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The Birefringent Property of an Optical Resinfor the Study of a Stress Field Developed in a Three Point Loading Beam

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ABSTRACT

The birefringent property, known also as the double-refraction phenomenon, is used in a polariscope to study a stress field developed in a three point loading beam. The model used for this analysis was made of an epoxy resin (PLM4) and a hardener (PLMH). The stress field was locked in the model by the stress freezing technique. Photoelastic fringes obtained on the analyzer of a regular polariscope were used to determine completely the stress field. A finite elements analysis was also conducted in order to determine the stress field numerically. A whole field comparison of the experimental photoelastic fringes and the simulated ones and a local analysis using the principal stresses difference showed very good agreement between the experimental solution and the numerical one.

KEYWORDS: *Birefringence, photoelasticity, fringe, stress.*

I. INTRODUCTION

The analysis of stress fields is of very high importance in the design of machinery components. Various methods can be used to solve this kind of problem [1-8], analytical as well as experimental. However, analytical solutions may, in some cases, be difficult to develop. Here, a numerical solution using the finite elements analysis was used to obtain the solution. Stresses can be obtained rapidly in any desired position of the model. To validate the finite elementsolution tests were conducted on a regular polariscope (fig.1) which uses the birefringence phenomenon to analyze stress fields. The isochromatic and isoclinic fringe patterns are used to determine stresses.

II. EXPERIMENTAL ANALYSIS

A three point loading beam is analyzed in a plan polariscope which allows observing two types of photoelastic fringes, the isochromatics and the isoclinics. The isochromatics are loci of points having the same shear stress and the isoclinics, which appear in dark color, are loci of points for which the principal stresses directions are parallel to the polarizer and the analyzer axes. The stress field can therefore be completely determined, principal stresses with their directions can be obtained in the whole model. The three point loading beam model (Figure 1) is mounted on a loading frame inside an oven at the stress freezing temperature of 130°C in order to lock the stress field inside the model. The model will then be analyzed on a regular polariscope with plan polarized light and with circularly polarized light in order to obtain the isochromatic fringe pattern and the isoclinic fringe pattern.

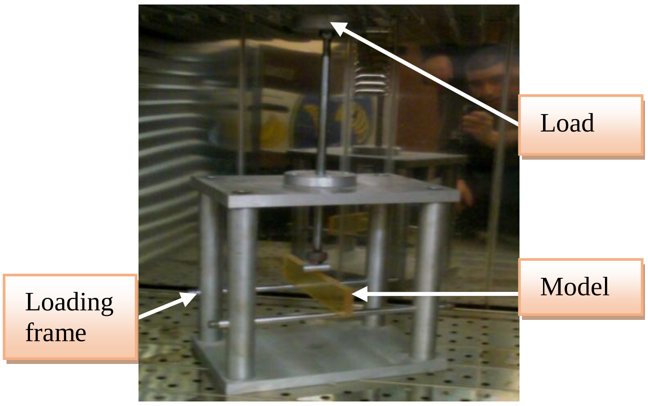

Figure 1: The model mounted on the loading frame inside an oven at the stress freezing temperature

To help the reader, a brief review of the experimental method is given bellow. Figure 2 shows the well-known photoelastic method based on the birefringent phenomenon. The light intensity obtained on the analyzer after traveling through the polarizer, the model and the analyzer is given by the following relation (1). The terms $\sin^2 2\alpha$ and $\sin^2\varphi/2$ give respectively the isoclinic fringes and the isochromatic fringes.

$$I = a^2 \sin^2 2\alpha \sin^2\varphi/2 \qquad (1)$$

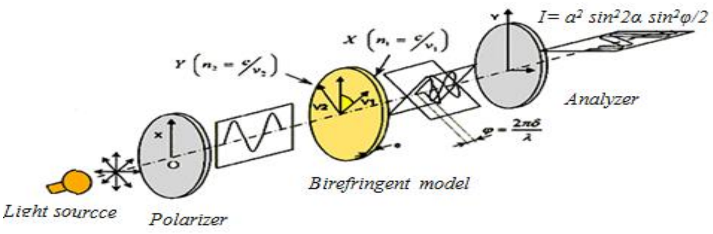

Figure 2: Light propagation through a photoelastic model

The light wave length used is λ=546nm.The isochromatic fringes allowed us to obtain the values of the principal stresses difference on the model by using the well known relation (2) where λ is the light wave length and C the optical characteristic of the birefringent material:

$$\sigma_1 - \sigma_2 = N\,(\lambda/C)/e \qquad (2)$$

This can only be done if the values of the fringe orders have been completely determined. The values of the fringe order N are determined either by the compensation technique or, whenever possible, by starting from a non stressed region on the model for which N=0. The fringe orders can then be easily deduced for the other fringes. The ratio f=λ/C called the fringe constant depends on the light wave used and the model material. Several solutions are available to obtain this value easily. Here, we subjected a disc (diameter D = 48 mm and thickness e = 14 mm) to a compressive load (P= 15N). The value of the principal stresses difference is given by the well known relation (3):

$$\sigma_1 - \sigma_2 = \frac{8P}{\pi e D}\frac{\left(D^4 - 4D^2x^2\right)}{\left(D^2 + 4x^2\right)^2} = Nf/e \qquad (3)$$

The value of the fringe constant can therefore be easily determined with the following relation (4) for different positions along the horizontal axis:

$$f = \frac{8P}{\pi D N}\frac{\left(D^4 - 4D^2x^2\right)}{\left(D^2 + 4x^2\right)^2} \qquad (4)$$

Where x represents the position of the isochromatic fringe. This distance x is taken along the diameter of the disc starting from the center of the disc (figure 3).
The obtained value (f=0.36 N/mm/fringe) was then used in the experimental solution as well as in the numerical solution to determine the stress values in the neighborhood of the contact zone along the direction of the applied load.

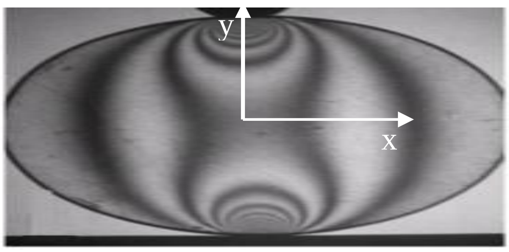

Figure 3: Isochromatics fringes obtained on the disc

2.1 Experimental results

The isochromatic fringes obtained with circularly polarized light (Figure 4), can be used for comparison purposes with the finite element solution and also to determine the experimental values of the principal stress difference in the neighborhood of the contact zones, along the vertical axis, for comparison purposes with the numerical values obtained with the finite element solution. We can see, as expected, a concentration of stresses close to the contact zone.

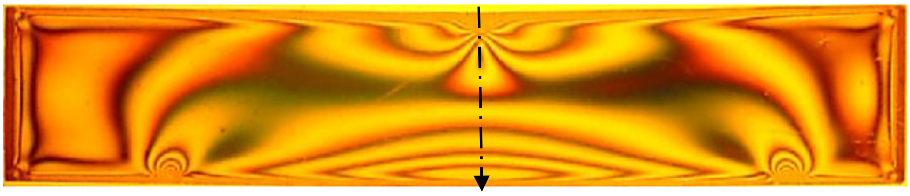

Figure 4: Experimental isochromatic fringes

A zoom (figure 5) allows to see clearly the different fringes in the neighborhood of the contact zone. We can therefore obtain the different values of the fringe orders which are necessary to obtain the graph of the principal stress difference along the vertical axis.

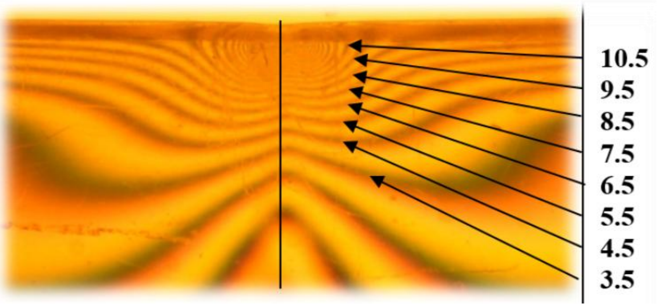

Figure 5: Fringe order values in the neighborhood of the contact zone

The isoclinics, which are the dark fringes, were obtained with plane polarized light for different polarizer and analyzer positions, the quarter wave plates were removed from the light path. We can see clearly the dark isoclinic fringe as it moves on the model. These isoclinics can be used to obtain the isostatics which are the principal stresses directions. The isostatics give a good understanding of how stresses are distributed in the volume of the model. This is very a important result for the design of mechanical components.

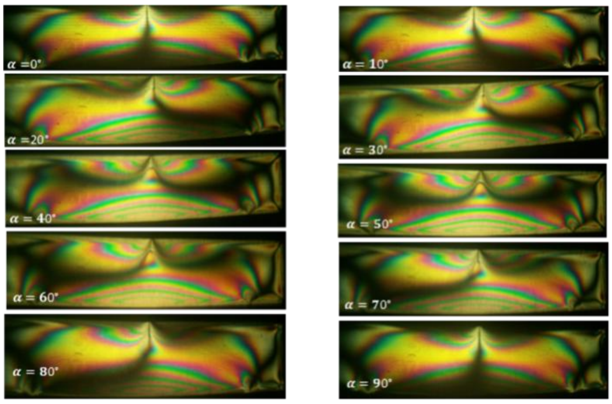

Figure 6: Isoclinics recorded for different polarizer and analyzer positions

III. NUMERICAL ANALYSIS

For a first approach of the solution we consider that the material behaves everywhere as a purely elastic isotropic material. Fringe constantf=0.36 N/mm/fringe, Young's modulus (E=15.9MPa) and Poisson's ratio (μ_2=0.4) were introduced in the finite element program. The mesh was refined in the neighborhood of the contact zone (figure 7) in order to achieve better approximation of stresses.

3.1. Numerical calculation of the isochromatic and the isoclinic fringes

The following relation (5)which can be obtained readily from Mohr's circle for stresses allows us to calculate the principal stresses difference at any point of a stressed model.

$$((\sigma_x - \sigma_y)^2 + 4\tau^2_{xy})^{0.5} = \sigma_1 - \sigma_2 = Nf/e \qquad (5)$$

The different values of the retardation angle φ can be calculated at any point on the model using the following relation (6):

$$\varphi = 2\pi N = 2\pi\ e/f\ ((\sigma_x - \sigma_y)^2 + 4\tau^2_{xy})^{0.5} \qquad (6)$$

The different values of $\sin^2\varphi/2$ which represents the simulated isochromatic fringes (figure 7) have been easily calculated. A comparison can then be made with the isochromatic fringes obtained experimentally with a monochromatic light (figure 4). We can see relatively good agreement; however in the neighborhood of the contact zone we can see some discrepancies. Another comparison using the principal stresses difference (figure 9) shows relatively good agreement eventhough close to the contact zone it was difficult to obtain experimentally the stresses.

The term $\sin^2 2\alpha$ represents the isoclinic fringes which are loci of points where the principal stresses directions are parallel to the polarizer and the analyzer. In the simulation program the different values of the isoclinic parameter α can be calculated with the following relation (eq. 7) which can be obtained readily from Mohr's circle of stresses:

$$\alpha = \text{arct}\ (2\tau_{xy} / (\sigma_x - \sigma_y)) \qquad (7)$$

The different values of $\sin^2 2\alpha$ can therefore be calculated and displayed (figure 8). The comparison is then possible with the experimental isoclinic fringes which are the dark fringes obtained experimentally on figure 6. Relatively good agreement can be observed between the experimental and the simulated isoclinic dark fringes. The simulated fringes were obtained only for α=60°.

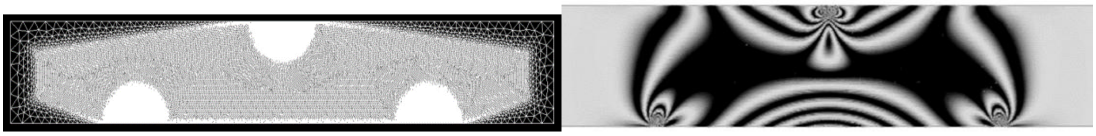

Figure 7: Finite element meshing and calculated isochromatic fringes

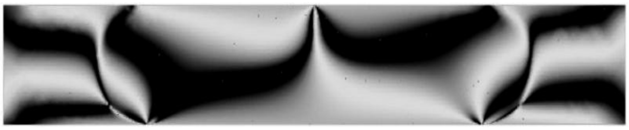

Figure 8: Calculated isoclinic for α=60°

Relatively good agreement are observed between the experimental fringes (figure 4) and the simulated fringes (figure 7 right).

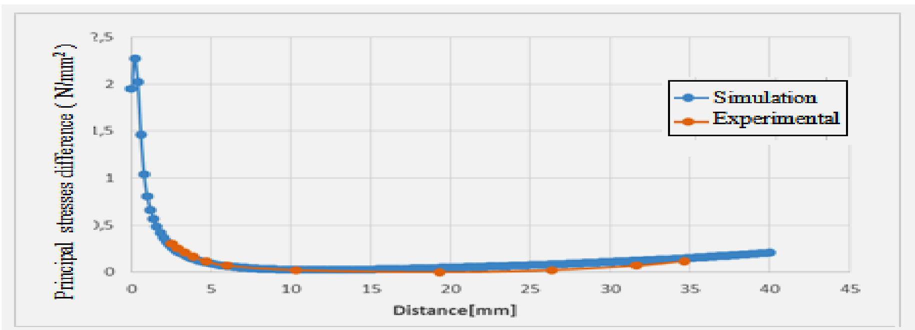

Figure 9: Principal stresses difference along the vertical axis osymmetry

IV. CONCLUSION

We have used the birefringent phenomenon to analyze stresses on a three point loading beam. The photoelastic fringes obtained on the analyzer were used to determine completely the stress field. A finite element solution was developed in order to simulate the photoelastic fringes and the stress values on the loaded model. Relatively good agreements were obtained between the experimental solution and the finite elements simulation.

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