Independent and Total Strong (Weak) Domination in Fuzzy Graphs

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ABSTRACT

Let G be a fuzzy graph. Then D V is said to be a strong (weak) fuzzy dominating set of G if every vertex v ε V- D is strongly (weakly) dominated by some vertex u in D. We denote a strong (weak) fuzzy dominating set by sfd-set(wfd-set). The minimum scalar cardinality of a sfd-set(wfd-set) is called the strong(weak) fuzzy domination number of G and is denoted by TGD- number γ_{sf} (G)(γ_{wf} (G)). In this paper we introduce the concept of total strong (weak) domination in fuzzy graphs and obtain some interesting results for this new parameter in fuzzy graphs.

KEY WORDS: *Fuzzy Graphs, Fuzzy Domination, Total domination,Fuzzy independent set, strong(weak)fuzzy domination number, Total strong(weak)fuzzy domination number.*

I. INTRODUCTION

The study of dominating sets in graphs was begun by Ore and Berge, the domination number, independent domination number are introduced by Cock-ayne and Hedetniemi. Rosenfeld [1] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness.

A.Somasundram and S.Somasundram [3] discussed domination in fuzzy graphs. They defined domination using effective edges in fuzzy graph. Nagoor Gani and Chandrasekaran [1] discussed domination, independent domination and irredundance in fuzzy graph using strong arcs. C.Natarajan and S.K.Ayyaswamy [4] introduced strong(weak) domination in fuzzy graphs. The concept of Strong (Weak) domination [8] in graphs was introduced by Sampathkumar and Pushpalatha. The notion of Domination of Domination in fuzzy graphs [3] was developed by A.Somasundaram and S.Somasundaram.

II. FUZZY INDEPENDENT SET:

Definition 1.1

Let G = (σ, μ) be a fuzzy graph. Two nodes in a fuzzy graph G are said to be fuzzy independent if there is no strong arc between them. A subset S of V is said to be fuzzy independent set for G if every two nodes of S are fuzzy independent.

Definition 1.2

Let G = (σ, μ) be a fuzzy graph. A fuzzy independent set S of G is said to be maximal fuzzy independent set if there is no fuzzy independent set whose cardinality is greater than the cardinality of S. The maximum cardinality among all maximal fuzzy independent set is called fuzzy independence number of G and it is denoted by γ (G).

FUZZY DOMINATING SET

Definition 1.3

Let G = (σ, μ) be a fuzzy graph. A subset D of V is said to be a dominating set of G if for every v ε V - D; there exists a u ε D such that u dominates v.

Definition 1.4

A dominating set D of a fuzzy graph G is called minimal dominating set of G is there does not exist any dominating set of G, whose cardinality is less than the cardinality of D. Minimum cardinality among all minimum dominating set in G is called domination number of G is denoted by $\gamma(G)$. The smallest cardinality of all independent fuzzy dominating set of G is called independent fuzzy domination number of G and is denoted by i (G).

FUZZY DOMINATION NUMBER:

Definition 1.5

Let G = (V,σ, μ) be a fuzzy graph. Then D V is said to be a fuzzy dominating set of G if for every v ε V- D, there exists u in D such that $\mu(u,v) = \sigma(u) \Lambda \sigma(v)$. The minimum scalar cardinality of D is called the fuzzy domination number and is denoted by $\gamma_f(G)$.

NEIGHBOURHOOD AND EFFECTIVE DEGREES OF A VERTEX:

Definition 1.6

Let G be a fuzzy graph. The neighbourhood of a vertex v in V is defined by N (v) = {u ε V; $\mu(u,v) = \sigma(u) \Lambda \sigma(v)$}. The scalar cardinality of N (v) is the neighbourhood degree of v, which is denoted by $d_N(v)$ and the effective degree of v is the sum of the weights of the edges incident on v, denoted by $d_E(v)$.

STRONG (WEAK) FUZZY VERTEX DOMINATION:

Definition 1.7

Let u and v be any two vertices of a fuzzy graph G. Then u strongly dominates v (v weakly dominates u) if

(i) $\mu(u,v) = \sigma(u) \Lambda \sigma(v)$ and (ii) $d_N(u) \geq d_N(v)$.

STRONG (WEAK) FUZZY DOMINATION NUMBER:

Definition 1.8

Let G be a fuzzy graph. Then Then D V is said to be strong(weak) fuzzy dominating set of G if for every vertex v ε V- D is strongly(weakly) dominated by some vertex u in D. We denote a strong(weak) fuzzy dominating set by sfd-set(wfd-set). The minimum scalar cardinality of a sfd-set(wfd-set) is called a strong(weak) fuzzy domination number of G and it is denoted by
$\gamma_{sf}(G)(\gamma_{wf}(G))$.

TOTAL DOMINATION:

Definition 1.9

The set S is said to be total dominating set if for every vertex v ε $\gamma(G)$, v is adjacent to atleast one vertex of S.

TOTAL STRONG (WEAK) FUZZY VERTEX DOMINATION:

Definition 1.10

Let u and v be any two vertices of a fuzzy graph G. Then u strongly dominates v (v weakly dominates u) if

(i) $\mu(u, v) \geq \sigma(u) \Lambda \sigma(v)$ and (ii) $d_N(u) \geq d_N(v)$ and (iii) for every vertex v ε $\gamma(G)$; v is adjacent to atleast one vertex of S.

Proposition 1.11

Let D be a Total minimal sfd-set of a fuzzy graph G. Then for each vεD, one of the following holds:

1. No vertex in D strongly dominates V.
2. There exists u ε V- D such that v is the only vertex in D which strongly dominates u.
3. If for every vertex v ε V (G), v is adjacent to atleast one vertex of S.

Proposition 1.12

For a fuzzy graph G of order p,

(1) $\gamma_{tf}(G) \leq \gamma_{tsf}(G) \leq p - \Delta_N(G) \leq p - \Delta_E(G)$.

(2) $\gamma_{tf}(G) \leq \gamma_{tsf}(G) \leq p - \delta_N(G) \leq p - \delta_E(G)$, where $\Delta_N(G)[\Delta_E(G)]$ and $\delta_N(G)[\delta_E(G)]$ denote the maximum and minimum neighbourhood degrees(effective degrees) of G.

Definition 1.13

A set D contained V of a fuzzy graph G is said to be Total Independent if

(i) $\mu(u,v)= \sigma(u) \Lambda \sigma(v)$ for all u,v in D

(ii) if for every vertex v ε V(G), v is adjacent to atleast one vertex of S.

MINIMUM AND MAXIMUM NEIGHBOURHOOD OF VERTEX DEGREE

Definition 1.14

$V_{\delta N} = \{v \varepsilon V : d_N(v) = \delta_N(G)\}$ and $V_{\Delta N} = \{v \varepsilon V : d_N(v) = \Delta_N(G)\}$.

Lemma 1.15

Let G is a fuzzy graph. If D is an TIWFDS (Total Independent Weak Fuzzy Dominating Set) of G, then $D \cap V_{\delta N} \neq \Phi$.

Proof:

Let v ε $V_{\delta N}$. Since D is an TIWFDS, V ε D or there exists a vertex u ε D such that $\mu(u,v)= \sigma(u) \Lambda \sigma(v)$ for which $d_N(u) \leq d_N(v)$.

If v ε D, then clearly $D \cap V_{\delta N} \neq \Phi$. On the other hand, $d_N(u) = d_N(v)$, since $d_N(v) = \delta_N(G)$ which implies u ε $V_{\delta N}$. Therefore, $D \cap V_{\delta N} \neq \Phi$.

Proposition 1.16:

Let G be a fuzzy graph. Then $i_{twf}(G) = p - \delta_N(G)$ iff V- (v) is Total independent for every v ε $V_{\delta N}$.

Proof:

Let D be an TIWFDS of G. Then by Lemma 1.15, $D \cap V_{\delta N} \neq \Phi$.

Let v ε $D \cap V_{\delta N}$. since D is independent, $D \cap N(v) = \Phi$ which implies D contained V – N(v) which implies $|D|_f \leq |V - N(v)|_f$

Then $i_{twf}(G) \leq |D|_f \leq |V - N(v)|_f$.

We state the following results without proving theorem, since they are analogous to the results on $i_{twf}(G)$.

Lemma 1.17

Let G be a fuzzy graph. If D is an TISFDS of G, then $D \cap V_{\delta N} \neq \Phi$.

Proposition 1.18

For a fuzzy graph G, $i_{tsf}(G) \leq p - \Delta_N(G)$.

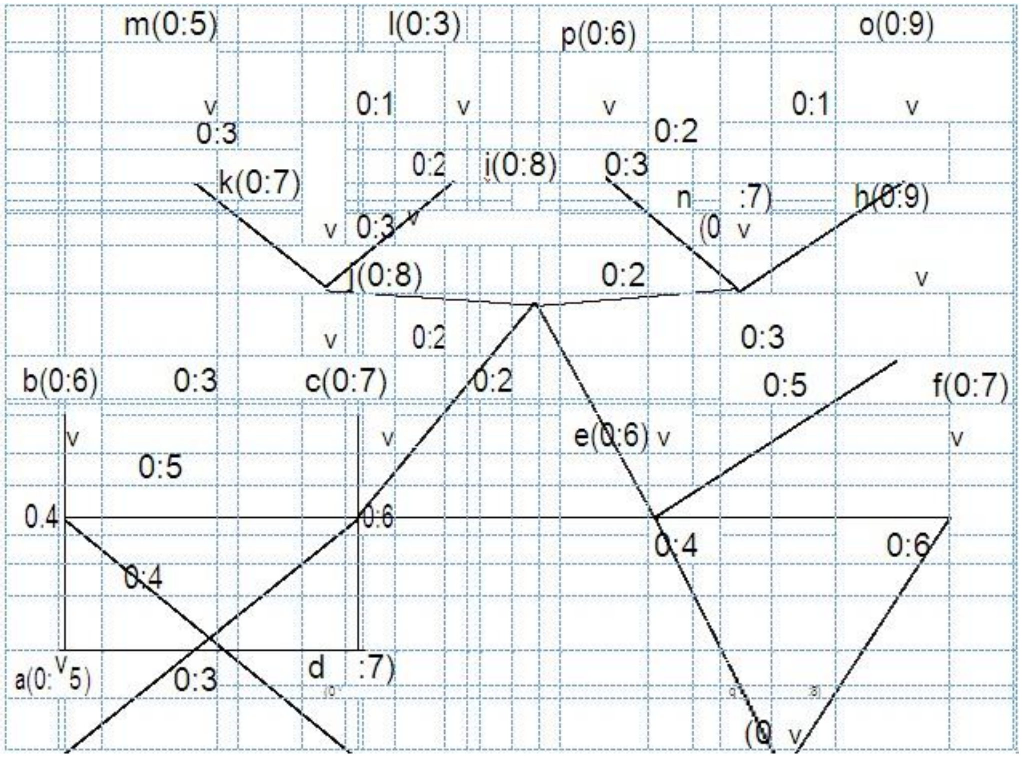
Proposition 1.19

Let G be a fuzzy graph with $i_{tsf}(G) \leq p - \Delta_N(G)$ and Let v ε $V_{\Delta N}$ Then V – N(v) is independent.

Proposition 1.20

Let G be a fuzzy graph. Then $i_{tsf}(G) \leq p - \Delta_N(G)$ iff V - N(v) is independent for every v ε $V_{\Delta N}$.

EXAMPLE:



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