

# Time-Cost Trade-Off Analysis in a Construction Project Problem: Case Study

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# ABSTRACT

In construction project, cost and time reduction is crucial in today's competitive market respect. Cost and time along with quality of the project play vital role in construction project's decision. Reduction in cost and time of projects has increased the demand of construction project in the recent years. Trade-off between different conflicting aspects of projects is one of the challenging problems often faced by construction companies. Time, cost and quality of project delivery are the important aspects of each project which lead researchers in developing time-cost trade-off model. These models are serving as important management tool for overcoming the limitation of critical path methods frequently used by company. The objective of time-cost trade-off analysis is to reduce the original project duration with possible least total cost. In this paper critical path method with a heuristic method is used to find out the crash durations and crash costs. A regression analysis is performed to identify the relationship between the times and costs in order to formulize an optimization problem model. The problem is then solved by Matlab program which yields a least cost of \$60937 with duration 129.50 ≈130 days. Applying this approach, the result obtained is satisfactory, which is an indication of usefulness of this approach in construction project problems.

Kevwords: Construction project: Critical path method: Trade-off analysis: Crashing

# I. INTRODUCTION

To stay competitive in the global market, completing the definite project compromising the cost and budget is very challenging. Several factors may cause delays such as labor related delay, political issues, contractor delay and some unseen delays which contribute to increase the uncertainty. So, proper planning and scheduling of all activities/jobs required for the completion of the project on time and within budget play a vital role to solve the problem. Project management can be defined as the process of utilizing resources, techniques etc. to get the job done properly within a specified timeframe at the lowest possible cost. It also ensures the performance standards to gain attraction to the buyers. Critical path method (CPM) and Project Evaluation and Review Technique (PERT) are both used for proper project planning and scheduling of large projects. They help the project managers in monitoring the progress of the stages. The major difference between them is that CPM considers deterministic durations to schedule the jobs.

To expedite the execution of a project, project managers need to reduce the scheduled execution time by hiring extra labor or using productive equipment. But this idea will incur additional cost hence shortening the completion time of jobs on critical path network is needed. For real life projects, decision makers always perform the trade-off between the time and cost of project through crashing/prolonging the duration of the activities. According to several researchers, time-cost trade problem (TCTP) is considered as one of the vital decisions in project accomplishment (Pour et al., 2012).

Research on the TCTP was first conducted by Kelly in 1961. Several researchers agreed that from 1961 the research mainly focused on the deterministic cases (Mobinia et al., 2011; Phillips and Dessouky, 1977; Weglarz et al., 2011). A variety of techniques were used to solve the time-cost trade problems which are classified into two areas: mathematical programming method and heuristic methods. Mathematical programming method includes linear programming, integer programming, and dynamic programming whereas heuristic methods use genetic algorithm, cost-loop method etc.

The aim of this study is to develop a hybrid a model for identifying the optimum total cost associated with project duration. A heuristic method named cost-loop method is used to find out the total crash costs at various durations. The time costs relation is then analyzed by regression analysis to obtain the objective function. A linear programming model has been developed and solved by using Matlab software.

The rest of this study is arranged as follows: The second section presents the literature review on time-cost trade-off analysis. Section 3 frameworks the developed methodology and provides a stepwise depiction of the anticipated steps for shortening project execution time. A prototype example is given to visualize the computational effectiveness in Section 4. And finally, in section five, results of the application are presented and suggestions for the future studies are clarified. This section wraps up this study.

# II. LITERATURE REVIEW

The objective of the time-cost trade-off problem (TCTP) is to reduce the original project duration obtained from the critical path analysis, to meet a specific deadline with the minimum direct and indirect cost of the project. Direct costs include costs of material, labor, equipment etc. whereas indirect costs are the necessary costs of doing work which can't be related to a particular task. There are enormous research works in the arena of TCTP. In 1991, Shouman et al. constructed a framework using mixed integer linear programming and CPM and utilized in natural gas projects. The value of the study is that, by the use of it, minimum total cost is achieved using crashing concept. A survey on forty seven papers conducted by Agarwal et al. (2013) revealed that about 41% work was performed in construction area during the 1990-2002. Liu et al. (1995) developed a hybrid method using linear and integer programming for time-cost trade-off problem. Several researchers used dynamic programming to adjust between two important aspects of the project (Hindelang et al., 1979; Prabuddha et al., 1995; Arauzo et al., 2009).

Recently, different computational optimization techniques such as genetic algorithms (GA), Evolutionary algorithms (EA), Particle swarm optimization (PSO) etc. have been used. Feng et al. (1997) adopted genetic algorithms to solve construction time-cost trade-off problems. Li et al. (1999) designed machine learning and genetic algorithms based system (MLGAS) in construction project and the system generated better results to nonlinear TCTP. In 2003, Poonambalam et al. applied genetic algorithms for sequencing problems in mixed model assembly lines of industrial arena and found better performance of GA. Genetic algorithm has been also used to optimize multi-objective time-cost-quality trade-off problem (Shahsavari Pour et al. 2010). Azaron et al. (2005) designed cost trade off problem as a multi-objective optimal problem consisting four objective functions and used genetic algorithm to solve it. Several researchers developed hybrid model based on genetic algorithms and other techniques and applied it to discrete TCTP problems (Azaron et al., 2005; Elazouni et al., 2007; Razek et al., 2010).

To deal with problems having uncertainties, different researchers have used fuzzy logic. Pathak et al. (2007) and Shahsavari Pour et al. (2012) applied fuzzy logic theory to consider affecting uncertainty in project quality. Pathak et al. (2008) proposed ANN with MOGA (multi-objective genetic algorithm) approaches for solving nonlinear TCTP for better project scheduling. In 2009, Chen et al. applied ant colony optimization algorithm in project scheduling to optimize the discounted cash flows. Zeinalzadeh, 2011 demonstrated a mathematical model using MILP-Lingo12 to minimize the total cost of a construction project.

The problem in this paper involves only deterministic time as a result dealing with uncertainty is avoided. Firstly, critical path method (CPM) is applied to determine the critical path and critical activities. A heuristic method named the cost-lope method is used to find out the total crash costs at various durations. The time costs relation is then analyzed by regression analysis and an optimization problem is formulized which is solved using Matlab software.

## III. METHODOLOGY

At first, necessary data for construction problem are taken from a secondary source. CPM is applied to find out the critical path and critical activities. The activities are shortened in order to get their lowest cost slopes using the heuristic method. As the duration of the activities on the shortest path are shortened, the project duration is also reduced. Eventually another path becomes critical, and a new list of activities on the critical path is prepared. Using this way, optimum schedules are identified. The time costs relation is then analyzed by regression analysis and an optimization problem is formulized which is solved using Matlab.

The procedure for shortening the project duration can be summarized in the following steps:

Step 1: Draw the project network.

*Step 2:* Perform CPM calculations and identify the critical path, using normal duration and costs for all activities.

*Step 3:* Compute the cost slope for each activity.

*Step 4:* Start by shortening the activity duration on the critical path which has the least cost slope and not been shortened to its crash duration.

*Step 5:* Reduce the duration of the critical activities with least cost slope until its crash duration is reached or until the critical path changes.

*Step 6:* When multiple critical paths are involved, the activity to shorten is determined by comparing the cost slope of the activity which lies on all critical path, with the sum of cost slope for a group of activities.

Step 7: Having shortened a critical path, adjust timings and floats.

Step 8: The cost increase due to activity shortening is calculated as the cost slope multiplied by the time units shortened.

Step 9: Continue until no further shortening is possible, and then the crash point is reached.

## IV. NUMERICAL EXAMPLE WITH CALCULATION

The proposed model has been applied on a construction project to demonstrate the practicality of the proposed methodology. It needs an estimate of how much time each activity takes in the normal way in order to schedule the activities in the network. It is required to crash the project duration from its original duration to a final duration of 110 days. The project data of a construction problem provided by the project manager is shown in table 1.Table 1 presents the details description of all activities required for the completion of the construction project. Here, there are five activities and construction process starts with activity A and ends with activity F. The associated cost in terms of dollar and required number of days to complete individual activity are demonstrated in table 1.The first column represents the activity identification code, immediate predecessors of each activity are shown in second column, third column shows the estimated normal cost and the fourth column provides the activity duration in days. Daily indirect cost of the project is assumed to be \$100.Table 2 depicts a project with hypothetical normal time - cost data and crash time –cost data of necessary activities.

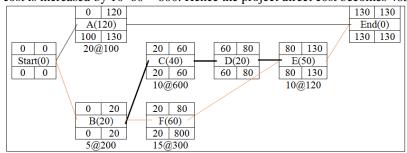
### 4.1 Computational steps

To determine the critical path consisting of activities with zero slack, different variables such as Earliest Start (ES), Earliest Finish (EF), Latest Start (LS), Latest Finish (LF) and Slack are computed. Table 3 shows the computation of determining the critical path of the project. Based on table 3 the total duration for the completion of the project is 140 days and the critical path is B-C-D-E. Project total normal direct cost is equal to sum of normal direct cost of all activities is \$48300.

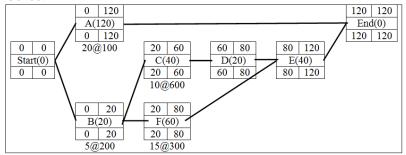
To expedite the project by reducing the expected project duration further down from 140 days, a process of crashing the duration of activities has been anticipated. Reduction of project duration incurs the extra cost. Project duration can be reduced by taking several measures such as overtime, hiring additional workers, using special time-saving materials, and special equipment. Table 4 shows the calculation of the cost-time slopes of the activities by a heuristic method named the cost-lope method. Both the cost slope and the crash ability are shown beneath each activity in the precedence diagram in fig. 1.

Now, the steps for shortening the activity duration on the critical path based on least cost-time slope are given as below:

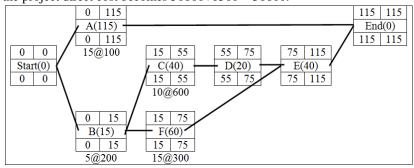
1. Activity "D" has the lowest cost slope on the critical path and this activity can be crashed by 10 days. By adjusting timing of the activities, the new critical path is B-F-E. New project duration is 130 days and the project direct cost is increased by 10\*60 = 600. Hence the project direct cost becomes 48300+600 = 48900.



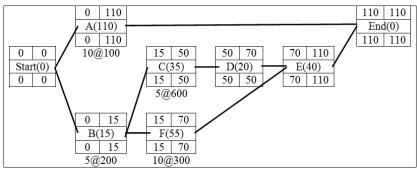
2. As activity "E" lies on both critical paths, at this step this activity can be crashed. Activity "E" can be crashed by 10 days. After this all activities has turned to critical activities. New project duration is 120 days and the project direct cost is increased by 10\*120 = 1200. Hence the project direct cost becomes 48900+1200 = 50100.



3. At this step, it is difficult to decrease one activity's duration and achieve decreasing in the project duration. So, either to crash an activity on all critical paths, otherwise, choose several activities on different critical paths. Activities "A" and "B" can be crashed together by 5 days which have the lowest cost slope (100+200). New project duration is 115 days and the project direct cost is increased by 5\*(100+200) = 1500. Hence the project direct cost becomes 50100+1500 = 51600.



4. In this final step, it is required to decrease the duration of an activity from path. "A" activity's duration can be crashed to 110 days, "C" to 35 days and "F" to 55 days. New project duration is 110 days and the project direct cost is increased by 5\*(100+600+300) = 5000. Hence the project direct cost becomes 51600+5000 = 56600



Finally, different cost data associated with project duration are obtained using the heuristic method which is presented in table 5. Figure 2 shows the scatter plot of different cost versus time required for completing the construction project. To visualize the relationship between total cost and time, the regression analysis is performed which is presented in figure 3. From figure 3, the obtained equation is Total cost (TC) =  $15.44 \text{ x}^2 - 3999 \text{ x} + 319875$  where 'X' represents the estimated duration (days) for the completion of project.

Now, the problem can be formulated as a linear programming model that has to be minimized. Here, Z is the total cost of the construction project obtained from the regression analysis. The aim of this model is to minimize the total cost (Z) subject to the constraint that the construction duration must be within 140 days to 110 days. The complete linear programming model can be given below:

 $\operatorname{Min} Z = 15.44x^2 - 3999x + 319875$ 

Subject to,

 $110 \le x \le 140;$ 

Where x represents duration of project completion.

Using Matlab software the solution of the mathematical model is obtained as the total cost is \$60937 with the duration of about 130 days.

## V. RESULT AND DISCUSSION

Crash times and crash costs are conflicting factors. Reducing one increases the other. The result shown in figure 2 indicates that total cost declines when project duration is increased. After some time, total cost reaches to its minimum value then it starts to increases with increasing duration. This analysis shows the real behavior of construction projects. This reveals the challenge of optimum resource utilization to compromise between different and usually conflicting aspects of projects. The regression analysis shows the relationship between the crash times and crash costs. The quadratic formula is best for representing the relationship. The time-cost trade-off analysis of this project results in a minimum cost of \$60937 with the duration of  $129.50 \approx 130$  days.

In this research, only deterministic values of activity duration and cost are used. Uncertainty behavior of project is avoided. For better prediction and result the parameter uncertainty can be considered.

## VI. CONCLUSION

The aim of this research was time-cost trade-off analysis for a construction project. The analysis has been done for an existing project. Critical path method (CPM) and a heuristic method have been used to find out the crash times and crash costs. Regression analysis has been done in order to develop the relationship between the crash times and the crash costs. The relation between crash times and crash costs has led to develop the optimization model. Matlab program has been used to get the minimum total costs of the project with the minimum durations. All these techniques utilized in this paper have shown a satisfactory result. The time-cost trade-off analysis of this project results in a minimum cost of \$60937 with the duration of 129.50  $\approx$ 130 days. This method is also capable of producing a satisfactory result with varying project activities which implies the usefulness of this approach presented in this paper.

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Table 1 Construction project data

#### List of Tables

Table I Construction project data						
Activity code	Immediate predecessors	Normal cost (\$)	Normal duration (days)			
Α	-	12000	120			
В	-	1800	20			
С	В	16000	40			
D	С	1400	30			
Ε	D,F	3600	50			
F	В	13500	60			

<b>List of normal &amp; crash cost-time data</b>						
Activity	Immediate	Normal cost	Normal duration	Crash cost	Crash duration	
code	predecessors	(\$)	(days)	(\$)	(days)	
Α	-	12000	120	14000	100	
В	-	1800	20	2800	15	
С	В	16000	40	22000	30	
D	С	1400	30	2000	20	
Ε	D,F	3600	50	4800	40	
F	В	13500	60	18000	45	

#### Table 2 List of normal & crash cost-time data

#### **Table 3** Computation of critical path method

<b>Tuble e</b> Computation of entited pain method						
Activity	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Slack	Critical
code	(ES)	(EF)	(LS)	(LF)	(LS-ES)	
Α	0	120	20	140	20	No
В	0	20	0	20	0	Yes
С	20	60	20	60	0	Yes
D	60	90	60	90	0	Yes
Ε	90	140	90	140	0	Yes
F	20	80	30	90	10	No

Table 4	Cost-time	slope of	the activities
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	Normal		Crash				
Activity	Duration	Costs	Duration	Costs	Crash cost-	Normal time-	Cost slope
Code	(days)	(\$)	(days)	(\$)	Normal cost ( $\Delta C$ )	Crash time ( $\Delta t$ )	$(\Delta C/\Delta t)$
Α	120	12000	100	14000	2000	20	100
В	20	1800	15	2800	1000	5	200
С	40	16000	30	22000	6000	10	600
D	30	1400	20	2000	600	10	60
Ε	50	3600	40	4800	1200	10	120
F	60	13500	45	18000	4500	15	300

#### Table 5 Duration-Cost Data

Duration (days)	Direct costs (\$)	Indirect costs (\$)	Total costs (\$)					
140	48300	14000	62300					
130	48900	13000	61900					
120	50100	12000	62100					
115	51600	11500	63100					
110	56600	11000	67600					

# **List of Figures**

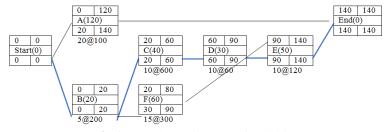


Fig.1.Precedence diagram of activities

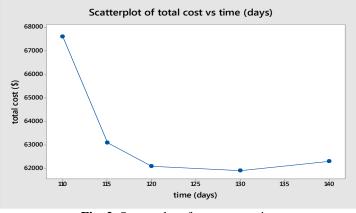


Fig. 2. Scatterplot of cost versus time

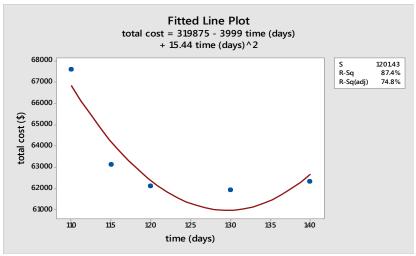


Fig.3. Regression analysis between costs versus time