$ = 2i-1, i = 1,2,-----------,2n-1
$f_{sdp}^*(v_i\, v_{n+i+1})$ = (n+1)(n+2i-1), i = 1,2,-----------,n-1
$f_{sdp}^*(v_1\, v_n)$ = $(n-1)^2$.
$f_{sdp}^*(v_{n+1}\, v_{2n})$ = (n-1)(3n-1).

Clearly f_{sdp}^* is an injection.

gcin of (v_1) = gcd of {$f_{sdp}^*(v_1\, v_2)$, $f_{sdp}^*(v_1\, v_n)$ = 1.
gcin of (v_{i+1}) = gcd of {$f_{sdp}^*(v_i\, v_{i+1})$, $f_{sdp}^*(v_{i+1}\, v_{i+2})$
= 1, i = 1,2,------------,2n-2
gcin of (v_{2n}) = gcd of {$f_{sdp}^*(v_{2n-1}\, v_{2n})$, $f_{sdp}^*(v_{n+1}\, v_{2n})$, $f_{sdp}^*(v_{n-1}\, v_{2n})$}
= gcd of {4n-3, (n-1)(3n-1), (n+1)(3n-3)}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence [Z- (C_n)] , admits square difference prime labeling.

Theorem 2.7 Duplication of a vertex in cycle C_n admits square difference prime labeling, if (n-2) $\not\equiv$ 0(mod3).

Proof: Let G be the graph and let v_1,v_2,----------------,v_{n+1} are the vertices of G.
Here |V(G) | = n+1 and | E(G) | = n+2
Define a function f : V → {0,1,2,-----------------,n } by
f(v_i) = i-1 , i = 1,2,------,n+1
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(v_i\, v_{i+1})$ = 2i-1, i = 1,2,------------,n
$f_{sdp}^*(v_1\, v_n)$ = $(n-1)^2$.
$f_{sdp}^*(v_2\, v_{n+1})$ = $(n-1)^2$.

Clearly f_{sdp}^* is an injection.

gcin of (v_1) = gcd of {$f_{sdp}^*(v_1\, v_2)$, $f_{sdp}^*(v_1\, v_n)$
= gcd of {1, $(n-1)^2$}= 1.
gcin of (v_{i+1}) = gcd of {$f_{sdp}^*(v_i\, v_{i+1})$, $f_{sdp}^*(v_{i+1}\, v_{i+2})$
= 1, i = 1,2,------------,n-1
gcin of (v_{n+1}) = gcd of {$f_{sdp}^*(v_n\, v_{n+1})$, $f_{sdp}^*(v_2\, v_{n+1})$
= gcd of {2n-1, n^2-1}
= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence G , admits square difference prime labeling.

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