ISSN (e): 2250 – 3005 || Volume, 05 || Issue, 07 || July – 2015 ||
International Journal of Computational Engineering Research (IJCER)

Effects of Temperature Dependent Viscosity and Thermal Conductivity in a Mixed Convection Boundary Layer Flow of a Micropolar Fluid towards a Heated Shrinking Sheet in presence of Magnetic Field

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Abstract:
Effects of temperature dependent viscosity and thermal conductivity in a mixed convection boundary layer flow of a micropolar fluid towards a heated shrinking sheet in presence of magnetic field along with stagnation flow have been studied in the present work. The boundary layer equations are transformed into ordinary differential equations using similarity transformations. The effects of variable viscosity, variable thermal conductivity and the parameters involved in the study on the velocity, micro-rotation and temperature distribution profiles are investigated by solving the governing transformed ordinary differential equations with the help of Runge-Kutta 4th order method with shooting technique. The numerical results are shown graphically and discussed in detail.

Keywords: Heat transfer, Magnetic field, Micro polar fluid, Shrinking sheet, Stagnation flow,Thermal conductivity, Variable viscosity.

I. Introduction

The problems of flow and heat transfer in the boundary layers of a continuous stretching/shrinking surface have attracted considerable attention of researchers due to their numerous applications in industrial manufacturing processes. Some of the applications are extraction of polymer sheets, paper production, hot rolling and glass-fiber production. Eringen [1] formulated the micropolar fluid theory as an extension of the Navier- Stokes model of classical hydrodynamics to facilitate the description of the fluids with complex molecules .The micropolar fluids are generally defined as isotropic, polar fluids in which deformation of molecules is neglected. Physically, a micropolar model can represent fluids whose molecules can rotate independently of the fluid stream flow and its local vortices. Micro polar fluids have important applications in colloidal fluids flow, blood flows, liquid crystals, lubricants and flow in capillaries, heat and mass exchangers etc.

Stagnation point flows have also applications in blood flow problems, the aerodynamics extrusion of plastic sheets, boundary-layer along material handling conveyers, the cooling of an infinite metallic plate in a cooling bath, and textile and paper industries. Flows over the tips of rockets, aircrafts, submarines and oil ships are some instances of stagnation flow applications[2]. Hiemenz [3] started the study of stagnation flow problem and reduced the Navier-Stokes equations for the forced convection problem to an ordinary differential equation of third order by using similarity transformation. Chamkha [4] solved the problem of the laminar steady viscous flows near a stagnation point with heat generation/absorbing. The steady two dimensional point flow of a power law fluid over a stretched surface was studied by Mahapatra and Gupta [5]. The numerical solution of unsteady boundary-layer flow of an incompressible viscous fluid in the stagnation point region over a stretching sheet was presented by Nazar*et.al* [6] (using Keller box method). The problem of steady two dimensional laminar MHD mixed convection stagnation point flow with mass transfer over a heated permeable surface was examined by Abdelkhalek [7]. Two dimensional steady incompressible mixed convection non orthogonal stagnation flow towards a heated or cooled stretching vertical plate was considered by Yian *et.al* [8]. The solution of hydro magnetic steady laminar two dimensional stagnation flow of a viscous incompressible electrically conducting fluid of variable thermal conductivity over a stretching sheet was obtained by Sharma and Shing [9] using shooting method. Effect of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet was investigated by Chen [10]. Heat transfer over a stretching surface with uniform or variable heat flux in micropolar fluids was studied by Ishak *et. Al* [11].

Mahanta *et .al*.[12] discussed the effects of radiation of flow of second grade fluid over a stretching sheet through porous medium with temperature dependent viscosity and thermal conductivity. A numerical study of steady incompressible micro polar fluid in a two dimensional stagnation point flow towards a stretching sheet was investigated by Nazar *et.al* [13]. Chang [14] presented the solution of flow and heat transfer characteristic of mixed convection in a micropolar fluid along a vertical flat plate with conduction effects. Ishak *et.al* [15] studied steady stagnation flow towards a vertical surface immersed in a micropolar fluid. Rashidi et.al [16] investigated the mixed convection boundary-layer flow of a micropolar fluid towards a heated shrinking sheet.

In the above literatures, in most of the studies, the viscosity and the thermal conductivity of the ambient fluid were assumed to be constant. When the effects of temperature dependent viscosity and thermal conductivity are taken into account, the flow characteristics are significantly changed compared to the constant property case. In this paper, an attempt has been made in this study to find the effects of temperature dependent viscosity and thermal conductivity on a mixed convection two dimensional stagnation point flow and heat transfer of a steady viscous incompressible micropolar fluid towards a heated shrinking sheet in presence of magnetic field with viscous dissipation. Viscosity and thermal conductivity are assumed to be inverse linear functions of temperature.

II. Formulation of the Problem:

We consider two dimensional stagnation point flow of a micro polar fluid impinging normally on a heated shrinking sheet at a fixed flat plat coinciding with the plane $y = 0$. The flow is assumed to be laminar, steady, viscous and incompressible and except the fluid viscosity and thermal conductivity all the fluid properties are assumed to be constant. Also a magnetic field of constant intensity is assumed to be applied normal to the surface and the electrical conductivity of the fluid is assumed to be small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. Under these assumptions the governing equations of the problem are as below:

CONTINUITY EQUATION:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

MOMENTUM EQUATION:

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\upsilon\frac{\partial u}{\partial y}\right) + \frac{\kappa}{\rho}\left(\frac{\partial N}{\partial y} + \frac{\partial^2 u}{\partial y^2}\right) \pm g\beta(T - T_\infty) - \frac{\sigma}{\rho}B^2 u \tag{2}$$

ANGULAR MOMENTUM EQUATION:

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \gamma\frac{\partial^2 N}{\partial y^2} - \kappa\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

ENERGY EQUATION:

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + (\mu + \kappa)\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

Where u, v are velocity components in the directions of x and y along and perpendicular to the surface respectively . N is the component of micro -rotation vector normal to the xy -plane, ρ is the density , κ is the vortex viscosity, g is the acceleration due to gravity, β is the coefficient of thermal expansion , γ is the spin gradient viscosity , j is the micro-inertia density , p is the pressure of the fluid , T is the temperature , υ and λ are the viscosity and thermal conductivity respectively which are the functions of x and y . c_p is the specific heat capacity at constant pressure of the fluid , B is the magnetic intensity and σ is the electrical conductivity . The term $\pm g\beta(T - T_\infty)$ of equation (2) indicates the buoyancy force, where "+" sign refers buoyancy assisting and "-''sign corresponds to the buoyancy opposing the flow regions.

The boundary conditions for the problem are:

$$\left.\begin{array}{l} u(x,0) = bx, \quad v(x,0) = 0, \quad N(x,0) = 0, \quad T(x,0) = T_\infty \\ \\ u(x,\infty) = U = ax, \quad N(x,\infty) = 0, \quad T(x,\infty) = T_\infty \end{array}\right\} \tag{5}$$

Where b < 0 for the shrinking sheet, T_w is temperature on the surface, T_∞ is temperature of the fluid at infinity and U is the free stream velocity of the fluid.

Following Lai and Kulacki [17] let us assume that,

$$\left.\begin{array}{l} \dfrac{1}{\mu} = \dfrac{1}{\mu_\infty}[1 + \delta(T - T_\infty)] \text{ or } \dfrac{1}{\mu} = \Delta(T - T_r) \text{ where } \Delta = \dfrac{\delta}{\mu_\infty} \text{ and } T_r = T_\infty - \dfrac{1}{\delta} \\ \\ \dfrac{1}{\lambda} = \dfrac{1}{\lambda_\infty}[1 + \xi(T - T_\infty)] \text{ or } \dfrac{1}{\lambda} = \varepsilon(T - T_c) \text{ where } \varepsilon = \dfrac{\xi}{\lambda_\infty} \text{ and } T_c = T_\infty - \dfrac{1}{\xi} \end{array}\right\} \tag{6}$$

where μ_∞ is the viscosity at infinity, Δ and T_∞ are constants , T_r is transformed reference temperature , δ is a constant based on thermal property of the fluid and $\Delta < 0$ for gas , $\Delta > 0$ for liquid. Similarly, ε and T_c are constants and their values depend on the reference state and thermal properties of the fluid i.e., ξ.

To solve equations (1) - (4) subject to the boundary conditions given in equation (5) we use the following similarity transformations,

$$\left.\begin{array}{l} \eta = \left(\dfrac{a}{\upsilon_\infty}\right)^{\frac{1}{2}} y, \quad p(x,\infty) = p_0 - \dfrac{\rho a^2}{2}(x^2 + y^2) \quad , \quad u(x,y) = axf'(\eta), \\ \\ v(x,y) = -(a\upsilon_\infty)^{\frac{1}{2}} f(\eta) \quad , \quad N(x,y) = -a\left(\dfrac{a}{\upsilon_\infty}\right)^{\frac{1}{2}} xg(\eta), \quad \theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty} \end{array}\right\} \tag{7}$$

Where η is the similarity parameter, p_0 is the stagnation pressure. Also from equations (6) and (7), we have,

$$\upsilon = -\upsilon_\infty \frac{\theta_r}{\theta - \theta_r}, \qquad \lambda = -\lambda_\infty \frac{\theta_c}{\theta - \theta_c} \tag{8}$$

Equation of continuity in equation (1) is identically satisfied using equation (6) and therefore the velocity field is compatible with continuity equation and represents the possible fluid motion. Using equations (7) and (8) in equations (2)-(4) we get the following differential equations:

$$(f')^2 - ff'' = 1 + \frac{\theta_r}{(\theta_r - \theta)^2}\theta' f'' + \frac{\theta_r}{\theta_r - \theta} f''' + K(f''' - g') + R\theta - M^2 f' \tag{9}$$

$$f'g - g'f = Cg'' + AK(f'' - 2g) \tag{10}$$

$$\frac{\theta_c}{(\theta_c - \theta)^2}(\theta')^2 + \frac{\theta_c}{\theta_c - \theta}\theta'' + P_r f\theta' + P_r E_c\left(\frac{\theta_r}{\theta_r - \theta} + K\right)(f'')^2 = 0 \tag{11}$$

Where

$K = \frac{\kappa}{\mu_\infty}$ is the coupling constant parameter , $C = \frac{\gamma}{\mu_\infty j}$ is the spin gradient viscosity parameter, $\Pr = \frac{\rho c_p \upsilon_\infty}{\lambda_\infty}$ is the Prandtl number, $M^2 = \frac{\sigma B^2}{\rho a}$ is the magnetic parameter,

$R = \pm \frac{G_{r_x}}{R_{e_x}^2}$ ('+' in assisting flow and '-' in opposing flow) is the buoyancy parameter ,

$E_c = \frac{a^2 x^2}{C_p (T_w - T_\infty)}$ is the Eckert number and $A = \frac{\mu_\infty}{\rho ja}$ is the micro inertia density parameter

Here $Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\upsilon_\infty x^2}$ is the local Grashoff number and $\mathrm{Re}_x = \frac{Ux}{\upsilon_\infty}$ is the local Reynolds number.

θ_r and θ_c are the dimensionless parameters characterizing the influence of viscosity and thermal conductivity respectively, can be written as:

$$\left.\begin{aligned} \theta_r &= \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\delta(T_w - T_\infty)} \\ \theta_c &= \frac{T_c - T_\infty}{T_w - T_\infty} = -\frac{1}{\xi(T_w - T_\infty)} \end{aligned}\right\} \tag{12}$$

The transformed boundary conditions are,

$$\left.\begin{aligned} &f(\eta) = 0, \quad f'(\eta) = b/a = \alpha, \quad g(\eta) = 0, \quad \theta(\eta) = 1 \quad \text{at} \quad \eta = 0 \\ &f'(\eta) = 1, \quad g(\eta) = 0, \quad \theta(\eta) = 0 \quad \text{as} \quad \eta \to 0 \end{aligned}\right\} \tag{13}$$

Here the case $\alpha = 0$ stands for Hiemenz flow towards a solid plate and the case $\propto > 0$ is for the stagnation point over a stretching sheet. In our case of stagnation flow towards a shrinking sheet, hence we take $\propto < 0$.

The two important physical quantities of our interest in the problem are skin friction coefficient (c_f) and Nusselt number (Nu) which are defined as,

$c_f = \dfrac{2\tau_w}{\rho U_0^{\,2}}$, where τ_w, the shear stress at the surface is given by, $\tau_w = \left[(\mu + \kappa)\dfrac{\partial u}{\partial y} + \kappa N \right]_{y=0}$

And $Nu = \dfrac{x q_w}{\lambda_\infty (T_w - T_\infty)}$, where q_w, the heat flux is given by, $q_w = -\lambda \left[\dfrac{\partial T}{\partial y} \right]_{y=0}$

Therefore,

$$c_f \, \mathrm{Re}_x^{1/2} = 2\left(\frac{\theta_r}{\theta_r - 1} + K \right) f''(0) \tag{14}$$

$$Nu \, \mathrm{Re}_x^{-1/2} = \frac{\theta_c}{\theta_c - 1}\theta'(0) \tag{15}$$

III. Results and Discussion:

The system of differential equations (9 - 11) together with the boundary conditions (13) is solved numerically by using the fourth order of Runge-Kutta integration accompanied with the shooting iteration scheme. The purpose of this study is to bring out the effects of the variable viscosity and variable thermal conductivity on the governing flow with the combinations of the other flow parameters.

The numerical computations have been carried out for various values of magnetic parameter (*M*), Prandtl number ($\Pr$), Eckert number (E_c), buoyancy parameter (R), coupling constant parameter (K), the variable viscosity parameter (θ_r), variable thermal conductivity parameter (θ_c) and micro inertia density parameter (A). In order to illustrate the results graphically, the numerical values of dimensionless velocity distribution $f'(\eta)$, dimensionless micro-rotation distribution $g(\eta)$ and temperature distribution $\theta(\eta)$ with variation of different parameters have been plotted in Figures 1 – 12.

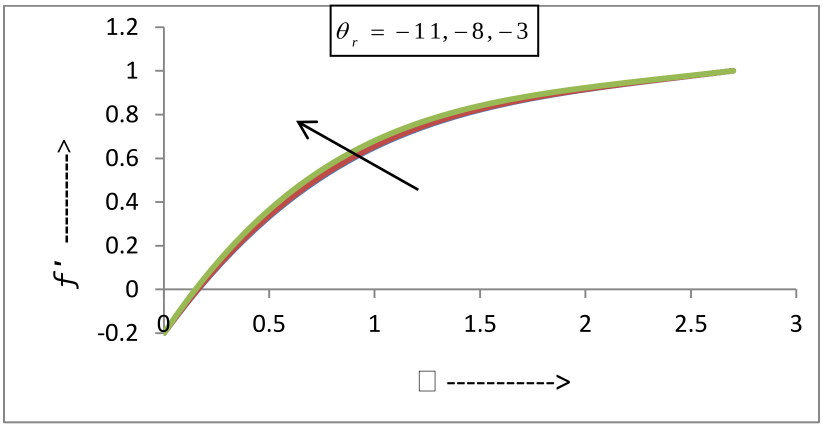

Figure 1. Variation of $f'(\eta)$ for different values of θ_r

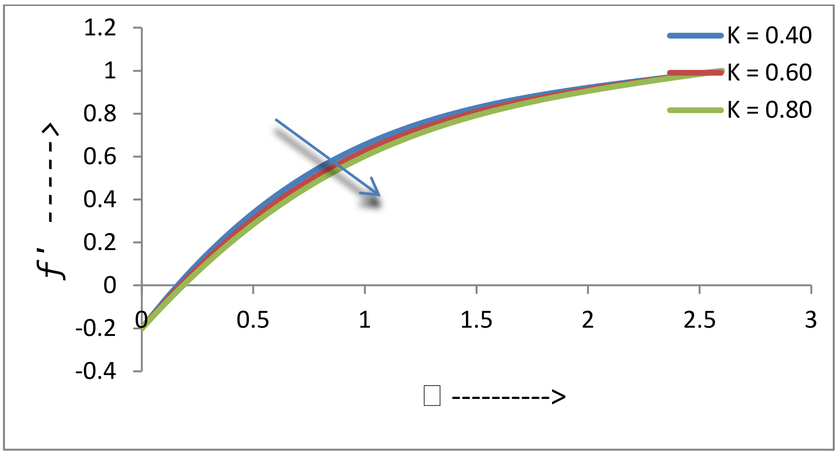

Figure 2. Variation of $f'(\eta)$ **for different values of** K

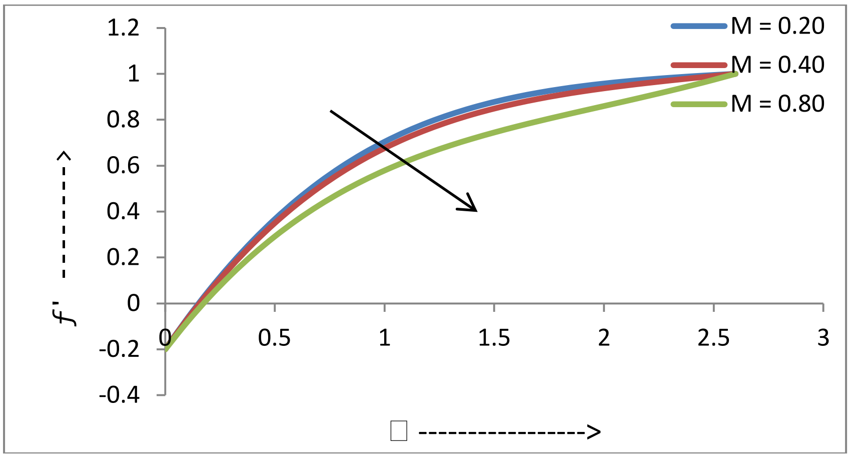

Figure 3. Variation of $f'(\eta)$ **for different values of** M

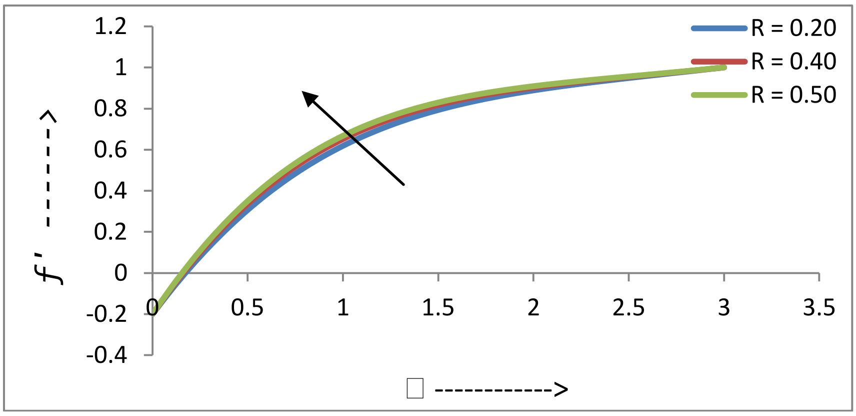

Figure 4. Variation of $f'(\eta)$ **for different values of** R

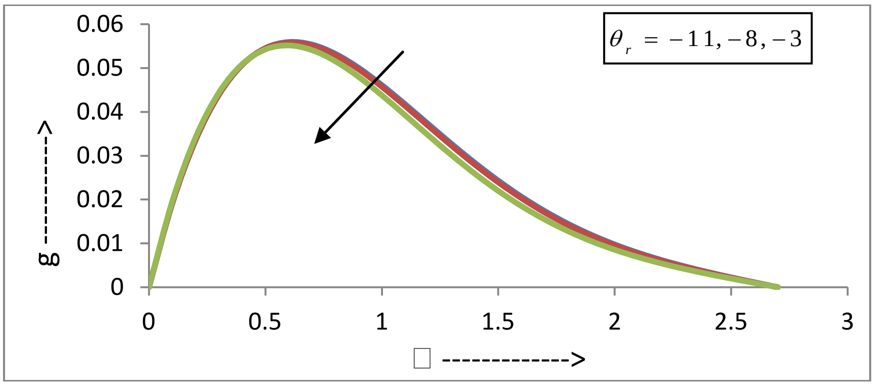

Figure 5. Variation of $g(\eta)$ **for different values of** θ_r

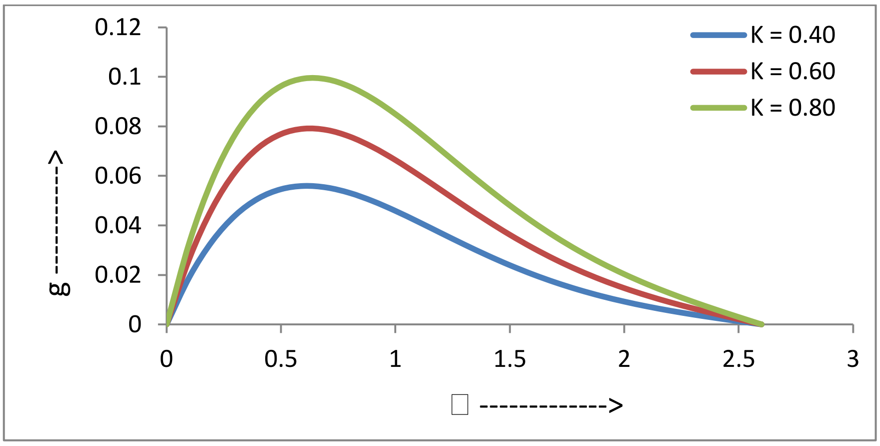

Figure 6. Variation of $g(\eta)$ **for different values of** K

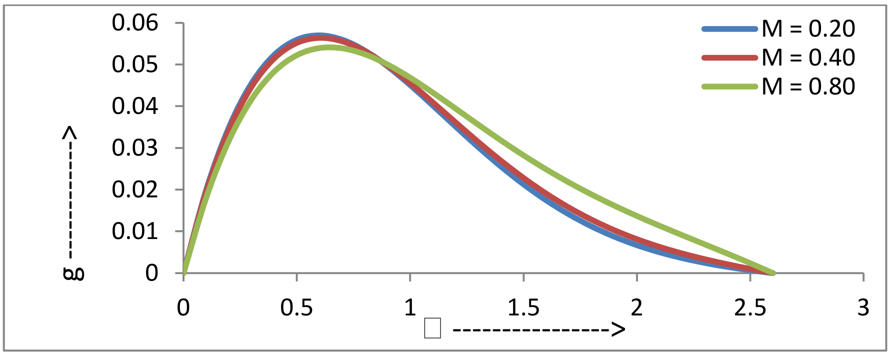

Figure 7. Variation of $g(\eta)$ **for different values of** M

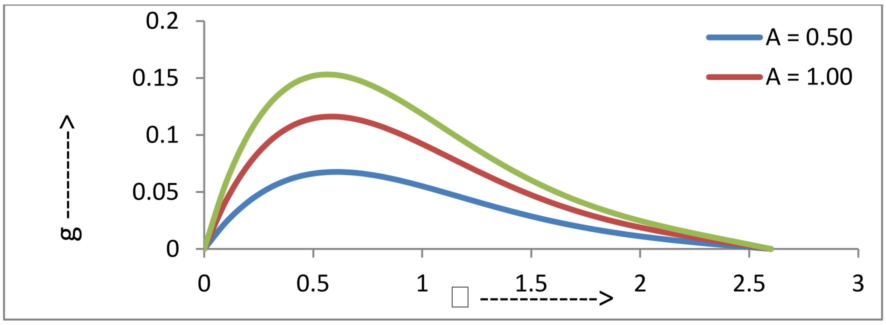

Figure 8. Variation of $g(\eta)$ **for different values of** A

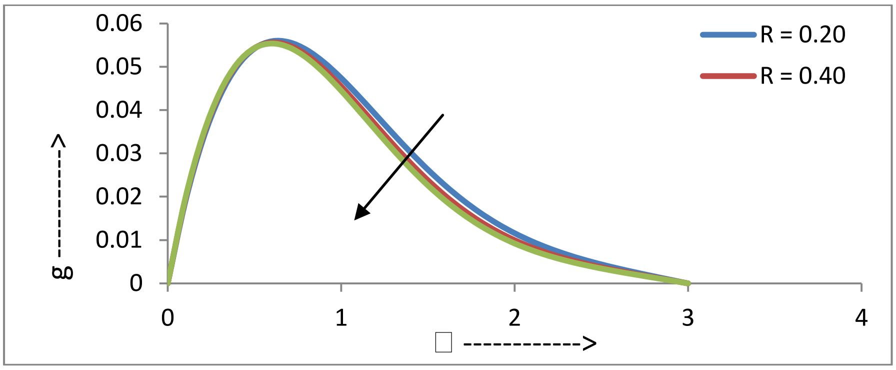

Figure 9. Variation of $g(\eta)$ **for different values of** R

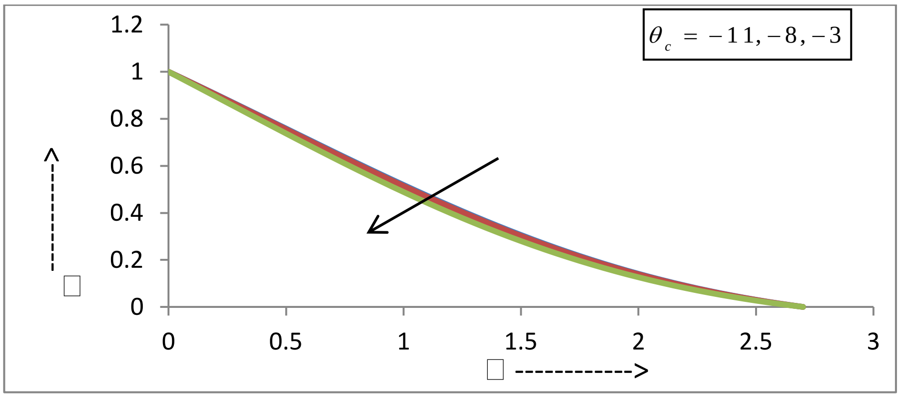

Figure 10. Variation of $\theta(\eta)$ **for different values of** θ_c

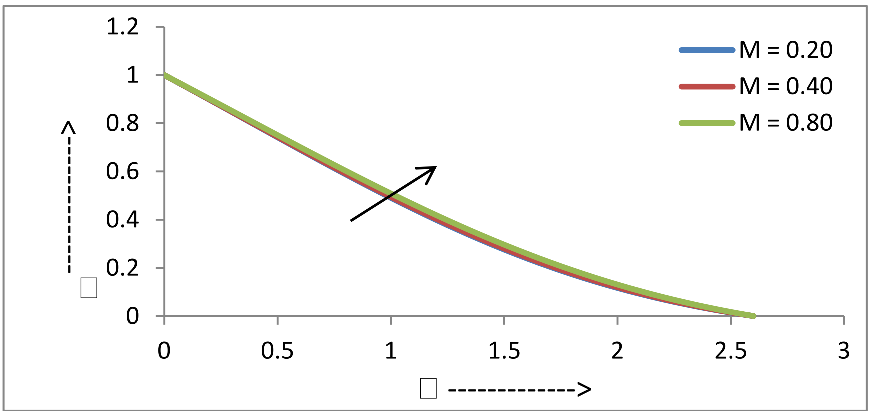

Figure 11. Variation of $\theta(\eta)$ **for different values of** M

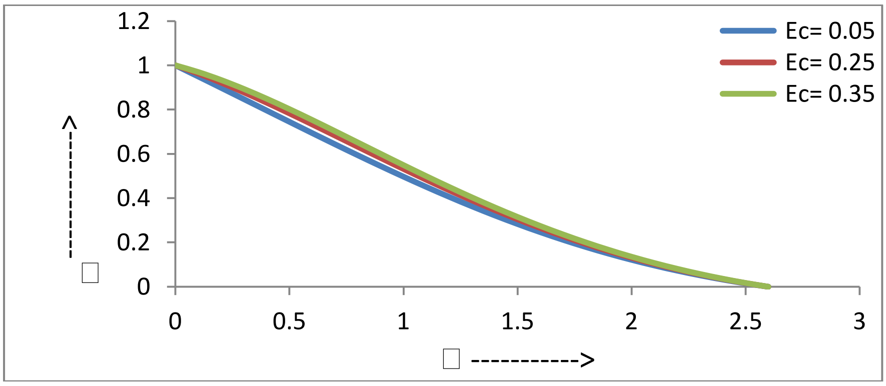

Figure 12. Variation of $\theta(\eta)$ **for different values of** E_c

The Figures 1- 4 represent the velocity distribution with the variation of viscosity parameter θ_r coupling constant parameter K, magnetic parameter M, Buoyancy parameter R respectively. It is seen that velocity increases with the increasing values of θ_r and R where as it decreases with the increasing values of M and K. It has been observed that with the variations of thermal conductivity parameter θ_c, micro-inertia density parameter A, Eckert number E_c and Prandtl number Pr the variation of velocity is not significant.

Figures 5 – 9 represents the variation in micro-rotation distribution with the variation of viscosity parameter θ_r, coupling constant parameter K, magnetic parameter M, micro-inertia density parameter A and Buoyancy parameter R respectively. It is observed that micro-rotation decreases with the increasing values of θ_r and R respectively. It is evident from the Figures 6, 7 and 8, the micro-rotation distribution increases with the increasing values of K, M and A respectively. In this case also variation of micro-rotation is not so much significant with the variations of thermal conductivity parameter θ_c, Eckert number E_c and Prandtl number Pr. The Figures (10-12) represents variation of temperature distribution with the variation of thermal conductivity parameter θ_c, Magnetic parameter M and Eckert number E_c. It is observed that temperature decreases as θ_c increases whereas it increases with the increasing values of M and E_c. In several practical applications, the surface characteristics such as friction factor and Nusselt number play important roles and hence, the missing values of $f''(0)$, $g'(0)$ and $\theta'(0)$ for various values of θ_r, θ_c, M, Pr, K and E_c have been derived in Table 1.

Table- 1.

θ_r	θ_c	M	Pr	K	E_c	$\alpha = -0.20$ $R = 0.40$ $C = 1.00$ $A = 0.40$		
						$f''(0)$	$g'(0)$	$\theta'(0)$
- 9	- 5	0.40	0.71	0.40	0.05	1.35553	0.106175	- 0.496973
- 7						1.371888	0.106582	- 0.498339
- 5						1.400093	0.107281	- 0.50069
- 8	**- 9**	0.40	0.71	0.40	0.05	1.364403	0.106458	- 0.478009
	- 6					1.363355	0.106393	- 0.490354
	- 5					1.362749	0.106355	- 0.497576
- 8	- 5	**0.40**	0.71	0.40	0.05	1.362749	0.106355	- 0.497576
		0.80				1.245599	0.098189	- 0.484796
		1.20				1.111045	0.087574	- 0.466789
- 8	- 5	**0.25**	**0.71**	0.40	0.05	1.390798	0.10821	- 0.500322
			1.71			1.375764	0.107211	- 0.589739
			2.21			1.370622	0.10688	- 0.619417
- 8	- 5	0.25	0.71	**0.40**	0.05	1.390798	0.10821	- 0.500322
				0.60		1.284479	0.153271	- 0.489207
				0.70		1.238793	0.173933	- 0.484205
- 8	- 5	0.25	0.71	0.40	**0.02**	1.389789	0.108154	- 0.525639
					0.05	1.390798	0.10821	- 0.500322
					0.08	1.391807	0.108266	- 0.474946

From the Table-1, it is observed that with the increasing values of θ_r and E_c, the values of $f''(0)$ and $g'(0)$ are increasing and the values of $\theta'(0)$ are decreasing for the increasing values of θ_r when all other parameters are fixed. It is seen that with the increasing values of θ_c and Pr, the values of $f''(0)$, $g'(0)$ and $\theta'(0)$ are decreasing. It is also indicated that when the magnetic parameter M and coupling constant parameter K increases, the values of $\theta'(0)$ increases but the reversal trend is observed for $f''(0)$ and $g'(0)$.

IV Conclusions

In this study , the effects of temperature dependent viscosity and thermal conductivity in a mixed convection boundary layer flow of a micropolar fluid towards a heated shrinking sheet in presence of magnetic field is examined. Numerical solutions are presented for the fluid flow and heat transfer characteristics for different values of parameters involved in the problem. The effects of temperature dependent viscosity and thermal conductivity on velocity, micro-rotation and temperature distribution are quite significant. Thus, the present study will serve as a scientific tool for understanding more complex flow problems concerning with the various physical parameters.

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