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Some Other Properties of Fuzzy Filters on Lattice Implication Algebras

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ABSTRACT

In this paper we discuss several properties of fuzzy filters are given in lattice implication algebra, the new equivalent conditions for fuzzy filters are also discuss in lattice implication algebra.

KEYWORDS- *Lattice implication algebra, fuzzy implication, filter, fuzzy implicative filter.*

I. INTRODUCTION

The concept of fuzzy set was introduced by zadeh[6]. Since then this concept has been Applied to other algebaric structure such as group, semigroup, ring, modules, vector space and topologies. With the development of fuzzy set, it is widely used in many fields. In order to research the logical system whose propositional value was given in a lattice , Xu. proposed the concept of lattice implication algebra and discussed their properties in [1]. Xu and Qin introduced the concept of filters and implicative filters in lattice implication algebra and investigate their properties in[2]. In [3] Xu. applied the concept of fuzzy sets to lattice implication algebra and proposed the notions of fuzzy filters and fuzzy implicative filters. In [7] [8] the notions of implicative filters, positive implicative and associative filters were studied. In[9][10] fuzzy filters, fuzzy positive implication and fuzzy associative filters were presented. In [11] Qin and Liu introduced a new class of filters known as v-Filters and they generated relation between v-filters and filters. Filter theory play an important role in studying the structure of algebra. In this paper we discuss several equivalent conditions for fuzzy filters are proved in lattice implication algebra. At the same time we discuss the relations of fuzzy filters and fuzzy implicative filters.

II. PRELIMANARIES

Definition 2.1. [5] A binary operator

$$I : [0, 1] \times [0, 1] \to [0, 1]$$

is said to be an implication function or an implication if it satisfies;
i) $I(x, y) \geq I(y, z)$ when $x \leq y$, $\forall z \in [0, 1]$.
ii) $I(x, y) \leq I(x, z)$ when $y \leq z$, $\forall x \in [0, 1]$.
iii) $I(0, 0) = I(1, 1) = I(0, 1)$ and $I(0, 1) = 0$ etc.

Definition 2.2. [1] Let $(L, \vee, \wedge, O, I)$be a bounded lattice with an order-reversing involution $'$, I and O the greatest and smallest element of L respectively, and
$\to: L \times L \to L$ be a mapping. $(L, \vee, \wedge,' , O, I))$ is called a quasi-lattice implication algebra if the following conditions hold for any $x, y, z \in L$
(I1) $x \to (y \to z) = y \to (x \to z)$;
(I2) $x \to x = I$;
(I3) $x \to y = y \to x$;
(I4) $x \to y = y \to x = I$ implies $x = y$
(I5) $(x \to y) \to y = (y \to x) \to x$

Definition 2.3. [1] A quasi-lattice implication algebra is called a lattice implication al-
gebra, if (I1) and (I2) hold for any $x, y, z \in L$
(I1) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$;
(I2) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$;

Theorem 2.1. [1] Let L be a quasi-lattice implication algebra, then for any x, y, x ∈ L
1. If $I \to x = I$, then $x = I$;
2. $I \to x = x$ and $x \to O = x$;
3. $O \to x = I$ and $x \to I = I$;
4. $(x \to y) \to ((y \to z) \to (x \to z)) = I$

Theorem 2.2. [1] Let L be a lattice implication algebra, then for any $x, y \in L$, $x \le y$ if and only if $x \to y = I$.

Theorem 2.3. [1] Let L be a lattice implication algebra, then for any $x, y, z \in L$, $x \le y$,
1. $(x \to z) \to (y \to z) = y \to (x \vee z) = (z \to x) \to (y \to x)$
2. $(z \to x) \to (z \to y) = (x \wedge z) \to y = (x \to z) \to (x \to y)$.

Theorem 2.4. [1] Let L be a lattice implication algebra, then for any $x, y, z \in L$,
1. $z \to (y \to x) \ge (z \to y) \to (z \to x)$;
2. $z \le y \to x$ if and only if $y \le z \to x$.

Theorem 2.5. [1] Let L be lattice implication algebra then the following statements are equivalent;
1. For any $x, y, z \in L$, $x \to (y \to z) = (x \wedge y) \to z$;
2. For any $x, y \in L$, $x \to (x \to y) = x \to y$;
3. For any $x, y, z \in L$,
   $(x \to (y \to z)) \to ((x \to y) \to (x \to z)) = I$

In lattice implication algebra L, we define binary operations $\oplus$ and $\otimes$ as follows: for any $x, y \in L$;
$x \otimes y = (x \to y')'$;
$x \oplus y = x \to y$;

Theorem 2.6. [1] Let L be a lattice implication algebra, then for any $x, y, z \in L$,
1. $x \otimes y = y \otimes x$, $x \oplus y = y \oplus x$;
2. $x \to (x \otimes y) = x \vee y = (x \oplus y) \to y$;
3. $(x \to y) \otimes x = x \wedge y$;
4. $x \to (y \to z) = (x \otimes y) \to z$;
5. $x \to (y \to z)$ if and only if $x \otimes y \le z$;

Definition 2.4. [2] A non-empty subset F of lattice implication algebra L is called a filter of L if it satisfies
(F1) $I \in F$
(F2) $(\forall x \in F)(\forall y \in L)(x \to y \in F \Rightarrow y \in F)$

Definition 2.5. [3] A fuzzy set A of lattice implication algebra L is called a fuzzy filter of L if it satisfies
(F3) $(\forall x \in L)(A(I) \ge A(x))$
(F4) $(\forall x, y \in L)(A(y) \ge \min\{A(x), A(x \to y)\})$

Definition 2.6. [3] Let L be a lattice implication algebra. A is a non-empty fuzzy set of L . A is called a fuzzy implicative filter of L if it satisfies:
(F5) $A(I) \ge A(x)$ for any $x \in L$;
(F6) $A(x \to z) \ge \min\{A(x \to y), A(x \to (y \to z))\}$ for any $x, y, z \in L$

Theorem 2.7. [3] Let L be a lattice implication algebra and A a fuzzy filter of L, then for any $x, y \in L$, $x \le y$ implies $A(x) \le A(y)$

III. PROPERTIES OF FUZZY FILTERS

Theorem 3.1. [9] Let A be a fuzzy set of L. A is a fuzzy filter of L if and only if it satisfies the following conditions; for any $x, y, z \in L$
1. $A(x) \le A(I)$;
2. $A(x \to z) \ge \min\{A(x \to y), A(y \to z)\}$.

Theorem 3.2. [9] Let A be a fuzzy set of L. A is a fuzzy filter of L if and only if it satisfies the following conditions; for any $x, y, z \in L$

1. $A(x) \le A(I)$;
2. $A(z) \ge \min\{A(x), A(y), A(x \to (y \to z))\}$.

Theorem 3.3. [9] Let A be a fuzzy set of L. A is a fuzzy filter of L if and only if it satisfies the following conditions; for any $x, y, z \in L$

1. $A(x) \le A(I)$;
2. $A(z \to x) \ge \min\{A((z \to y) \to x), A(y)\}$.

Theorem 3.4. [9] Let L be a lattice implication algebra and A be a fuzzy set of L, if A is a fuzzy implicative filter, the following statements are satisfied and equivalent:

1. A is a fuzzy filter for any $x, y \in L$,
$A(x \to y) \ge A(x \to (x \to y))$;
2. A is a fuzzy filter and for any $x, y, z \in L$,
$A((x \to y) \to (x \to z)) \ge A(x \to (y \to z))$
3. $A(x) \le A(I)$ and for any $x, y, z \in L$
$A(x \to y) \ge \min\{A(z \to (x \to (x \to y))), A(z)\}$

Theorem 3.5. [9] Let A be a fuzzy filter of L if $x \le y \to z$for any $x, y, z \in L$ then
$A(z) \ge \min\{A(x), A(y)\}$
Corollary 3.1. [9] Let A be a fuzzy filter of L. If $(x \otimes y) \to z = I$ for any $x, y, z \in L$, then

$$A(z) \ge \min\{A(x), A(y)\}.$$

Theorem 3.6. [9] Let A be a fuzzy set of L. A is a fuzzy filter of L iff it satisfies the following conditions: for any $x, y \in L$.

1. If $x \le y$, then $A(x) \le A(y)$;
2. $A(x \otimes y) \ge \min\{A(x), A(y)\}$

IV. SOME OTHER PROPERTIES OF FUZZY FILTERS

Theorem 4.1. Let A be a fuzzy set of L. A is a fuzzy filter of L iff it satisfies the following conditions, for any $x, y, z \in L$

1. $A(x) \le A(I)$;
2. $A(y \to (x \vee z)) \ge \min\{A(x \to z), A(x \to z) \to y))\}$

Proof: Assume that A is a fuzzy filter of L. then (1) is trivial and for any $x, y, z \in L$
$(y \to (x \vee z)) \ge \min\{A(x \to z), A(x \to z) \to y)\}$
by theorem 2.2 it can suffices to prove that
$(x \to (x \vee z)) \to ((x \to z) \to y) = I$
for this
$= (y \to (x \vee z)) \to ((x \to z) \to y)$
$= ((z \to x) \to (y \to x)) \to ((x \to x) \to y)$
$= ((z \to y) \to (z \to y)) = I$ if $x = I$
then $(y \to (x \vee z)) \ge ((x \to z) \to y)$
Then form theorem 2.6 hence
$A(y \to (x \vee z)) \ge \min\{A(x \to z), A((x \to z) \to y)\}$ conversely by (1) and (2) it follows that $A(x) \le A(I)$ and for any $x \in L$
By (2) it follows that for any $x, y, z \in L$ if we take $x = I$ then
$A(y) \ge \min(A(z), A(z \to y))$
Hence A is a fuzzy filter of L.

Theorem 4.2. Let A be a fuzzy set of L. A is a fuzzy filter of L iff it satisfies the following conditions for any $x, y, z \in L$

1. $A(y) \le A(I)$
2. $A(y \to z) \ge \min\{A(x), A(y), A(x \to (y \to z))\}$

Proof: Assume that A is a fuzzy filter of L, then (1) is trivial and for any $x, y, z \in L$
$A(y \to z) \ge \min\{A(x), A(y), A(x \to (y \to z))\}$
By theorem 2.2 it can suffices to prove that

$((x \to (y \to z)) \to (y \to z)) = I$
For this we have
$= ((x \to (y \to z)) \to (y \to z))$
$= ((x \to z) \to z$ if $y = I$
$= (x \to (z \to z)$
$= (x \to I) = I$
Then $((x \to (y \to z)) \leq (y \to z)$
Then form theorem 2.6 hence
$A((x \to (y \to z)) \leq A(y \to z)$
It follows that
$A(y \to z) \geq \min\{A(x), A(y), A(x \to (y \to z)\}$
conversely by (1) and (2) it follows that $A(x) \leq A(I)$ and for any $x \in L$
By (2) it follows that for any x, y, $z \in L$ if we take $y = I$ then
$A(z) \geq \{A(x), A(x \to z))\}$
Hence A is a fuzzy filter of L.

Theorem 4.3. Let A is a fuzzy filter of L, if $x \leq y$ for any x, $y \in L$ then
1. $A(x) \leq A(I)$
2. $A(x \vee y) \geq \min\{A(x), A(x \oplus y)\}$

Proof:Suppose that A is a fuzzy filter of L then (1) is trivial any for any x, $y \in L$
$A(x \vee y) \geq \min\{A(x), A(x \oplus y)\}$
for this we have to show that
$(x \to (x \oplus y)) \to (x \vee y) = I$
By theorem 2.2 . Hence
$= (x \oplus y) \to (x \to (x \vee y))$
$= (x \oplus y) \to ((y \to x) \to (x \to x))$
By theorem 2.3, and
$= (x \oplus y) \to ((y \to x) \to I)$
$= (x \oplus y) \to (y \to (x \to I))$
$= (x \oplus y) \to (y \to I)$
By theorem 2.1 and
$= (x \oplus y) \to I$ by theorem 2.5 and
$= (x \to y) \to I$ because $x \leq y$ hence
$(x \to (x \oplus y)) \to (x \vee y) = I$
this implies that
$(x \oplus y) \leq (x \to (x \vee y))$ Then by theorem 2.2 we get
$A(x \oplus y) \leq A(x \to (x \vee y))$
$A(x \to (x \vee y)) \geq \min\{A(x \oplus y)\}$ by theorem 2.7
Hence
$A(x \vee y) \geq \min\{A(x), A(x \oplus y)\}$.

V. CONCLUSION

In this paper we define some other equivalent conditions for fuzzy filters in lattice implication algebra.The relation between fuzzy filters and fuzzy implicative filters are also defined, and we also prove that fuzzy implicative filters are fuzzy filters in lattice implication algebra.

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