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Solitons Solutions to Some Evolution Equations by ExtendedTan-Cot Method

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ABSTRACT:
The proposed extended Tan-Cot method is applied to obtain new exact travelling wave solutions to evolution equation. The method is applicable to a large variety of nonlinear partial differential equations, the Fifth-order nonlinear integrable equation, the symmetric regularized long wave equation, the higher-order wave equation of Kdv type, and Benney-Luke equation. The Extended Tan-Cot method seems to be powerful tool in dealing with nonlinear physical models.

Keywords:*Exact solution, extended Tan-Cot method, Fifth-order nonlinear integrable equation, the symmetric regularized long wave equation (SRLW), the higher-order wave equation of Kdv type, and Benney-Luke equation(B-L), nonlinear partial differential equations.*

I. INTRODUCTION

In modern science nonlinear phenomena are one of the most impressive fields of research. Nonlinear phenomena occur in numerous branches of science and engineering, such as, plasma physics, fluid mechanics, gas dynamics, elasticity, relativity, chemical reactions, ecology, optical fiber, solid state physics, biomechanics, etc., all are essentially governed by nonlinear equations. NLEEs are frequently used to illustrate the motion of isolated waves. Since the appearance of solitary wave in natural sciences is expanding every day, it is important to seek for exact traveling wave solutions to NLEEs. The exact solutions to NLEEs help us to provide information about the structure of complex physical phenomena. Therefore, exploration of exact traveling wave solutions to NLEEs turns into an essential task in the study of nonlinear physical phenomena. Travelling waves are the solitary waves, which are localized travelling waves, asymptotically zero at large distances, the periodic waves, which rise or descend from one asymptotic state to another. It is notable to observe that there is no unique method to solve all kind of NLEEs. Methods are proposed to obtain exact travelling wave solutions for a large variety of nonlinear partial differential equations (PDEs) that are the Jacobi elliptic function method [1], the homogenous balance method [2], the modified simple equation method [3], the (G'/G)-expansion method [4], the improved (G'/G) expansion method [5], the truncated Painleve expansion method [6], the homotopy perturbation method [7], the variational method [8], the Backlund transformation [9], the Exp-function method [10], the asymptotic method [11], the non-perturbative method [12], the Hirota's bilinear transformation method [13], the tanh-function method [14], the F-expansion method [15], the generalized Riccati equation [16], the ansatz method [17], the perturbation method [18], the He's semi-inverse variational method [19], the Lie symmetry method [20], the method of integrability [21], and the mapping method [22]. Other methodsare proposed to obtain exact travelling wave solutions such as sine-cosine function method [23], tanh-cothmethod[24], Tan-Cot- function method [25], and sech method [26].In this paper the proposed method is applicable solve to a large variety of nonlinear partial differential equations, theFifth-order nonlinear integrable equation, the symmetric regularized long wave equation, the higher-order wave equation of Kdv type, and Benney-Luke equation.

II. DESCRIPTION OF EXTENDED TAN-COT FUNCTION METHOD

This method proposed by Anwar [27] to obtain new exact travelling wave solutions to the (3+1)-dimensional Kadomtsev-Petviashvili equation and (2+1)-dimensional equation.

For a given nonlinear evolution equation, say, in two variables (1+1) - dimensional

$$P(u, u_t, u_x, u_{xx} \ldots\ldots\ldots) = 0 \quad (1)$$

We seek a travelling wave solution of the form:

$u(x,t) = U(\xi)$, and $\xi = x - \omega t$ (2)

Where ω isconsidered constant. The following chain rule

$$\frac{\partial U}{\partial t} = -\omega \frac{dU}{d\xi}, \quad \frac{\partial U}{\partial x} = k \frac{dU}{d\xi}, \quad \frac{\partial^2 U}{\partial x^2} = k^2 \frac{d^2U}{d\xi^2}$$

converted the PDE Eq.(1), to an ordinary differential equation ODE

$$Q(U, U', U'', U''', \ldots\ldots\ldots) = 0 \quad (3)$$

with Q being another polynomial form of their argument, which will be called the reduced ordinary differential equations of Eq.(3). Integrating Eq.(3) as long as all terms contain derivatives, the integration constants are considered to be zeros in view of the localized solutions. However, the nonzero constants can be used and handled as well. Now finding the traveling wave solutions to Eq.(3) is equivalent to obtaining the solution to the reduced ordinary differential equation Eq.(3).

Introduce the ansatz, the new independent variable

$$T = \tan(\xi) \quad (4)$$

that leads to the change of variables:

$$\frac{dU}{d\xi} = (1 + T^2)\frac{dU}{dT}$$

$$\frac{d^2U}{d\xi^2} = 2T(1 + T^2)\frac{dU}{dT} + (1 + T^2)^2 \frac{d^2U}{dT^2}$$

$$\frac{d^3U}{d\xi^3} = 2(1 + 3T^2)(1 + T^2)\frac{dU}{dT} + 6T(1 + T^2)^2\frac{d^2U}{dT^2} + (1 + T^2)^3\frac{d^3U}{dT^3} \quad (5)$$

The next step is that the solution is expressed in the form

$$U(\xi) = \sum_{i=0}^{m} a_i T^i + \sum_{i=1}^{m} b_i T^{-i} \quad (6)$$

where the parameter m can be found by balancing the highest-order linear term with the nonlinear terms in Eq.(3), and $\omega, a_0, a_1, \ldots, a_m, b_1, \ldots, b_m$ are to be determined. Substituting Eq.(6) into Eq.(3) will yield a set of algebraic equations for $\omega, a_0, a_1, \ldots, a_m, b_1, \ldots, b_m$ because all coefficients of T have to vanish. Having determined these parameters, knowing that m is positive integer in most cases, and using Eq.(6) we obtain analytic solutions $u(x,t)$,in a closed form.

The trigonometric functions can be extended to hyperbolic functions by using the complex form. So that a tanh-function expansion solution generates from a tan function expansion solution for $T = \tan(i\xi) = i\tanh(\xi)$, and a cot-function expansion solution generates from a coth function expansion solution for $T^{-1} = \cot(i\xi) = -i\coth(\xi)$.

III. APPLICATIONS

In this section, we will bring to bear the new tan- cotmethod discussed in Section 2 to theFifth-order nonlinear integrable equation, the symmetric regularized long wave equation, the higher-order wave equation of Kdv type, and Benney-Luke (BL) which are very important in the field of nonlinear mathematical physics.

3.1. Fifth-order nonlinear integrable equation

In this section, we solve the fifth-order nonlinear evolution equation introduced by Wazwaz [28]:

$$u_{ttt} - u_{txxxx} - 4(u_x u_t)_{xx} - 4(u_x u_{xt})_x = 0 \quad (7)$$

In thissection, we will use the extended Tan-Cot method in its standard form aspresented to solve Eq.(3.1). The technique is basedon the a priori assumption that the traveling wave solutionscan be expressed in terms of tan functions.

By means of the method, we first use the wavevariable $\xi = x - \omega t$ that transforms Eq.(7) into an ODE

$$-\omega^2 U''' + U^{(5)} + 4(U'^2)'' + 4(U'U'')' = 0 \quad (8)$$

Integrating Eq.(8) twice with zero constants to get the following ODE;

$$-\omega^2 U' + U''' + 6U'^2 = 0 \quad (9)$$

The next step is that the solution is expressed in the form of Eq.(6)

$$-\omega^2(1+T^2)\frac{dU}{dT}+2(1+3T^2)(1+T^2)\frac{dU}{dT}+6T(1+T^2)^2\frac{d^2U}{dT^2}+(1+T^2)^3\frac{d^3U}{dT^3}$$

$$+6(1+T^2)^2[\frac{dU}{dT}]^2=0 \qquad (10)$$

Now, to determine the parameter m, we balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq.(10) we balance $U^{/2}$ with $U^{///}$, to obtain:
m+3 = (m+1)(m+1) , then m = 1. The Tan-Cot method admits the use of the finite expansion for:

$U=a_0+a_1T+b_1T^{-1}$, $U^{/}=a_1-b_1T^{-2}$, $U^{//}=2b_1T^{-3}$, and $U^{///}=-6b_1T^{-4}$

Substituting $U^{/},U^{//},U^{//},U^{///}$ from Eq.(6) in Eq.(10),

$$-\omega^2(1+T^2)[a_1-b_1T^{-2}]+2(1+3T^2)(1+T^2)[a_1-b_1T^{-2}]+6T(1+T^2)^2 2b_1T^{-3}$$

$$-(1+T^2)^3 6b_1T^{-4}+6(1+T^2)^2[a_1-b_1T^{-2}]^2=0 \qquad (11)$$

Equating the coefficients of Ti i=-4,-2, 0, and 2 for both sides, then a set of nonlinear equations occurred as in the following

$T^{-4}: -6b_1+6b_1^2=0$

$T^{-2}: \omega^2-2-12a_1+6b_1=0$

$T^0: -\omega^2a_1+2a_1+6a_1^2-12a_1b_1=0$

$T^2: 6a_1+6a_1^2=0$ (12)

Solving the set (12), to get:

$a_1=-1$, $b_1=1$, and $\omega=\mp 4$

Then the solution

$$u(x,t)=a_0-[\tan(x\pm 4t)-\cot(x\pm 4t)] \qquad (13)$$

Figure (1) represents $u(x,t)=-[\tan(x+4t)-\cot(x+4t)]$, while figure (2) illustrates the solitary of $u(x,t)=-[\tan(x-4t)-\cot(x-4t)]$.

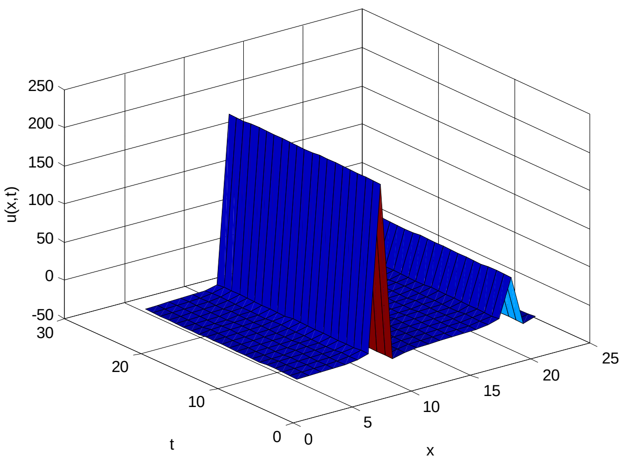

Fig.(1)

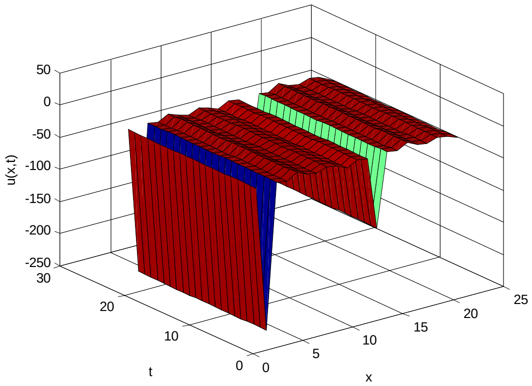

Fig.(2)

3.2. The Symmetric Regularized Long Wave (SELW) equation

In this section, we study the solution of the symmetric regularized long wave equation (SRLW) equation [29] given by:

$$u_{xxtt} + u_{tt} + u_{xx} + uu_{xt} + u_x u_t = 0 \quad (14)$$

Substituting $u(x, y, t) = U(\xi)$,and $\xi = x - \omega t$ into Eq. (14)

$$\omega^2 U^{(4)} + (\omega^2 + 1)U'' - \omega UU'' - \omega U'^2 = 0 \quad (15)$$

or

$$\omega^2 U^{(4)} + (\omega^2 + 1)U'' - \omega[UU']' = 0 \quad (16)$$

Integrating Eq.(16) twice with zero constant, we have:

$$\omega^2 U'' + (\omega^2 + 1)U - \frac{\omega}{2}U^2 = 0 \quad (17)$$

we postulate Tan-Cot series in Eq.(5), then Eq.(17) reduces to:

$$\omega^2[2T(1+T^2)\frac{dU}{dT} + (1+T^2)^2\frac{d^2U}{dT^2}] + (\omega^2+1)(1+T^2)\frac{dU}{dT} - \frac{\omega}{2}[(1+T^2)\frac{dU}{dT}]^2 = 0 \quad (18)$$

Now, to determine the parameter m, we balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq. (18) we balance U^2 with U'' , to obtain 2m=m+2, then m=2. The Tan-Cot method admits the use of the finite expansion for :

$$U = a_0 + a_1 T + a_2 T^2 + b_1 T^{-1} + b_2 T^{-2}$$

and

$$U' = a_1 + 2a_2 T - b_1 T^{-2} - 2b_2 T^{-3}$$

and

$$U'' = 2a_2 + 2b_1 T^{-3} + 6b_2 T^{-4} \quad (19)$$

Substituting U , U' , U'' from Eq.(19) in Eq. (18),

$$\omega^2[2T(a_1 + 2a_2T - b_1T^{-2} - 2b_2T^{-3}) + (1+T^2)(2a_2 + 2b_1T^{-3} + 6b_2T^{-4})]$$

$$+ (\omega^2+1)[a_1 + 2a_2T - b_1T^{-2} - 2b_2T^{-3}] - \frac{\omega}{2}(1+T^2)[a_1 + 2a_2T - b_1T^{-2} - 2b_2T^{-3}]^2 = 0 \quad (20)$$

then equating the coefficient of T^i , i= 0, 2, 4, -2,-4 leads to the following nonlinear system of algebraic equations:

$T^{-6} : -2\omega b_2^2 = 0$

$T^{-5} : -2\omega b_1 b_2 = 0$

$T^{-4} : 12\omega b_2 - b_1^2 - 4b_2^2 = 0$

$T^{-3} : 2\omega^2 b_1 - 2(\omega^2 + 1)b_2 + 2\omega b_2 a_1 - 2\omega b_1 b_2 = 0$

$$T^{-2}: 2\omega^2 b_2 - (\omega^2+1)b_1 + \omega a_1 b_1 + 4\omega a_2 b_2 - \frac{\omega}{2} b_1^2 = 0$$
$$T^{-1}: a_2 b_1 + a_1 b_2 = 0$$
$$T^{0}: 2\omega^2 a_2 + (\omega^2+1)a_1 - \frac{\omega}{2} a_1^2 + \omega a_1 b_1 + 4\omega a_2 b_2 = 0$$
$$T: \omega^2 a_1 + (\omega^2+1)a_2 - \omega a_1 a_2 + \omega a_2 b_1 = 0$$
$$T^2: 12\,\omega a_2 - 4a_2^2 - a_1^2 = 0$$
$$T^3: 2a_1 a_2 = 0$$
$$T^4: 2a_2^2 = 0 \qquad (21)$$

Solving the nonlinear systems of equations (21) we can get:

$b_1 = 0 \;\; b_2 = \mp 3i \;\; a_1 = 0 \;\; a_2 = \mp i \;\; \omega = \mp i$

$$u(x,t) = a_0 \mp i\left[\tan^2(x \pm it) + 3\cot^2(x \pm it)\right] \qquad (22)$$

Figure (3) illustrates the solitary shape of u(x, t) in Eq. (22).

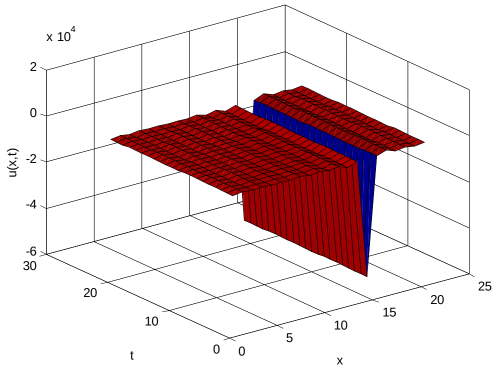

Fig.(3)

3.3. A higher-order wave equation of Korteweg–de Vries type

The solution will be handling to the higher-order wave equation of Kdv type:

$$u_t + u_x + \alpha u u_x + \beta u_{xxx} + \alpha^2 \rho_1 u^2 u_x + \alpha\beta(\rho_2 u u_{xxx} + \rho_3 u_x u_{xx}) = 0 \qquad (23)$$

By means of the method, we first use the wavevariable $\xi = x - \omega t$ that transforms Eq.(23) into an ODE

$$-\omega U' + U' + \alpha U U' + \beta U''' + \alpha^2 \rho_1 U^2 U' + \alpha\beta(\rho_2 U U''' + \rho_3 U' U'') = 0 \qquad (24)$$

eq.(24) can be written as

$$(1-\omega)U' + \frac{\alpha}{2} U^{2\prime} + \beta U''' + \frac{\alpha^2}{3}\rho_1 U^{3\prime} + \alpha\beta\rho_2(UU''' + U'U'') + \alpha\beta(\rho_3 - \rho_2)U'U'' = 0 \qquad (25)$$

Integrating Eq.(25) once with zero constant to get

$$(1-\omega)U + \frac{\alpha}{2}U^2 + \beta U'' + \frac{\alpha^2}{3}\rho_1 U^3 + \alpha\beta\rho_2 U U'' + \frac{\alpha\beta}{2}(\rho_3 - \rho_2)U'^2 = 0 \qquad (26)$$

The next step is that the solution is expressed in the form of Eq.(6)

$$(1-\omega)U + \frac{\alpha}{2}U^2 + \beta[2T(1+T^2)\frac{dU}{dT} + (1+T^2)^2\frac{d^2U}{dT^2}] + \frac{\alpha^2}{3}\rho_1 U^3 + \alpha\beta\rho_2 U[2T(1+T^2)\frac{dU}{dT} + (1+T^2)^2\frac{d^2U}{dT^2}] \qquad (27)$$
$$+ \frac{\alpha\beta}{2}(\rho_3 - \rho_2)(1+T^2)^2[\frac{dU}{dT}]^2 = 0$$

Now, to determine the parameter m, we balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq. (27) we balance U^3 with U'' , to obtain 3m=m+2, then m=1. The Tan-Cot method admits the use of the finite expansion for :

$$U = a_0 + a_1 T + b_1 T^{-1}, \; U' = a_1 - b_1 T^{-2}, \text{ and } U'' = 2b_1 T^{-3} \qquad (28)$$

Substituting $U, U^{/}, U^{//}$ from Eq.(28) in Eq. (27),

$$(1-\omega)[a_0 + a_1T + b_1T^{-1}] + \frac{\alpha}{2}[a_0 + a_1T + b_1T^{-1}]^2 + \beta[2T(1+T^2)(a_1 - b_1T^{-2}) + (1+T^2)^2 2b_1T^{-3}]$$
$$+\frac{\alpha^2}{3}\rho_1[a_0 + a_1T + b_1T^{-1}]^3 + \alpha\beta\rho_2[a_0 + a_1T + b_1T^{-1}][2T(1+T^2)(a_1 - b_1T^{-2}) + (1+T^2)^2 2b_1T^{-3}]$$
$$+\frac{\alpha\beta}{2}(\rho_3 - \rho_2)(1+T^2)^2[a_1 - b_1T^{-2}]^2 = 0 \tag{29}$$

$$(1-\omega)[a_0 + a_1T + b_1T^{-1}] + \frac{\alpha}{2}[a_0^2 + 2a_1b_1 + 2a_0a_1T + 2a_0b_1T^{-1} + a_1^2T^2]$$
$$+2\beta(a_1T + b_1T^{-1} + a_1T^3)$$
$$+\frac{\alpha^2}{3}\rho_1[a_0^3 + 6a_0a_1b_1 + 3a_0^2a_1T + 3a_0^2b_1T^{-1} + a_0a_1^2T^2 + 2a_1a_1b_1T + 2a_0a_1a_1T^2 + a_1^3T^3 + 3a_1b_1^2T^{-1} + a_1^2b_1T]$$
$$+2\alpha\beta\rho_2[a_0a_1T + a_0b_1T^{-1} + a_0a_1T^3 + a_1^2T^2 + 2a_1b_1 + a_1a_1T^4 + a_1b_1T^2]$$
$$+\frac{\alpha\beta}{2}(\rho_3 - \rho_2)(a_1^2 + 2a_1^2T^2 - 4a_1b_1 + a_1^2T^4 - 2a_1b_1T^2 + b_1^2) = 0$$

then equating the coefficient of T^{i}, i= -4,-3,-2,-1,0,1, 2,3, 4 leads to the following nonlinear system of algebraic equations:

$$T^{-4}: 3\rho_2 + \rho_3 = 0$$
$$T^{-3}: 6\beta + \alpha^2\rho_1b_1^2 + 6\alpha\beta\rho_2a_0 = 0$$
$$T^{-2}: \frac{\alpha}{2}b_1 + \alpha^2\rho_1a_0b_1 + 2\alpha\beta\rho_2[a_1 + b_1] + \frac{\alpha\beta}{}(\rho_3 - \rho_2)(b_1 - a_1) = 0$$
$$T^{-1}: (1-\omega) + \alpha a_0 + 2\beta + \alpha^2\rho_1[a_0^2 + a_1b_1] + 2\alpha\beta\rho_2a_0 = 0$$
$$T^{0}: (1-\omega)a_0 + \frac{\alpha}{2}[a_0^2 + 2a_1b_1] + \frac{\alpha^2}{3}\rho_1[a_0^3 + 6a_0a_1b_1] + 4\alpha\beta\rho_2a_1b_1 + \frac{\alpha\beta}{2}(\rho_3 - \rho_2)(a_1^2 - 4a_1b_1 + b_1^2) = 0$$
$$T: (1-\omega) + \alpha a_0 + 2\beta + \alpha^2\rho_1[a_0^2 + a_1b_1] + 2\alpha\beta\rho_2a_0 = 0$$
$$T^{2}: \frac{1}{2}a_1 + \alpha\rho_1a_0a_1 + 2\beta\rho_2[a_1 + b_1] + \beta(\rho_3 - \rho_2)(a_1 - b_1) = 0$$
$$T^{3}: 6\beta + \alpha^2\rho_1a_1^2 + 6\alpha\beta\rho_2a_0 = 0$$
$$T^{4}: 3\rho_2 + \rho_3 = 0 \tag{30}$$

Solving the nonlinear systems of equations (30) we can get:

$$\rho_3 = -3\rho_2 \quad a_0 = -\frac{1+8\beta\rho_2}{2\alpha\rho_1} = -\frac{A}{\alpha} \quad a_1 = b_1 = \frac{\sqrt{6\beta\rho_1[A\rho_2 - 1]}}{\alpha\rho_1}$$

$$\omega = [1-4\beta] - A(1-4\beta\rho_2) + \frac{A^2}{\alpha^2}$$

$$u(x,t) = -\frac{A}{\alpha} + \frac{\sqrt{6\beta\rho_1[A\rho_2 - 1]}}{\alpha\rho_1}\left[\tan(x-\omega t) + \cot(x-\omega t)\right] \tag{31}$$

Where:

$$A = \frac{(1+8\beta\rho_2)}{2\rho_1}$$

For $\alpha = \beta = \rho_1 = \rho_2 = 1$, $A = 4.5,\ \omega = 30.75$

$$u(x,t) = -4.5 + \sqrt{21}\left[\tan(x - 30.75t) + \cot(x - 30.75t)\right] \tag{32}$$

Figure (4) represents the shape of the solitary u(x,t) in Eq.(32).

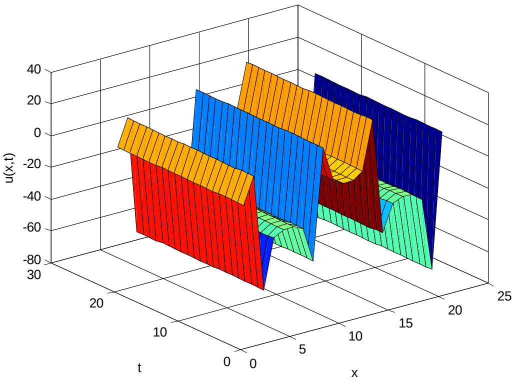

Fig.(4)

3.4 TheBenney–Luke equation

The NLEE that is going to be studied in this paper is called Benney-Luke (BL) equation and is given by [30] I will make use of the method to find exact solitary wave solutions to the (BL) equation. Let consider theBenney–Luke equation in the form:

$$u_{tt} - u_{xx} + \alpha u_{xxxx} + \beta u_{xxtt} + \delta u_t u_{xx} + \gamma u_x u_{xt} = 0 \qquad (33)$$

This equation is an approximation of the full water wave equations and formally suitable for describing two-way water wave propagation in presence of surface tension. The positive parameters $\alpha$and $\beta$are related to the inverse bond number$\alpha - \beta = \sigma - 1/3$, which capture the effects of surface tension and gravity forces.

BL equation is a NLEE that has been around and studied for a very long time. There are various analyses that were conducted for this equation. These are the stability analysis [30], Cauchy problem [31], existence and analyticity of solutions [32], traveling wave solutions [33], the generalized two-dimensional BL equation [34] and so forth.

Using the traveling wave variable $\xi = x - \omega t$ Eq. (33) convertsinto the following ODE for $u(x,t) = u(\xi)$:

$$(\omega^2 - k^2)u^{\prime\prime} + (\alpha + \beta\omega^2)u^{(4)} + (\delta + \gamma)\omega u^{\prime} u^{\prime\prime} = 0 \qquad (34)$$

Eq. (34) is integrable, therefore integrating with respect to ξonce and choosing the integration constant to zero, we obtain

$$(\omega^2 - k^2)u^{\prime} + (\alpha + \beta\omega^2)u^{\prime\prime\prime} + \frac{(\delta + \gamma)\omega}{2}u^{\prime 2} = 0 \qquad (35)$$

The next step is that the solution is expressed in the form of Eq.(6)

$$(\omega^2 - k^2)(1 + T^2)\frac{dU}{dT} + (\alpha + \beta\omega^2)[2(1 + 3T^2)(1 + T^2)\frac{dU}{dT} + 6Y(1 + T^2)^2\frac{d^2U}{dT^2} + (1 + T^2)^3\frac{d^3U}{dT^3}]$$
$$+ \frac{(\delta + \gamma)\omega}{2}(1 + T^2)^2[\frac{dU}{dT}]^2 = 0 \qquad (36)$$

Now, to determine the parameter m, we balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq. (36) we balance $U^{\prime 2}$ with $U^{\prime\prime\prime}$, to obtain (m+1) (m+1) = m+3, then m=1. The Tan-Cot method admits the use of the finite expansion for :

$$U = a_0 + a_1 T + b_1 T^{-1},\ U^{\prime} = a_1 - b_1 T^{-2},\ U^{\prime\prime} = 2b_1 T^{-3},\ U^{\prime\prime\prime} = -6b_1 T^{-4} \qquad (37)$$

Substituting $U, U^{\prime}, U^{\prime\prime}, U^{\prime\prime\prime}$ from Eq.(37) in Eq. (36)

$$(\omega^2 - k^2)(a_1 - b_1 T^{-2}) + (\alpha + \beta\omega^2)[2(1 + 3T^2)(a_1 - b_1 T^{-2}) + 6T(1 + T^2)(2b_1 T^{-3})$$
$$+ (1 + T^2)^2(-6b_1 T^{-4})] + \frac{(\delta + \gamma)\omega}{2}(1 + T^2)(a_1 - b_1 T^{-2})^2 = 0 \qquad (38)$$

then equating the coefficient of T^i , i= -4,-2,0,2 leads to the following nonlinear system of algebraic equations:

$$T^{-4}: 12(\alpha + \beta\omega^2) - (\delta + \gamma)\omega b_1 = 0$$
$$T^{-2}: 2(\omega^2 - k^2) + 4(\alpha + \beta\omega^2) + (\delta + \gamma)\omega(2a_1 - b_1) = 0$$
$$T^{0}: 2(\omega^2 - k^2) + 4(\alpha + \beta\omega^2) + (\delta + \gamma)\omega(a_1 - 2b_1) = 0$$
$$T^{2}: 12(\alpha + \beta\omega^2) + (\delta + \gamma)\omega(a_1) = 0 \qquad (39)$$

Solving the nonlinear systems of equations (39) we can get:

$$b_1 = \frac{12(\alpha + \beta k^2)}{(\delta + \gamma)\sqrt{(16\alpha + k^2)(1 - 16\beta)}}, \quad \omega = \sqrt{\frac{16\alpha + k^2}{1 - 16\beta}}, \quad a_1 = -b_1 \qquad (40)$$

$$u(x,t) = a_0 - \frac{12(\alpha + \beta k^2)}{(\delta + \gamma)\sqrt{(16\alpha + k^2)(1 - 16\beta)}}\left[\tan\left(x - \sqrt{\frac{16\alpha + k^2}{1 - 16\beta}}t\right) - \cot\left(x - \sqrt{\frac{16\alpha + k^2}{1 - 16\beta}}t\right)\right] \qquad (41)$$

Where:

$(16\alpha + k^2)(1 - 16\beta) > 0$, $(1 - 16\beta) > 0$, and $(\delta + \gamma \neq 0)$

for $a_0 = 0, \alpha = k = \delta = \gamma = 1, \beta = 1/20$

$$u(x,t) = -3.4166\left[\tan(x - \sqrt{85}\,t) - \cot(x - \sqrt{85}\,t)\right] \qquad (42)$$

Figure (5) represents the solitary shape for u(x, t) in Eq.(42)

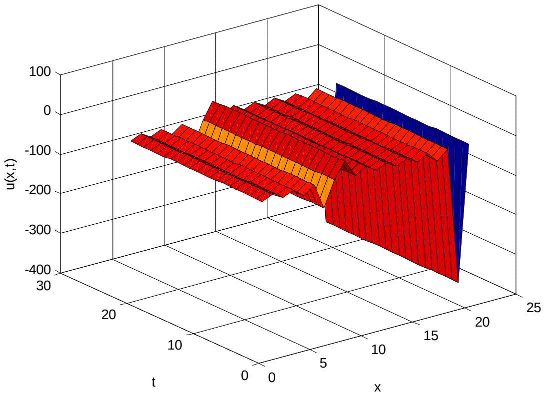

Fig.(5)

IV. CONCLUSION

The exact travelling wave solutions to different types of nonlinear partial differential equations have been studied by means of the extendedTan-Cot method. It can be easily seen that the implemented method used in this paper is powerful and applicable to a large variety of nonlinear partial differential equations.

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