Optimal Synthesis of a Single-Dwell 6-Bar Planar Linkage

Galal A. Hassaan
Mechanical Design & Production Department, Faculty of Engineering, Cairo University, Giza, Egypt

ABSTRACT
Six-bar linkage of single or double dwell linkage are used to overcome the problems of cam-follower mechanisms. Optimal synthesis of dwell mechanisms provides accurate synthesis fulfilling most of the functional constraints imposed to satisfy the mechanism specifications for proper operation.In the present work, the synthesis problem is formulated in the form of an objective function and three functional constraints. The synthesis problem incorporates seven parameters to be optimally evaluated covering all the links of the mechanism. The transmission angle of the 4-bar linkage and at the output slider are both considered to control the performance quality of the synthesized mechanism.In an application on the methodology presented, it was possible to synthesize 6-bar planar linkage for a single dwell across 60 degrees of crank rotation with a maximum error of 0.229 %.

KEYWORDS: *optimal mechanism design – 6-bar planar linkage – single-dwell mechanism.*

I. INTRODUCTION

It is difficult to synthesize 6-bar planar dwell mechanisms using the geometrical-graphical analysis approach. This simply because this approach requires exactly circular parts of the coupler point within a specific crank angle. Optimal mechanism synthesis is the proper solution which provides accurate and straight forward synthesis. Some approaches are available in the literature in this concern.

Sandgren (1985) applied the nonlinear programming technique to a class of 6-bar linkage to produce single and multi-dwell mechanisms [1]. Kota, Erdman and

Riley (1987) studied the 4-bar linkage generating straight line, arc and symmetrical curves [2]. In a second part of their work they explained the development of an expert system for designing linkage-type dwell mechanisms [3].

Danian, Yuanji and Shaowei (1988) studied the synthesis of 6-bar long dwell mechanisms [4]. **Iyer (1996)** developed a computer-aided design approach to design both circular-arc and straight line dwell mechanisms [5].

Ogot (1996) developed a design tool to support the design of a linkage-type dwell mechanism required for a vertical filler. He presented single and double dwell examples [6].

Chen and Yang (2005) proposed a procedure to synthesize optimal mechanisms using the multi-level decomposition approach. They applied their approach to design a 4-bar linkage and 6-bar dwell mechanisms [7].

Shiakolas, Koladiya and Kebrle (2005) presented a methodology for the synthesis of 6-bar linkages dwell and dual-dwell mechanisms with prescribed timing and transmission angle constraints [8].

Sonmez (2007) introduced new classes of compliant long dwell mechanism designs incorporating the buckling motion of flexible links [9].

Jagannath (2011) introduced an approach for the design of planar 6-bar linkage with rotating joints producing two dwells. He used a genetic algorithm-based optimization scheme for solving the resulting optimization problem [10].

II. ANALYSIS

A 6-bar single-dwell planar mechanism with translational output has the structure shown in Fig.1 [11].

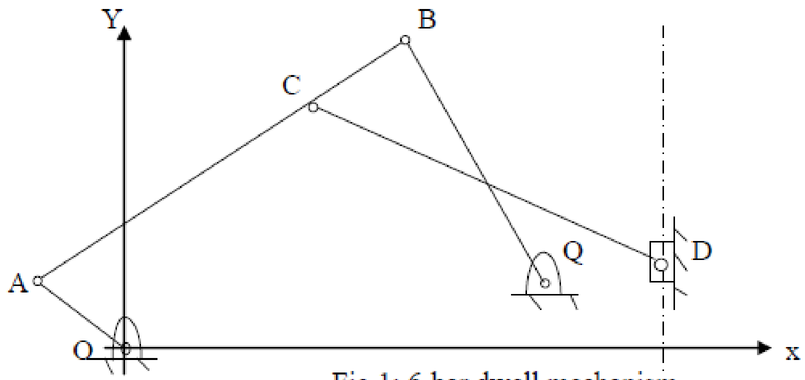

Fig.1: 6-bar dwell mechanism.

Mathematical models are build for the whole mechanism using two displacement polygons as shown in Fig.2.

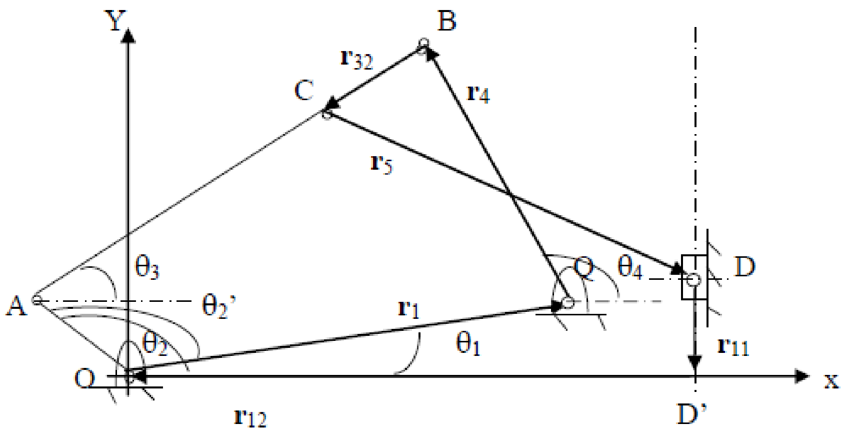

Fig.2: Displacement analysis of the 6-bar dwell mechanism.

Polygon 1: OABQO

This polygon reveals two kinematical equations relating the orientation of vectors AB and BQ [12]. Those equations are nonlinear and can be solved numerically or using MATLAB through its command “fsolve”.To avoid solving nonlinear equations, geometrical relationships can be used revealing θ_3 (orientation of AB) and θ_4 (orientation of QB) as [13]:

$$\theta_3 = \mu - \beta - \theta_1$$

where:

$$\mu = \tan^{-1} \{r_4 \sin\gamma / (r_3 - r_4 \cos\gamma)\}$$

$$\beta = \tan^{-1} \{r_2 \sin\theta_2' / (r_1 - r_2 \cos \theta_2')\}$$

$$\theta_2' = \theta_2 - \theta_1$$

$$\gamma = \cos^{-1} (A_1 + B_1)$$

$$A_1 = (r_3^2 + r_4^2 - r_1^2 - r_2^2) / (2r_3r_4)$$

$B_1 = [r_1 r_2/(r_3 r_4)] \cos \theta_2'$

And $\theta_4 = \square - \beta - \sigma + \theta_1$

Where $\sigma = \square - \mu - \gamma$

Polygon: OQBCDD'O

The displacement analysis vector equation across this polygon is (see Fig.2):

$\mathbf{r}_1 + \mathbf{r}_4 + \mathbf{r}_{32} + \mathbf{r}_5 + \mathbf{r}_{11} + \mathbf{r}_{12} = 0$

Resolving in the x- direction gives:

$$r_{1x} + r_{4x} + r_{32x} + r_{5x} + r_{11x} + r_{12x} = 0 \qquad (1)$$

where: $r_{1x} = r_1 \cos \theta_1$, $r_{4x} = r_4 \cos \theta_4$

$r_{32x} = r_{32} \cos \theta_{32}$, $r_{11x} = 0$

$r_{12x} = r_{12}$

Resolving in the y-direction gives:

$$r_{1y} + r_{4y} + r_{32y} + r_{5y} + r_{11y} + r_{12y} = 0 \qquad (2)$$

where: $r_{1y} = r_1 \sin \theta_1$, $r_{4y} = r_4 \sin \theta_4$

$r_{32y} = r_{32} \sin \theta_{32}$, $r_{11y} = r_{11}$

$r_{12y} = 0$

The orientation θ32 is related to θ3 through:

$$\theta_{32} = \theta_3 + \square$$

The vector component r_{5x} is given from Eq. 1 by:

$$r_{5x} = - r_{1x} - r_{4x} - r_{32x} - r_{11x} - r_{12x}$$

As r_5 is the length of link 5, the component r_{5y} is given by (Fig.2):

$$r_{5y} = - \sqrt{\{r_5^2 - r_{5x}^2\}}$$

Now, Eq. 2 gives r_{11} as:

$$r_{11} = - r_{1y} - r_{4y} - r_{32y} - r_{5y} \qquad (3)$$

Eq.3 gives the output slider displacement, y_D as:

$$y_D = - r_{11} \qquad (4)$$

Mechanism transmission angle:Any mechanism has to be synthesized such that its transmission angle is not far away from an optimal value [14]. In the present 6-bar mechanism, transmission angle can be considered in 2-locations in the mechanism:

[1] Transmission angle between the coupler and rocker of the 4-bar linkage (OABQ).
[2] Transmission angle between link 5 and the slider D (output link).

4-bar linkage transmission angle:

This is angle ABQ in Fig.1. It has a minimum and maximum values ($TA_{4barmin}$ and $TA_{4barmax}$) when the crank is co-linear with the coupler. That is [15]:

$TA_{4barmin} = \cos^{-1} \{[r_3^2 + r_4^2 - (r_1 - r_2)^2] / (2r_3r_4)\}$ (5)

And $TA_{4barmax} = \cos^{-1} \{[r_3^2 + r_4^2 - (r_1 + r_2)^2] / (2r_3r_4)\}$ (6)

Output link transmission angle: This transmission angle, TA_{out}, is related to the orientation of the position vector $\mathbf{r}_5$ (θ_5) through:

$TA_{out} = 2\pi - \theta_5$

III. REQUIREMENTS

It is required that:

[1] The mechanism output, y_D dwells once during the rotation of the crank for one revolutions.
[2] The dwell motion of the slider is through a specific crank angle θ_2.
[3] The transmission angle of the 4-bar linkage OABQ and at the output slider are within the recommended range (between 45 and 135 degrees [12]).

IV. OPTIMAL MECHANISM SYNTHESIS

(a) Objective function

An error function is used to define the objective function of the optimization process. The error, e in this case is between the slider displacement y_D and the desired slider displacement, y_{Ddes} at specific crank angles. That is:

$e = y_D - y_{Ddes}$ (8)

The objective function F is the integral of absolute error (IAE). That is:

$F = \int |e|\, dt$ (9)

(b) Functional constraints

The performance of the 6-bar planar dwell mechanism is controlled using a three functional constraints:

(i) The minimum transmission angle of the 4-bar linkage, $TA_{4barmin}$:

$C_1 = (45*\pi/180) - TA_{4barmin}$ (10)

(ii) The maximum transmission angle of the 4-bar linkage, $TA_{4barmax}$:

$C_2 = TA_{4barmax} - 135*\pi/180$ (11)

(iii) The minimum transmission angle of the output slider, TA_{outmin}:

$C_3 = (45*\pi/180) - TA_{outmin}$ (12)

(c) Design parameters

The design parameters of the 6-bar planar dwell mechanism are:

- Ground orientation: θ_1.
- Ground link: r_1.
- Crank: r_2.
- Coupler: r_3.
- Rocker: r_4.
- Mini-coupler: r_{32}.
- Output coupler: r_5.

(d) Optimazation technique

MATLAB optimization toolbox is used to provide the optimal design of the mechanism under study [16,17].

APPLICATION

Suppose that it is required to synthesise a 6-bar planar single dwell mechanism having:

- Slider line of action at 800 mm from the fixed origin O.
- Slider dwells at 100 mm during a crank angle from 300 to 360 degrees.

A MATLAB code is written using the methodology presented in this paper. The code results are as follows:

- Ground orientation: θ_1 = 100 degrees
- Ground link: r_1 = 617.9 mm
- Crank: r_2 = 100 mm
- Coupler: r_3 = 698.805 mm
- Rocker: r_4 = 511.840 mm
- Mini-coupler: r_{32} = 0
- Output coupler: r_5 = 347.180 mm

The output slider displacement for one revolution of the crank of the optimally synthesized mechanism is shown in Fig.3.

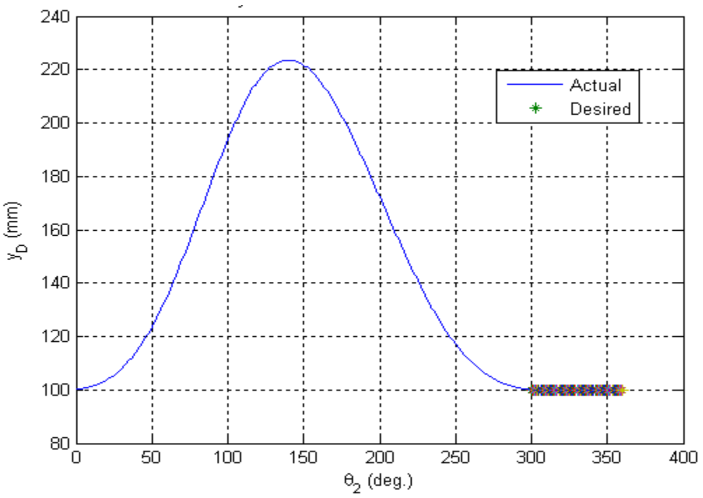

Fig.3 Output slider displacement.

Characteristics of the optimally synthesized mechanism:

- 4-bar minimum transmission angle: 47.63 degrees
- 4-bar maximum transmission angle: 70.83 degrees
- Output slider minimum transmission angle: 123.0 degrees
- Output slider maximum transmission angle: 144.5 degrees
- The 4- bar linkage is a Grashof crank-rocker one allowing complete rotation of the crank (shortest link).
- The transmission angle of the 4-bar linkage against the crank angle is shown in Fig.4.
- The transmission angle at the output slider against the crank angle is shown in Fig.5.

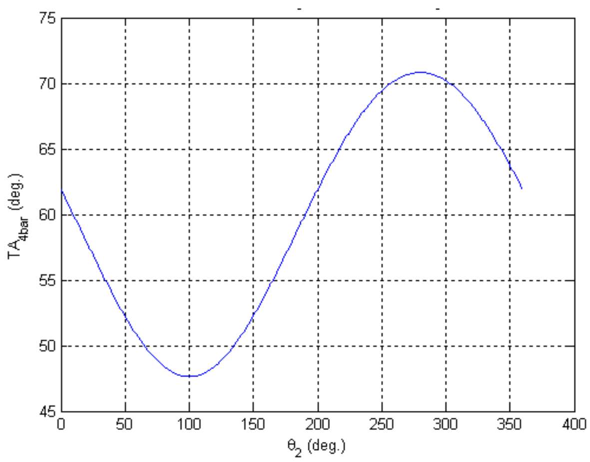

Fig.4: The 4-bar linksge transmission angle.

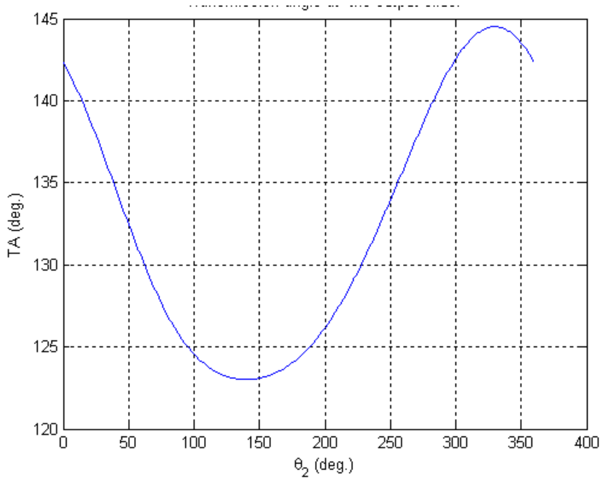

Fig.5: The mechanism transmission angle at the output slider.

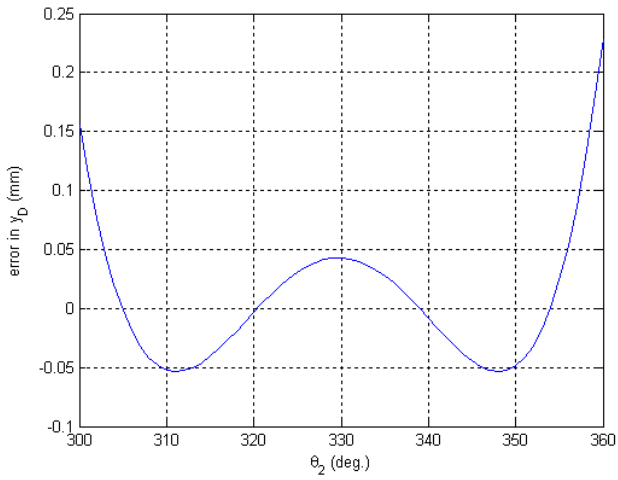

Fig.6: Error in the slider displacement.

V. DISCUSSIONS

- It is possible to synthesize accurately a 6-bar planar single dwell mechanism using nonlinear optimization.
- The proposed approach relied on defining an IAE objective function and three functional constraints.
- Constrains were set on the transmission angles of the 4-bar linkage of the mechanism and on its output slider transmission angle.
- All the values of the transmission angle during operation of the optimally synthesized mechanism were within the recommended range.
- It was possible in the application presented to attain a single dwell motion over 60 degrees of the crank rotation with a maximu error of 0.229 %.

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