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Study on M/M/2 Transient Queue with Feedback under Catastrophic Effect

S. Shanmugasundaram[1], S.Chitra[2]
[1,2]Department of Mathematics, Government Arts College, Salem – 636 007.
[2]Department of Mathematics, Sengunthar College of Engineering, Thiruchengode, Namakkal - 637 205.

ABSTRACT
In this paper we study the time dependent solution of feedback customer with two servers along with catastrophic effect. We derive the asymptotic behavior of the queue length and the stationary probability distributions. The numerical examples are also given to test the effectiveness of the queue length under the catastropic conditions.
Keywords: *Asymptotic behavior, Catastrophes, Departure Process, Feedback customer, Poisson process, Stationary probability distribution, Transient probability.*
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I. INTRODUCTION

In the year of 1909, queueing theory originated in telephony with the work of Erlang [2]. After his work many authors to develop different types of queueing models, incorporating different arrival patterns, different service time distributions and various service disciplines. In the year of 1963, Takacs [13] first introduced queues with feedback mechanism which includes the possibility for a customer return to the counter for additional service. In the year of 2000, Santhakumaran and Thangaraj [9] have studied a single server queue with impatient and feedback customers. In the year of 2008, Santhakumaran and Shanmugasundaram [9] have focused to study a Preparatory Work on Arrival Customers with a Single Server Feedback Queue

In queueing theory, the time independent solutions only derived for a long time. According to the theory and applications of queueing theory time dependent solution is necessary. Parthasarathy [8] and Parthasarathy and Sharafali [7] have discussed single and multiple server poisson queues of transient state solution in easiest manner. Krishna Kumar and Arivudainambi [4] has proposed a transient state solution for the mean queue size of M/M/1 queueing model when catastrophes occurred at the service station. Krishna Kumar and Pavai Madheswari [6] have introduced transient solution of M/M/2 queue with catastrophes. Krishna Kumar and Pavai Madheswari [5] have focussed to study transient solution of M/M/2 queue with heterogeneous servers subject to catastrophes.Shanmugasundaram and Shanmugavadivu [11] have discussed a time dependent solution of single server queue with Markovian arrival and Markovian service. Shanmugasundaram and Chitra [12] have discussed time dependent solution of M/G2/1 retrial queue and feedback on Non Retrail customers with catastrophes. Thangaraj and Vanitha [14] have studied transient solution of single server queue with feedback using continued fraction method. Chandrasekaran and Saravanarajan [1] has proposed a transient and reliability analysis of single server queue with feedback subject to catastrophes also discussed server failures and repairs. In this paper we analyze the time dependent solution of M/M/2 feedback queue with catastrophes.

II. MODEL DESCRIPTION AND ANALYSIS

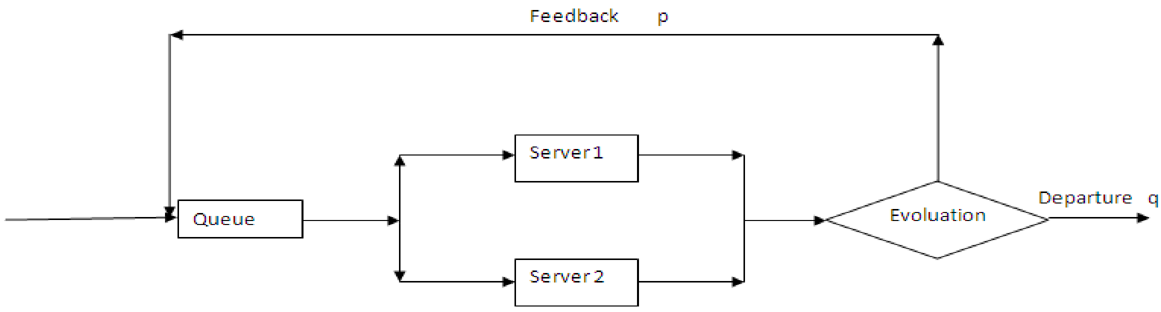


Fig1. Schematic flow of customer

Figure 1 illustrate that the external customers arrive according to a poisson process with rate λ_t. If the server is idle upon an arrival, service of an arriving customer starts instantaneously. After receiving service the customer a decision is made whether or not feedback. If a customer has feedback he joins the end of the original queue, there is no difference between normal customer and feedback customer. If a customer does feedback he joins the feedback stream with probability p. The feedback is assumed to occur instantaneously. If a customer does not feedback; he joins the departure process with probability q and leaves the system forever. The queue discipline is FIFO and infinite in capacity, the service times are non-negative, independent and identically distributed random variable with parameter μ_t. Catastrophes occurs from the arrival and service process follows poisson process with rate v_t.All the available customers are destroyed immediately when the catastrophes occurred in the system, the service gets inactivated. The server is ready for service when a new arrival happens. The motivation for this model is comes from Production system, Bank, Hospital, etc.

Let $Q_n(t) = Q\,[X(t) = n]$, $n = 0,1,2, \ldots$ denote the probability that there are n customers in the system at time t.

Let $Q(x,t) = \sum_{n=0}^{\infty} Q_n(t)\, x^n$ be the probability generating function.

A system of differential difference equation satisfied by M/M/2 feedback queue subject to catastrophe is

$$Q'_0(t) = -\,\lambda_t\, Q_0(t) + \mu_t q Q_1(t) + v_t[1 - \,Q_0(t)]$$
(1)
$$Q'_1(t) = -(\lambda_t + \mu_t \mathrm{q} + v_t)\, Q_1(t) + 2\mu_t q Q_2(t) + \lambda_t Q_0(t)$$
(2)

$$Q'_n(t) = -(\lambda_t + 2\mu_t \mathrm{q} + v_t) Q_n(t) + 2\mu_t q Q_{n+1}(t) + \lambda_t Q_{n-1}(t), for\ n = 2,3,4, \ldots$$
(3)

Taking Laplace transform of equation (1) we get,

$$Q_0^*(z)(z + \lambda_t + v_t) = q_0 + \mu_t q Q_1^*(z) + \frac{v_t}{z}$$

$$Q_0^*(z) = \frac{q_0}{(z+\lambda_t+v_t)} + \mu_t q \frac{Q_1^*(z)}{(z+\lambda_t+v_t)} + \frac{v_t}{z(z+\lambda_t+v_t)}$$
(4)

Taking inverse Laplace transform on both sides we get,

$$Q_0(t) = \mu_t q \int_0^t Q_1(u)\, e^{-[\lambda_t+v_t](t-u)} du + \frac{v_t}{\lambda_t+v_t}\left[1 - e^{-[\lambda_t+v_t]t}\right] + q_0 e^{-[\lambda_t+v_t]t}$$
(5)

Initially the number of customers present in the system is random and its probability generating function is $m(x) = \sum_{i=0}^{\infty} q_i x^i$

The probability generating function $Q(x,t)$ is derived from the above equations (1), (2) & (3)

Consider

$$Q'_n(t) = -(\lambda_t + 2\mu_t \mathrm{q} + v_t) Q_n(t) + 2\mu_t q Q_{n+1}(t) + \lambda_t Q_{n-1}(t), for\ n = 2,3,4, \ldots$$

Multiply both sides by x^n and taking the summation we get the partial differential equation is

$$\frac{\partial Q(x,t)}{\partial t} = \left[\lambda_t x - (\lambda_t + 2\mu_t \mathrm{q} + v_t) + 2\frac{\mu_t q}{x}\right] Q(x,t) + 2\mu_t \mathrm{q}\left(1 - \frac{1}{\mathrm{x}}\right) Q_0(t) + \mu_t \mathrm{q}(\mathrm{x} - 1) Q_1(t) + v_t$$
(6)

The solution of partial differential equation (6) is

$$Q(x,t)\, e^{-\int\left[\lambda_t x - (\lambda_t + 2\mu_t \mathrm{q} + v_t) + 2\frac{\mu_t q}{x}\right]dt}$$
$$= \int [2\mu_t \mathrm{q}\left(1 - \frac{1}{\mathrm{x}}\right) Q_0(t) + \mu_t \mathrm{q}(\mathrm{x} - 1) Q_1(t) + v_t]\, e^{-\int\left[\lambda_t x - (\lambda_t + 2\mu_t \mathrm{q} + v_t) + 2\frac{\mu_t q}{x}\right]dt} + c$$

$$Q(x,t)\, e^{-\left[\lambda_t x-(\lambda_t+2\mu_t q+v_t)+2\frac{\mu_t q}{x}\right]t} = \int_0^t \left[2\mu_t q\left(1-\frac{1}{x}\right)Q_0(t)+\mu_t q(x-1)Q_1(t)+v_t\right]e^{-\left[\lambda_t x-(\lambda_t+2\mu_t q+v_t)+2\frac{\mu_t q}{x}\right]dt}+c$$

using the initial condition $Q(x,0)=m(x)$, $\quad \therefore\ c=m(x)=\sum_{i=0}^{\infty} q_i x^i$

$$Q(x,t)=\sum_{i=0}^{\infty} q_i x^i\, e^{\left[\lambda_t x-(\lambda_t+2\mu_t q+v_t)+2\frac{\mu_t q}{x}\right]t}+v_t\int_0^t e^{\left[\lambda_t x-(\lambda_t+2\mu_t q+v_t)+2\frac{\mu_t q}{x}\right]u}\,du + \int_0^t \left[2\mu_t q\left(1-\frac{1}{x}\right)Q_0(u) + \mu_t q(x-1)Q_1(u)\right]e^{\left[\lambda_t x-(\lambda_t+2\mu_t q+v_t)+2\frac{\mu_t q}{x}\right](t-u)}\,du \quad (7)$$

$$\sum_{n=0}^{\infty} Q_n(t)\,x^n=\sum_{i=0}^{\infty} q_i x^i\, e^{\left[\lambda_t x+2\frac{\mu_t q}{x}\right]t}e^{-(\lambda_t+2\mu_t q+v_t)t}+v_t\int_0^t e^{\left[\lambda_t x+2\frac{\mu_t q}{x}\right]u}\,e^{-(\lambda_t+2\mu_t q+v_t)t}du + \int_0^t \left[2\mu_t q\left(1-\frac{1}{x}\right)Q_0(u) + \mu_t q(x-1)Q_1(u)\right]e^{\left[\lambda_t x+2\frac{\mu_t q}{x}\right](t-u)}\,e^{-(\lambda_t+2\mu_t q+v_t)(t-u)}du \quad (8)$$

Let $\alpha=2\sqrt{2\lambda_t\mu_t q}$ and $\beta=\sqrt{\frac{\lambda_t}{2\mu_t q}}$ Then $e^{\left[\lambda_t x+2\frac{\mu_t q}{x}\right]t}=\sum_{n=-\infty}^{\infty}(\beta x)^n I_n(\alpha t)$ we get,

$$Q_n(t)=\sum_{i=0}^{\infty} q_i I_{n-i}(\alpha t)\beta^{n-i}\,e^{-(\lambda_t+2\mu_t q+v_t)t}+v_t\beta^n\int_0^t e^{-(\lambda_t+2\mu_t q+v_t)t}\,I_n(\alpha u)du + 2\mu_t q\int_0^t e^{-(\lambda_t+2\mu_t q+v_t)(t-u)}\left[I_n(\alpha(t-u))\beta^n - I_{n+1}(\alpha(t-u))\beta^{n+1}\right]Q_0(u)du + \mu_t q\int_0^t e^{-(\lambda_t+2\mu_t q+v_t)(t-u)}\left[I_{n-1}(\alpha(t-u))\beta^{n-1} - I_n(\alpha(t-u))\beta^n\right]Q_1(u)du \quad (9)$$

Equating β^n on both sides we get,

$$0=\sum_{i=0}^{\infty} q_i I_{n-i}(\alpha t)\beta^{-i}e^{-(\lambda_t+2\mu_t q+v_t)t}+v_t\int_0^t e^{-(\lambda_t+2\mu_t q+v_t)t}\,I_n(\alpha u)du + 2\mu_t q\int_0^t e^{-(\lambda_t+2\mu_t q+v_t)(t-u)}\left[I_n(\alpha(t-u)) - I_{n+1}(\alpha(t-u))\beta\right]Q_0(u)\,du + \mu_t q\int_0^t e^{-(\lambda_t+2\mu_t q+v_t)(t-u)}\left[I_{n-1}(\alpha(t-u))\beta^{-1} - I_n(\alpha(t-u))\right]Q_1(u)du \quad (10)$$

Substitute equation (10) in (9) we get,

$$Q_n(t) = \sum_{i=0}^{\infty} q_i\beta^{n-i}\left[I_{n-i}(\alpha t) - I_{n+i}(\alpha t)\right]e^{-(\lambda_t+2\mu_t q+v_t)t}+n\beta^n\int_0^t Q_0(u)\,\frac{I_n(\alpha(t-u))}{(t-u)}e^{-(\lambda_t+2\mu_t q+v_t)(t-u)}du + \frac{n\beta^{n-2}}{2}\int_0^t Q_1(u)\,\frac{I_n(\alpha(t-u))}{(t-u)}e^{-(\lambda_t+2\mu_t q+v_t)(t-u)}du \quad (11)$$

Let $n=1$ and taking Laplace transform on both side we get,

$$
\begin{aligned}
Q_1^*(z) &= \left\{1 - \frac{(z+\lambda_t+2\mu_t\mathrm{q}+v_t)\sqrt{(z+\lambda_t+2\mu_t\mathrm{q}+v_t)^2-\alpha^2}}{2\alpha\beta}\right\} \\
&= Q_0^*(z)\frac{\beta}{\alpha}\left[(z+\lambda_t+2\mu_t\mathrm{q}+v_t)\sqrt{(z+\lambda_t+2\mu_t\mathrm{q}+v_t)^2-\alpha^2}\,\right] \\
&+ \sum_{i=1}^{\infty} q_i\,\beta^{-i+1}\left\{\frac{\left[(z+\lambda_t+2\mu_t\mathrm{q}+v_t)\sqrt{(z+\lambda_t+2\mu_t\mathrm{q}+v_t)^2-\alpha^2}\right]^{i-1}}{\alpha^{i-1}\sqrt{(z+\lambda_t+2\mu_t\mathrm{q}+v_t)^2-\alpha^2}}\right. \\
&\left. - \frac{\left[(z+\lambda_t+2\mu_t\mathrm{q}+v_t)\sqrt{(z+\lambda_t+2\mu_t\mathrm{q}+v_t)^2-\alpha^2}\right]^{i+1}}{\alpha^{i+1}\sqrt{(z+\lambda_t+2\mu_t\mathrm{q}+v_t)^2-\alpha^2}}\right\} \qquad (12)
\end{aligned}
$$

Substitute equation $Q_0^*(z)$ we get,

$$
Q_1^*(z) = \frac{\left\{\left(\frac{q_0 z+v_t}{z(z+\lambda_t+v_t)}\right)\frac{\beta}{\alpha}\left[w-\sqrt{w^2-\alpha^2}\,\right]+\sum_{i=1}^{\infty} q_i\beta^{-i+1}\left[\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{i-1}}{\alpha^{i-1}\sqrt{w^2-\alpha^2}}-\frac{\left[w-\sqrt{w^2-\alpha^2}\,\right]^{i+1}}{\alpha^{i+1}\sqrt{w^2-\alpha^2}}\right]\right\}}{\left(1-\frac{z+2\lambda_t+v_t}{4\lambda_t(z+\lambda_t+v_t)}\right)\left[w-\sqrt{w^2-\alpha^2}\,\right]}
$$

(13)

Where $w = z+\lambda_t+2\mu_t\mathrm{q}+v_t$

$$
\begin{aligned}
Q_1^*(z) = \frac{q_0\beta}{\alpha}\sum_{n=0}^{\infty}\frac{1}{(4\lambda_t)^n}\sum_{m=0}^{\infty}\binom{n}{m}\frac{{\lambda_t}^m}{(z+\lambda_t+v_t)^{m+1}}\left[w-\sqrt{w^2-\alpha^2}\right]^{n+1} \\
+\frac{v_t\beta}{z\alpha}\sum_{n=0}^{\infty}\frac{1}{(4\lambda_t)^n}\sum_{m=0}^{\infty}\binom{n}{m}\frac{{\lambda_t}^m}{(z+\lambda_t+v_t)^{m+1}}\left[w-\sqrt{w^2-\alpha^2}\right]^{n+1} \\
+\sum_{i=0}^{\infty}\sum_{n=0}^{\infty}\frac{q_i}{\beta^{i-1}}\frac{1}{(4\lambda_t)^n}\sum_{m=0}^{\infty}\binom{n}{m}\frac{{\lambda_t}^m}{(z+\lambda_t+v_t)^{m+1}} \\
\times\left\{\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n+i-1}}{\alpha^{i-1}\sqrt{w^2-\alpha^2}}\right. \\
\left.-\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n+i+1}}{\alpha^{i+1}\sqrt{w^2-\alpha^2}}\right\} \qquad (14)
\end{aligned}
$$

Taking inverse Laplace transform on both side

$$
\begin{aligned}
Q_1(t) = \frac{q_0\beta}{\alpha}\sum_{n=0}^{\infty}\frac{1}{(4\lambda_t)^n}\sum_{m=0}^{\infty}\binom{n}{m}(n+1)\alpha^{n+1}{\lambda_t}^m\int_0^t\frac{e^{-(\lambda_t+v_t)u}u^m}{m!}\frac{I_{n+1}\big(\alpha(t-u)\big)}{t-u}e^{-(\lambda_t+2\mu_t\mathrm{q}+v_t)(t-u)}du+ \\
\frac{v_t\beta}{\alpha}\int_0^t\sum_{n=0}^{\infty}\frac{1}{(4\lambda_t)^n}\sum_{m=0}^{\infty}\binom{n}{m}(n+\ 1)\alpha^{n+1}{\lambda_t}^m\int_0^t\frac{e^{-(\lambda_t+v_t)u}v^m}{m!}\frac{I_{n+1}\big(\alpha(u-v)\big)}{u-v}e^{-(\lambda_t+2\mu_t\mathrm{q}+v_t)(u-v)}dvdu \\
+\sum_{i=0}^{\infty}\sum_{n=0}^{\infty}\frac{q_i}{\beta^{i-1}}\frac{1}{(4\lambda_t)^n}\sum_{m=0}^{\infty}\binom{n}{m}\alpha^n{\lambda_t}^m\int_0^t\frac{e^{-(\lambda_t+v_t)u}u^{m-1}}{(m-1)!}e^{-(\lambda_t+2\mu_t\mathrm{q}+v_t)(t-u)}\Big[I_{n+i-1}\big(\alpha(t \\
-u)\big)-I_{n+i+1}\big(\alpha(t-u)\big)\Big]du \\
+\textstyle\sum_{i=1}^{\infty}\sum_{n=0}^{\infty}\frac{q_i\beta^{1-i}}{(2\beta)^n}\left[I_{n+i-1}(\alpha t)-I_{n+i+1}(\alpha t)\right]
\end{aligned}
$$

(15)

Therefore we have the state probabilities of $Q_0(t)$ *and* $Q_1(t)$. Substituting equations (5) & (15) in equation (11) we get the state probability $Q_n(t)$.

III. ASYMPTOTIC BEHAVIOR OF AVERAGE QUEUE LENGTH

Theorem 1: If $v > 0$, the asymptotic behavior of the probability of the server being idle is

$$
Q_0(t) = \frac{v\left[4\lambda-\left((\lambda+\mu\mathrm{q}+v)-\sqrt{(\lambda+2\mu\mathrm{q}+v)^2-\alpha^2}\right)\right]}{\left[4\lambda(\lambda+v)-(2\lambda+v)\left((\lambda+2\mu\mathrm{q}+v)-\sqrt{(\lambda+2\mu\mathrm{q}+v)^2-\alpha^2}\right)\right]}
$$

as $t \to \infty$ (16)

Proof:
From equation (4) & (13) we get

$$Q_0^*(z) = \frac{q_0}{(z+\lambda_t+\upsilon_t)} + \frac{\upsilon_t}{z(z+\lambda_t+\upsilon_t)} + \frac{\mu_t q}{(z+\lambda_t+\upsilon_t)}$$

$$\times \frac{\left\{\left(\frac{q_0 z+\upsilon_t}{(z+\lambda_t+\upsilon_t)}\right)\frac{\beta}{\alpha}[w-\sqrt{w^2-\alpha^2}] + \sum_{i=1}^{\infty} q_i \beta^{-i+1}\left[\frac{[w-\sqrt{w^2-\alpha^2}]^{i-1}}{\alpha^{i-1}\sqrt{w^2-\alpha^2}} - \frac{[w-\sqrt{w^2-\alpha^2}]^{i+1}}{\alpha^{i+1}\sqrt{w^2-\alpha^2}}\right]\right\}}{\left(1-\frac{z+2\lambda_t+\upsilon_t}{4\lambda_t(z+\lambda_t+\upsilon_t)}\right)[w-\sqrt{w^2-\alpha^2}]}$$

By final value theorem

$$\lim_{t\to\infty} Q_0(t) = \lim_{z\to 0} z\, Q_0^*(z)$$

$$\lim_{t\to\infty} Q_0(t) = \frac{\upsilon_t\left\{1+[(\lambda_t+2\mu_t q+\upsilon_t)-\sqrt{(\lambda_t+2\mu_t q+\upsilon_t)^2-\alpha^2}]\left(\frac{1}{4\lambda_t}+\frac{\beta}{\alpha}\frac{\upsilon_t}{(\lambda_t+\upsilon_t)}\right)\right\}}{\left\{1-[(\lambda_t+2\mu_t q+\upsilon_t)-\sqrt{(\lambda_t+2\mu_t q+\upsilon_t)^2-\alpha^2}]\left(\frac{1}{4\lambda_t}+\frac{\beta}{\alpha}\frac{\upsilon_t}{(\lambda_t+\upsilon_t)}\right)\right\}}$$

$$+\frac{\frac{\beta}{\alpha}\frac{\mu_t q\,\upsilon_t}{(\lambda_t+\upsilon_t)}[(\lambda_t+2\mu_t q+\upsilon_t)-\sqrt{(\lambda_t+2\mu_t q+\upsilon_t)^2-\alpha^2}]}{\left\{1-[(\lambda_t+2\mu_t q+\upsilon_t)-\sqrt{(\lambda_t+2\mu_t q+\upsilon_t)^2-\alpha^2}]\left(\frac{1}{4\lambda_t}+\frac{\beta}{\alpha}\frac{\upsilon_t}{(\lambda_t+\upsilon_t)}\right)\right\}} \quad as\ z\to 0$$

By Tauberian theorem

$$Q_0(t) = \frac{\upsilon[4\lambda-((\lambda+\mu q+\upsilon)-\sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2})]}{[4\lambda(\lambda+\upsilon)-(2\lambda+\upsilon)((\lambda+2\mu q+\upsilon)-\sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2})]} \quad \text{as } t\to\infty$$

Theorem 2: If $\upsilon > 0$, the asymptotic behavior of the mean system size h(t) is give by

$$h(t) = \frac{(\lambda-2\mu q)}{\upsilon_t} + \frac{8\lambda\mu q+(\lambda-2\mu q)((\lambda+2\mu q+\upsilon)-\sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2})}{4\lambda(\lambda+\upsilon)-(\upsilon+2\lambda)((\lambda+2\mu q+\upsilon)-\sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2})} \quad as \quad t\to\infty$$

(17)

Proof:
The average queue length $h(t) = \sum_{n=1}^{\infty} nQ_n(t) = \left.\frac{\partial Q(x,t)}{\partial t}\right]$ at $x = 1$ we get,

$$\frac{dh(t)}{dt} + \upsilon_t h(t) = (\lambda_t - 2\mu_t q) + 2\mu_t q Q_0(t) + \mu_t q Q_1(t)$$

Solving for h(t) with $h(0) = \sum_{i=1}^{\infty} ip_i$, we get

$$h(t) = \frac{(\lambda_t-2\mu_t q)}{\upsilon_t}(1-e^{-t\upsilon_t})2\mu_t q\int_0^t Q_0(u)\, e^{-(t-u)\upsilon_t}du + \mu_t q\int_0^t Q_1(u)\, e^{-(t-u)\upsilon_t}du + \sum_{i=1}^{\infty} ip_i\, e^{-t\upsilon_t}$$

Taking Laplace transform on both sides we get,

$$h^*(z) = \frac{(\lambda_t-2\mu_t q)}{\upsilon_t(z+\upsilon_t)} + \frac{\sum_{i=1}^{\infty} ip_i\, e^{-t\upsilon_t}}{(z+\upsilon_t)} + \frac{2\mu_t q}{(z+\upsilon_t)}\left\{\frac{\upsilon_t+zq_0}{z(z+\lambda_t+\upsilon_t)}\right\} + \left\{\frac{2\mu_t{}^2 q^2}{(z+\upsilon_t)(z+\lambda_t+\upsilon_t)} + \frac{\mu_t q}{(z+\upsilon_t)}\right\}Q_1^*(z)$$

By final value theorem

$$\lim_{t\to\infty} h(t) = \lim_{z\to 0} z\, h^*(z)$$

$$= \frac{(\lambda_t-2\mu_t q)}{\upsilon_t} + \lim_{z\to 0} z\frac{Q_0{}^*(z)}{(z+\upsilon_t)}\left\{2\mu_t q + \frac{[w-\sqrt{w^2-\alpha^2}]}{4\left[1-\frac{1}{4\lambda_t}[w-\sqrt{w^2-\alpha^2}]\right]}\right\}$$

By Tauberian theorem

$$h(t) = \frac{(\lambda-2\mu q)}{\upsilon} + \frac{8\lambda\mu q+(\lambda-2\mu q)((\lambda+2\mu q+\upsilon)-\sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2})}{4\lambda(\lambda+\upsilon)-(\upsilon+2\lambda)((\lambda+2\mu q+\upsilon)-\sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2})} \quad as \quad t\to\infty$$

Particular case:
If $q = 1$ then the equations (16) & (17) exactly coincides with the result of Krishna Kumar and Pavai Madheswari [6].

IV. STATIONARY PROBABILITY DISTRIBUTION

The stationary probability distribution $\{\pi_n , n \geq 0\}$ for the M/M/2 feedback queue with catastrophes $\upsilon > 0$ is

$\pi_0 = 1 - \rho$

$\pi_n = (1-\rho)\rho^n$

Where $\rho = \frac{(\lambda+2\mu q+\upsilon)-\sqrt{\lambda^2+4\mu^2 q^2+\upsilon^2+4\mu q\upsilon+2\lambda\upsilon-4\lambda\mu q}}{4\mu q}$

Proof:

$\pi_0 = \lim_{t\to\infty} Q_0(t)$

$$= \frac{\upsilon\left[4\lambda - \left((\lambda+2\mu q+\upsilon) - \sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2}\right)\right]}{\left[4\lambda(\lambda+\upsilon) - (2\lambda+\upsilon)\left((\lambda+2\mu q+\upsilon) - \sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2}\right)\right]}$$

$$= 1 - \frac{(\lambda+2\mu q+\upsilon) - \sqrt{(\lambda+2\mu q+\upsilon)^2-\alpha^2}}{4\mu q}$$

$$= 1 - \frac{(\lambda+2\mu q+\upsilon) - \sqrt{\lambda^2+4\mu^2 q^2+\upsilon^2+4\lambda\mu q+4\mu q\upsilon+2\lambda\upsilon-8\lambda\mu q}}{4\mu q}$$

$$\pi_0 = 1 - \frac{(\lambda+2\mu q+\upsilon)-\sqrt{\lambda^2+4\mu^2 q^2+\upsilon^2+4\mu q\upsilon+2\lambda\upsilon-4\lambda\mu q}}{4\mu q}$$

(18)

By taking Laplace transform of equation (11) we get

$$Q_n^*(z) = \left(\frac{\beta}{\alpha}\right)^n \left[w-\sqrt{w^2-\alpha^2}\right]^n Q_0^*(z) + \frac{\beta^{n-2}}{2\alpha^n}\left[w-\sqrt{w^2-\alpha^2}\right]^n Q_1^*(z) + \sum_{i=0}^{\infty} p_i\, \beta^{n-i}\left[\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n-i}}{\alpha^{n-i}\sqrt{w^2-\alpha^2}} - \frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n+i}}{\alpha^{n+i}\sqrt{w^2-\alpha^2}}\right] \qquad (19)$$

Substitute equation (13) in (18) we get,

$$Q_n^*(z) = \sum_{i=0}^{\infty} q_i\, \beta^{n-i}\left[\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n-i}}{\alpha^{n-i}\sqrt{w^2-\alpha^2}} - \frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n+i}}{\alpha^{n+i}\sqrt{w^2-\alpha^2}}\right] + \left(\frac{\beta}{\alpha}\right)^n \frac{\left[w-\sqrt{w^2-\alpha^2}\right]^n}{\beta} \frac{\dfrac{\sum_{i=1}^{\infty} i q_i\left[w-\sqrt{w^2-\alpha^2}\right]^i}{(\alpha\beta)^i\sqrt{w^2-\alpha^2}}}{\left[1-\dfrac{\left[w-\sqrt{w^2-\alpha^2}\right]}{2\alpha\beta}\right]}$$
$$+ Q_0^*(z)\left\{\left(\frac{\beta}{\alpha}\right)^n\left[w-\sqrt{w^2-\alpha^2}\right]^n + \frac{\beta^{n-1}}{2\alpha^{n+1}}\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n+1}}{\left[1-\dfrac{\left[w-\sqrt{w^2-\alpha^2}\right]}{2\alpha\beta}\right]}\right\} \qquad (20)$$

Multiply equation (19) by z and taking limit as $z \to 0$ we get,

$$\lim_{z\to 0} zQ_n^*(z) = \lim_{z\to 0} zQ_0^*(z)\left(\frac{\beta}{\alpha}\right)^n\left[w-\sqrt{w^2-\alpha^2}\right]^n + \frac{\beta^{n-1}}{2\alpha^{n+1}}\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n+1}}{\left[1-\dfrac{\left[w-\sqrt{w^2-\alpha^2}\right]}{2\alpha\beta}\right]}$$

By Tauberian theorem we get,

$$\pi_n = \pi_0 \left(\frac{\beta}{\alpha}\right)^n \frac{\left((\lambda + 2\mu q + \upsilon) - \sqrt{\lambda^2 + 4\mu^2 q^2 + \upsilon^2 + 4\mu q\upsilon + 2\lambda\upsilon - 4\lambda\mu q}\right)^n}{\left\{1 - \frac{(\lambda + \mu q + \upsilon) - \sqrt{(\lambda + 2\mu q + \upsilon)^2 - \alpha^2}}{2\alpha\beta}\right\}}$$

Thus the stationary probability distribution exists if and only if $\upsilon > 0$ and $\lambda \neq 2\mu q$.

V. NUMERICAL ILLUSTRATIONS

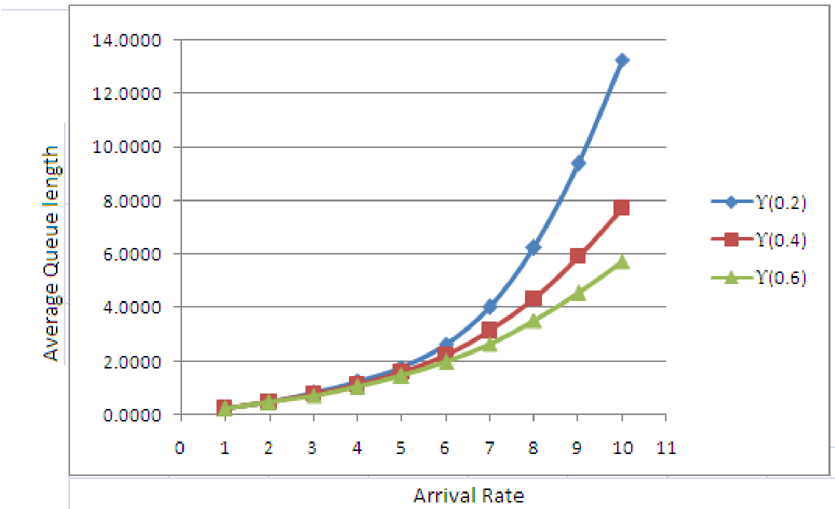
In this section some numerical analysis along with its related graphs based on average queue length are shown. The main intention is to illustrate the influence of the parameters $q = 0.4$ the average queue length of the system for varying values of λ and ν.

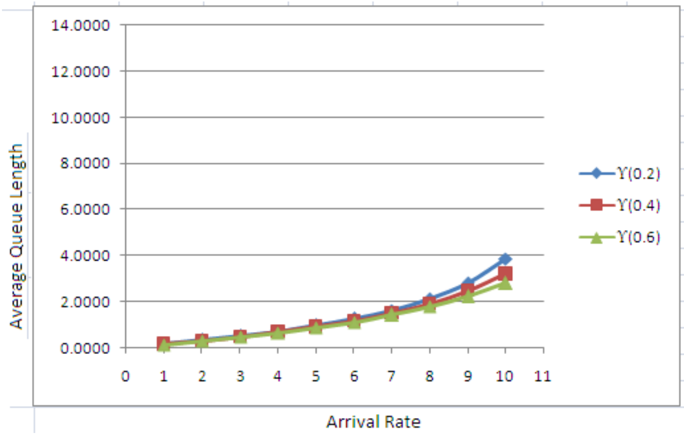
Table:1 Computed values of average queue length of the system $h(t)$ for $\mu = 10$ and q = 0.4 with the catastrophic effect of $\nu = 0.2,\ 0.4,\ 0.6$

λ	υ (0.2)	υ (0.4)	υ (0.6)
1	0.2415	0.2302	0.2199
2	0.5043	0.4785	0.4553
3	0.8154	0.7663	0.7236
4	1.2161	1.1230	1.0464
5	1.7783	1.5920	1.4521
6	2.6356	2.2358	1.9753
7	4.0223	3.1326	2.6515
8	6.2331	4.3478	3.5041
9	9.3750	5.8909	4.5326
10	13.2354	7.7091	5.7134

Table:2 Computed values of average queue length of the system $h(t)$ for $\mu = 15$ and q = 0.4 with the catastrophic effect of $\nu = 0.2,\ 0.4,\ 0.6$

λ	υ (0.2)	υ (0.4)	υ (0.6)
1	0.1623	0.1572	0.1524
2	0.3311	0.3202	0.3100
3	0.5136	0.4954	0.4785
4	0.7188	0.6904	0.6644
5	0.9589	0.9150	0.8758
6	1.2520	1.1829	1.1230
7	1.6259	1.5129	1.4198
8	2.1262	1.9323	1.7835
9	2.8292	2.4788	2.2358
10	3.8613	3.2015	2.8007





VI. CONCLUSION

The asymptotic behavior of the system and stationary probability distributions are derived with catastrophic effect and feedback. The numerical example illustrates that under asymptotic behavior when a customer does the feedback, queue length increases with arrival rate on different catastrophic effect and on different service rate.

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