

Extended Kalman Filter based Missile Tracking

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ABSTRACT:

Kalman filter has been successfully used as an estimator in recursive linear systems. For nonlinear systems extended Kalman filter is more suitable for use. In case the target trajectory is steady, tracking can be achieved by Kalman filter. But target maneuvering makes tracking process more complicated. Extended Kalman filter gives real time tracking for such targets, provided we have high speed processor. This work describes and examines the use of this algorithm in missile and target tracking.

KEYWORDS: nonlinear, non-Gaussian, missile, tracking, estimator, extended Kalman filter, Algorithm.

I. INTRODUCTION:

The Kalman filter is an algorithm which uses a series of measurements observed over time, containing noise (random variations) and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those that would be based on a single measurement alone [1]. In tracking problems for example, with the assumption that the system parameters change linearly, Kalman filter equations operate properly as has been shown in a previous work [2]. Many practical systems have non-linear state update or measurement equations. The Kalman filter can be applied to a linearised version of these equations with loss of optimality [4]. In this research extended Kalman filter is used to estimate the position of a missile without losing optimality if the system is nonlinear. A simulation is conducted to test assumptions made.

II. KALMAN FILTER METHOD AND PHASES:

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. Equations for the Kalman filter fall into two phases: time update and measurement update [3].

The Kalman filter model assumes the true state at time k is evolved from the state at $(k - 1)$ according to

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k \dots\dots\dots [1]$$

where

- \mathbf{F}_k is the state transition model which is applied to the previous state \mathbf{x}_{k-1} ;
- \mathbf{B}_k is the control-input model which is applied to the control vector \mathbf{u}_k ;
- \mathbf{w}_k is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance \mathbf{Q}_k .

$$\mathbf{w}_k \sim N(0, \mathbf{Q}_k) \dots\dots\dots [2]$$

At time k an observation (or measurement) \mathbf{z}_k of the true state \mathbf{x}_k is made according to

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \dots\dots\dots [3]$$

where \mathbf{H}_k is the observation model which maps the true state space into the observed space and \mathbf{v}_k is the observation noise which is assumed to be zero mean Gaussian white noise with covariance \mathbf{R}_k .

$$\mathbf{v}_k \sim N(0, \mathbf{R}_k) \dots\dots\dots [4]$$

The initial state, and the noise vectors at each step $\{\mathbf{x}_0, \mathbf{w}_1, \dots, \mathbf{w}_k, \mathbf{v}_1 \dots \mathbf{v}_k\}$ are all assumed to be mutually independent. In practice, their matrices might change with each time step or measurement, however sometimes, as it is considered in this case, they are assumed to be constant.

The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback—i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

In short the time update projects the current state estimate ahead in time. The measurement update adjusts the projected estimate by an actual measurement at that time.

After each time and measurement update pair, the process is repeated with the previous a posteriori estimates used to project or predict the new a priori estimates. This recursive nature is one of the very appealing features of the Kalman filter.

III. EXTENDED KALMAN FILTER IMPLEMENTATION:

If the process to be estimated and (or) the measurement relationship to the process is non-linear then a Kalman filter that linearizes about the current mean and covariance is referred to as an extended Kalman filter or EKF [4]. The EKF utilizes the first term in a Taylor expansion of the nonlinear function [5].

The approach adopted for implementing Kalman Filter is also valid in the case of extended Kalman Filter ie.,the two phases: Time Update (“Predict”) and Measurement Update (“Correct”) [2] are the same the difference will be in the equations. For the extended Kalman Filter the equations obtained are as follows:

$$X_k = f (X_{k-1}, U_{k-1}, W_{k-1}) \dots\dots\dots [5]$$

with a measurement z that is

$$Z_k = h (X_k, V_k) \dots\dots\dots [6]$$

where the random variables w_k and v_k again represent the process and measurement noise. In this case the non-linear function f in the difference equation [1] relates the state at the previous time step $k-1$ to the state at the current time step k . It includes as parameters any driving function u_{k-1} and the zero-mean process noise w_k . The non-linear function h in the measurement equation [2] relates the state X_k to the measurement z_k .

In practice of course one does not know the individual values of the noise w_k and v_k at each time step. However, one can approximate the state and measurement vector without them as

$$\tilde{X}_k = f (\tilde{X}_{k-1}, U_{k-1}, 0) \dots\dots\dots [7]$$

and

$$\tilde{Z}_k = h (\tilde{X}_k, 0) \dots\dots\dots [8]$$

where \tilde{x}_k is some a posteriori estimate of the state (from a previous time step k) [4].

IV. ALGORITHM:

The algorithm used in Kalman filter approach and Radar detection with a little modification is valid for the Extended Kalman Filter implementation, the new algorithm is as follows:

Start

- generate a hypothetical target .
(azimuth = 45 degrees, range = 200 km., height = 4000meter, heading = 90 degrees east, speed = 800 km/hr).
- generate a tracker aircraft (or missile).
(azimuth = 45 digress, range = 70 km., heading = 90 degrees east, speed = 1200 km/hr).
(The tracker height should follow the data acquired from the height finder Radar in order to Approximate the real height of the target.)
- asuume that both the target and the tracker are detected by three dimensional Radar or azimuth and range Radar plus height finder Radar..
- apply Extended Kalman filter prediction formula to give the estimated target position.
- maneuver the heading of the tracker based on the estimated coordinates of the target.
- if the distance between the target and the tracker approaches zero is true.
then collision is assumed.
- otherwise the target and the tracker are in visual contact.
- end

V. RESULTS:

With the assumptions made above and values of F, G, H are known the Kalman filter equations resulted in values presented in table 1. Estimated values are a bit far from the measured values at the beginning of the observation but they are very close to the measured values when the object is close to the target. The implementation of the extended kalman Filter equations produced a more accurate values specially at the early stages of the process.

Table 1: results

	True Values		Meas.	Estimates		Errors in Estimates	
	Position	Velocity		Position	Velocity	Position	Velocity
T=Kt	X₁	X₂	Z(k)	X₁(K)	X₂(k)	P₁₁(k)	P₂₂(k)
0	100	0		95	1	10	1
1	99.5	- 1	100	99.6	0.4	0.9	0.9
2	97.0	- 2.4	96.4	97.2	- 2.1	0.7	0.6
3	95.0	- 3.2	94.0	94.8	- 3.0	0.7	0.3
4	92.0	- 4	92.7	92.4	- 3.7	0.6	0.2
5	89.0	- 4.8	89.2	89.4	- 4.6	0.5	0.1

The results shown in Table 1 assumes that both target and the tracker are approximately close in height.

VI. CONCLUSION:

Kalman Filters and Extended Kalman Filter (EKF) produce estimates that are very near to measured values which means both can be used for the estimation of the position of an attacking missile. The EKF implementation results in values that are much close to the real values than those produced by KF estimates and hence more reliable positioning and tracking can be achieved.

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