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On intuitionistic fuzzy β generalized closed sets

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ABSTRACT
In this paper, we have introduced the notion of intuitionistic fuzzy β generalized closed sets, and investigated some of their properties and characterizations

KEYWORDS:*Intuitionistic fuzzy topology, intuitionistic fuzzyβ closed sets, intuitionistic fuzzy β generalized closed sets.*

I. Introduction

The concept of fuzzy sets was introduced byZadeh [12] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we have introduced the notion of intuitionistic fuzzy β generalized closed sets, and investigated some of their properties and characterizations.

II. Preliminaries

Definition 2.1: [1**]** An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \epsilon X$ to the set A respectively, and$0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.
An intuitionistic fuzzy set A in X is simply denoted by $A=\langle x, \mu_A, \nu_A\rangle$instead of denoting$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

Definition 2.2: [1**]** Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \epsilon X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle: x \in X\}$,
(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle: x \in X\}$,
(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0\sim = \langle x, 0, 1\rangle$ and $1\sim =\langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [3**]**An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSsin X satisfying the following axioms:

(i) $0\sim, 1\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X,τ) is called *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[5] An IFS $A = \langle x, \mu_{A,} \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) intuitionistic fuzzy β closed set (IFβCS for short) if int(cl(int(A))) $\subseteq$ A,

(ii) intuitionistic fuzzy β open set (IFβOS for short) if A $\subseteq$ cl(int(cl(A))).

Definition 2.5: [6]Let A be an IFS in an IFTS (X,τ). Then the β-interior and β-closure of A are defined as

βint(A) = $\cup$ {G / G is an IFβOS in X and G $\subseteq$ A}.

βcl(A) = $\cap${K / K is an IFβCS in X and A $\subseteq$ K}.

Note that for any IFS A in (X,τ), we have $\beta cl(A^c) = (\beta int(A))^c$ and$\beta int(A^c) = (\beta cl(A))^c$.

Result 2.6: Let A be an IFS in (X,τ), then

(i) βcl(A)$\supseteq$ A $\cup$ int(cl(int(A)))

(ii) βint(A) $\subseteq$ A $\cap$ cl(int(cl(A)))

Proof: (i)Now int(cl(int(A))) $\subseteq$ int(cl(int(βcl(A))) $\subseteq$ βcl(A), since A $\subseteq$ βcl(A) and βcl(A) is an IFβCS. Therefore A $\cup$ int(cl(int(A))) $\subseteq$ βcl(A).
(ii) can be proved easily by taking complement in (i).

III. Intuitionistic fuzzyβgeneralized closed sets

In this section we have introduced intuitionistic fuzzy β generalized closed sets and studied some of their properties.

Definition 3.1: An IFS A in an IFTS (X,τ) is said to be an *intuitionistic fuzzy β generalized closed set* (IFβGCS for short) if βcl(A) $\subseteq$ U whenever A$\subseteq$ U and U is an IFβOS in (X,τ).
The complement A^c of an IFβGCS A in an IFTS (X,τ) is called an intuitionistic fuzzy β generalized open set (IFβGOS in short) in X.

The family of all IFβGCSs of an IFTS (X,τ) is denoted by IFβGC(X).

Example 3.2: Let X = {a, b} and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ be an IFS in X.
Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \epsilon[0,1], \mu_b \epsilon [0,1], \nu_a\epsilon[0,1], \nu_b\epsilon [0,1] / 0\leq \mu_a+\nu_a\leq 1$ and$0 \leq \mu_b+\nu_b\leq 1\}$.
We have A $\subseteq$ G. As βcl(A) = A,βcl(A) $\subseteq$ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X.

Theorem 3.3: EveryIFCSin (X, τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Let A be an IFCS. Thereforecl(A) = A. Let A $\subseteq$ U and U be an IFβOS. Since βcl(A) $\subseteq$ cl(A) =A $\subseteq$ U, we have βcl(A) $\subseteq$ U. Hence A is an IFβGCS in (X, τ).

Example 3.4: Let X = {a, b} and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT onX. Let $A = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ be an IFS in X.
Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a\epsilon[0,1], \mu_b \epsilon [0,1], \nu_a\epsilon[0,1], \nu_b\epsilon [0,1] / 0\leq \mu_a+\nu_a\leq 1$ and $0 \leq \mu_b+\nu_b\leq 1\}$.
We have A $\subseteq$ G. As βcl(A) = A,βcl(A) $\subseteq$ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFCS, sincecl(A) = $G^c \neq$ A.

Theorem 3.5: EveryIFRCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Let A be an IFRCS[10]. Since every IFRCS is an IFCS [9], by Theorem 3.3, A is an IFβGCS.

Example 3.6: Let X = {a, b} and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$.Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT onX. Let $A = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ be an IFS in X.
Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a\epsilon [0,1], \mu_b \epsilon [0,1], \nu_a\epsilon [0,1], \nu_b\epsilon [0,1] / 0 \leq \mu_a+\nu_a\leq 1$ and $0 \leq \mu_b+\nu_b\leq 1\}$.

We have A ⊆ G. As βcl(A) = A,βcl(A) ⊆ G, where G is an IFβOS inX. This implies that A is an IFβGCS in X, but not an IFRCS, sincecl(int(A)) = cl(0∼) = 0∼ ≠ A.

Theorem 3.7: EveryIFSCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFSCS [5]. Let A ⊆ U and U be an IFβOS. Sinceβcl(A) ⊆scl(A) = A and A ⊆ U, by hypothesis, we haveβcl(A) ⊆ U. Hence A is an IFβGCS.

Example 3.8: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩.Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ be an IFS in X.
Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.
We have A ⊆ G.As βcl(A) = A, βcl(A) ⊆ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFSCS, since int(cl(A)) = int(G^c) = G ⊈ A.

Theorem 3.9: EveryIFαCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFαCS [5]. Let A ⊆ U and U be an IFβOS. Sinceβcl(A) ⊆ αcl(A) =A and A ⊆ U, by hypothesis, we have βcl(A) ⊆ U. Hence A is an IFβGCS.

Example 3.10: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Thenτ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ be an IFS in X.
Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.
We have A ⊆ G. As βcl(A) = A, βcl(A) ⊆ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFαCS, since cl(int(cl(A))) = cl(int(G^c)) = cl(G) = G^c⊈ A.

Theorem 3.11: EveryIFPCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFPCS [5]. LetA ⊆ U and U be an IFβOS.Sinceβcl(A) ⊆ pcl(A) = Aand A⊆ U, by hypothesis, we have βcl(A) ⊆ U. Hence A is an IFβGCS.

Example 3.12: Let X = {a, b} and G = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩ be an IFS in X.
Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /μ_b <0.6 whenever $\mu_a \geq 0.5$, μ_a <0.5 whenever $\mu_b \geq 0.6$,$0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.
NowA⊆ 1∼. As βcl(A) = 1∼ ⊆ 1∼, we have A is an IFβGCS in X, but not an IFPCS since cl(int(A)) = cl(G) =1∼ ⊈ A.

Remark 3.13: Every IFGCS and every IFβGCS are independent to each other.

Example 3.14: Let X = {a, b} and G_1 = ⟨x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)⟩ and G_2 = ⟨x,(0.3_a, 0.1_b),(0.7_a, 0.8_b)⟩. Then τ = {0∼, G_1, G_2,1∼} is an IFT on X. Let A = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ be an IFS in X. Then A ⊆ G_1 andcl(A) = G_1^c⊆ G_1.Therefore A is an IFGCS in X.
Now IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_a \geq 0.5$ and $\mu_b \geq 0.5$ or$\mu_a < 0.3$ and $\mu_b < 0.1$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.
Since A ⊆ G_1 where G_1 is an IFβOS in X, but βcl(A) =⟨x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)⟩ ⊈ A, A is not an IFβGCS.

Example 3.15: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ be an IFS in X.
Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.
We have A ⊆ G. As βcl(A) = A,βcl(A) ⊆ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFGCS in X, sincecl(A) = G^c⊈ G.

Theorem 3.16: EveryIFβCSin (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFβCS [5] then βcl(A) = A. Let A ⊆ U and U be an IFβOS. Thenβcl(A) ⊆ U, by hypothesis. Therefore A is an IFβGCS.

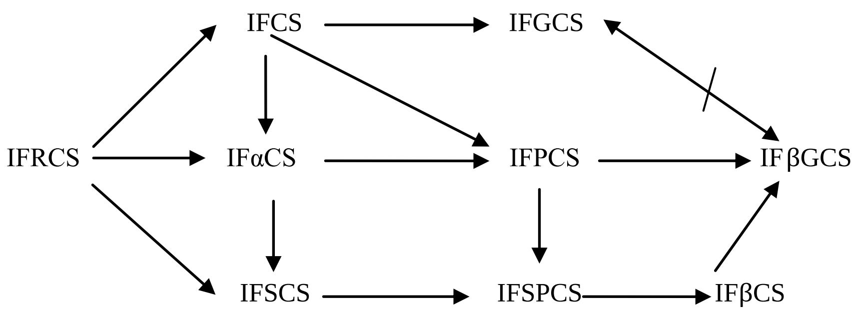
Example 3.17: Let X = {a, b} and G = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩.Thenτ = {0∼, G, 1∼} is an IFT on X. LetA = ⟨x, (0.5_a, 0.8_b), (0.5_a, 0.2_b)⟩ be an IFS in X.
Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / provided μ_b <0.7 whenever$\mu_a \geq 0.5$,μ_a <0.5 whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Now A ⊆ 1∼ and βcl(A) = 1∼ ⊆ 1 ∼. This implies that A is an IFβGCS in X,but not an IFβCS,sinceint(cl(int(A))) = int(cl(G)) = int(1 ∼) = 1 ∼ ⊈ A.

Theorem 3.18: Every IFSPCS in (X,τ) is an IFβGCS in (X,τ) but not conversely.
Proof: Assume A is an IFSPCS[11]. Since every IFSPCS is an IFβCS [7], by Theorem 3.16, A is an IFβGCS.

Example 3.19: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ be an IFS in X.
Then, IFβC(X) = {0∼, 1∼, $\mu_a \epsilon$ [0,1], $\mu_b \epsilon$ [0,1], $\nu_a \epsilon$ [0,1], $\nu_b \epsilon$ [0,1] /0≤ $\mu_a+\nu_a$≤1 and 0 ≤ $\mu_b+\nu_b$≤ 1}.
Here A is an IFβCS in X. As int(cl(int(A))) = 0∼ ⊆ A. Therefore A is an IFβGCS in X.
Since IFPC(X) = {0∼, 1∼, $\mu_a \epsilon$ [0,1], $\mu_b \epsilon$ [0,1], $\nu_a \epsilon$ [0,1], $\nu_b \epsilon$ [0,1] /either $\mu_b \geq$ 0.6 or μ_b < 0.4 whenever $\mu_a \geq$ 0.5, 0≤ $\mu_a+\nu_a$≤1 and 0 ≤ $\mu_b+\nu_b$ ≤ 1}.
But A is not an IFSPCS in X, as we cannot find any IFPCS B such that int(B) ⊆ A ⊆ B in X.

In the following diagram, we have provided relations between various types of intuitionistic fuzzy closedness.



The reverse implications are not true in general in the above diagram.

Remark 3.20: The union of any two IFβGCS is not an IFβGCS in general as seen from the following example.

Example 3.21: Let X = {a, b} and τ = {0∼, G_1, G_2, 1∼} whereG_1 = ⟨x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)⟩and G_2 = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩.Then the IFSsA = ⟨x, (0.6_a, 0.5_b), (0.4_a, 0.3_b)⟩ and B = ⟨x, (0.4_a, 0.8_b), (0.4_a, 0.2_b)⟩are IFβGCSs in (X,τ) but A ∪ B is not an IFβGCS in (X,τ).
Then IFβC(X) = {0∼, 1∼, $\mu_a \epsilon$ [0,1], $\mu_b \epsilon$ [0,1], $\nu_a \epsilon$[0,1], $\nu_b \epsilon$ [0,1] / provided μ_b <0.7 whenever$\mu_a \geq$0.6, μ_a <0.6 whenever $\mu_b \geq$0.7, 0≤ $\mu_a+\nu_a$≤1 and 0 ≤ $\mu_b+\nu_b$≤ 1}.
As βcl(A) = A, we have A is an IFβGCS in X and βcl(B) = B, we haveB is an IFβGCS in X. NowA ∪ B = ⟨x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)⟩ ⊆ G_1, where G_1 is an IFβOS, but βcl(A ∪ B) = 1∼ ⊈ G_1.

Theorem 3.22: Let (X,τ) be an IFTS. Then for every A ϵ IFβGC(X) and for every B ϵ IFS(X), A ⊆ B ⊆ βcl(A) ⇒ B ϵ IFβGC(X).
Proof: Let B ⊆ U and U be an IFβOS. Then since, A ⊆ B, A ⊆ U. By hypothesis, B ⊆ βcl(A). Therefore βcl(B) ⊆ βcl(βcl(A)) = βcl(A) ⊆ U, since A is an IFβGCS. Hence B ϵ IFβGC(X).

Theorem 3.23: An IFS A of an IFTS (X,τ) is an IFβGCS if and only if$A_q{}^c$ F ⇒ βcl(A) $_q{}^c$ F for every IFβCS F of X.
Proof: (Necessity): Let F be an IFβCS and A $_q{}^c$ F, then A ⊆ F^c [9], where F^c is an IFβOS. Then βcl(A) ⊆ F^c, by hypothesis. Hence again [9] βcl(A) $_q{}^c$ F.
Sufficiency: Let U be an IFβOS such that A ⊆ U. Then U^c is an IFβCS and A ⊆ $(U^c)^c$. By hypothesis, $A_q{}^c$ U^c⇒ βcl(A) $_q{}^c$ U^c. Hence by [9], βcl(A) ⊆ $(U^c)^c$ = U. Therefore βcl(A) ⊆ U. Hence A is an IFβGCS.

Theorem 3.24: Let (X,τ) be an IFTS. Then every IFS in (X,τ) is an IFβGCS if and only if IFβO(X) = IFβC(X).

Proof: (Necessity): Suppose that every IFS in (X,τ) is an IFβGCS. Let U ϵ IFβO(X), and by hypothesis, βcl(U) $\subseteq$ U $\subseteq$ βcl(U). This implies βcl(U) = U. Therefore U ϵ IFβC(X). Hence IFβO(X) $\subseteq$ IFβC(X). Let Aϵ IFβC(X), then $A^c \epsilon$ IFβO(X) $\subseteq$ IFβC(X). That is, $A^c \epsilon$ IFβC(X). Therefore Aϵ IFβO(X). Hence IFβC(X) $\subseteq$ IFβO(X). Thus IFβO(X) = IFβC(X).

Sufficiency: Suppose that IFβO(X) = IFβC(X). Let A $\subseteq$ U and U be an IFβOS. By hypothesis βcl(A) $\subseteq$ βcl(U) =U, since U ϵ IFβC(X). Therefore A is an IFβGCS in X.

Theorem 3.25: If A is an IFβOS and an IFβGCS in (X,τ) then A is an IFβCS in (X,τ).

Proof: Since A $\subseteq$ A and A is an IFβOS, by hypothesis, βcl(A) $\subseteq$ A. But A $\subseteq$ βcl(A). Therefore βcl(A) = A. Hence A is an IFβCS.

Theorem 3.26: Let A be an IFβGCS in (X,τ) and $p_{(\alpha,\beta)}$be an IFP in X such that $\text{int}(p_{(\alpha,\beta)})_q\beta$cl(A), then $\text{int}(\text{cl}(\text{int}(p_{(\alpha,\beta)})))\ _q$ A.

Proof: Let A be an IFβGCSand let $(\text{int}(p_{(\alpha,\beta)}))_q\beta$cl(A).

Suppose $\text{int}(\text{cl}(\text{int}(p_{(\alpha,\beta)})))\ _q{}^c$ A, since by [9] A $\subseteq$ $[\text{int}(\text{cl}(\text{int}(p_{(\alpha,\beta)})))]^c$.This implies $[\text{int}(\text{cl}(\text{int}(p_{(\alpha,\beta)})))]^c$ is an IFβOS. Then by hypothesis,

$\beta\text{cl}(A) \subseteq[\text{int}(\text{cl}(\text{int}(p_{(\alpha,\beta)})))]^c$

$= \text{cl}(\text{int}(\text{cl}[(p_{(\alpha,\beta)})]^c.$

$\subseteq \text{cl}(\text{cl}[(p_{(\alpha,\beta)})]^c.$

$= \text{cl}[(p_{(\alpha,\beta)})]^c.$

$=(\text{int}(p_{(\alpha,\beta)}))^c$. This impliesint$(p_{(\alpha,\beta)})_q{}^c\beta$cl(A),which is a contradiction to the hypothesis. Hence $\text{int}(\text{cl}(\text{int}(p_{(\alpha,\beta)})))\ _q$ A.

Theorem 3.27: Let F $\subseteq$ A $\subseteq$ X where A is an IFβOS and an IFβGCS in X. Then F is an IFβGCS in A if and only if F is an IFβGCS in X.

Proof: Necessity: Let U be an IFβOS in X and F $\subseteq$ U. Also let F be an IFβGCS in A. Then clearly F $\subseteq$ A $\cap$ U and A $\cap$ U is an IFβOS in A. Hence the β closure of F in A, βcl_A(F) $\subseteq$A $\cap$ U. By Theorem 3.25, A is an IFβCS. Therefore βcl(A) = A and the β closure of F in X, βcl(F) $\subseteq$ βcl(F) $\cap$ βcl(A) = βcl(F) $\cap$ A = βcl_A(F) $\subseteq$ A $\cap$ U $\subseteq$ U. That is, βcl(F) $\subseteq$ U whenever F $\subseteq$ U. Hence F is an IFβGCS in X.

Sufficiency: Let V be an IFβOS in A such that F $\subseteq$ V. Since A is an IFβOS in X, V is an IFβOS in X. Therefore βcl(F) $\subseteq$ V, since F is an IFβGCS in X. Thus βcl_A(F) =βcl(F) $\cap$ A $\subseteq$ V $\cap$ A $\subseteq$ V. Hence F is an IFβGCS in A.

Theorem 3.28: For an IFS A, the following conditions are equivalent:

(i) A is an IFOS and an IFβGCS

(ii) A is an IFROS

Proof: (i) ⇒ (ii) Let A be an IFOS and an IFβGCS. Then βcl(A) $\subseteq$ A and A $\subseteq$ βcl(A) this implies that βcl(A) = A. Therefore A is an IFβCS, since int(cl(int(A))) $\subseteq$ A. Since A is an IFOS, int(A) = A. Therefore int(cl(A)) $\subseteq$ A. Since A is an IFOS, it is an IFPOS. Hence A $\subseteq$int(cl(A)). Therfore A = int(cl(A)). Hence A is an IFROS.

(ii) ⇒ (i) Let A be an IFROS. Therefore A = int(cl(A)). Since every IFROS in an IFOS and A$\subseteq$ A. This implies int(cl(A)) $\subseteq$ A. That is int(cl(int(A))) $\subseteq$ A. Therfore A is an IFβCS. Hence A is an IFβGCS.

Theorem 3.29: For an IFOS A in (X,τ), the following conditions are equivalent.

(i) A is an IFCS

(ii) A is an IFβGCS and an IFQ-set

Proof: (i) ⇒ (ii) Since A is an IFCS, it is an IFβGCS. Now int(cl(A)) = int(A) = A = cl(A) = cl(int(A)), by hypothesis. Hence A is an IFQ-set[8].

(ii) ⇒ (i) Since A is an IFOS and an IFβGCS, by Theorem 3.28, A is an IFROS. ThereforeA = int(cl(A)) = cl(int(A)) = cl(A), by hypothesis. A is an IFCS.

Theorem3.30: Let (X,τ) be an IFTS, then for every A ∈ IFSPC(X) andfor every B in X, int(A) ⊆ B ⊆ A ⇒B ∈ IFβGC(X).

Proof: Let A be an IFSPCS in X. Then there exists an IFPCS, (say) C such that int(C) ⊆ A ⊆ C. By hypothesis, B ⊆A. Therefore B ⊆ C. Since int(C) ⊆ A, int(C) ⊆int(A) and int(C) ⊆ B, by hypothesis. Thus int(C) ⊆ B ⊆ C and by [5], B ∈ IFSPC(X). Hence by Theorem 3.18,B ∈ IFβGC(X).

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