Modelling Mutualism: A Mathematical Model Of Plant Species Interactions In A Harsh Climate

[1],E.N. Ekaka , [2,]E.O.Chukwuocha , Eziaku Osarolube, E.H. Amadi,

ABSTRACT.

Ecologists Have Observed That, In A Harsh Environment, Plant Species May Cease To Compete For Resources, But Display Positive Interactions Such As Mutualism. This Means That The Impact Of Climate Change, Where Harsh Envi- Ronments May Become More Benign, Could Be To Change The Nature Of Interac- Tions Between Plant Species And This May Be One Influence On Biodiversity. In This Talk, We Have Considered Whether The Observed Positive Interactions May Be Explained Through A Combination Of Existing Models Of Competition With Other Known Features Of The Environment In The Arctic (Where These Observa- Tions Have Been Made). This Provides A Prototype Model Where Sensitivity Of The Ecosystem To Different Types Of Environmental Change May Be Considered.

***KEY WORDS** : And Phrases. Mathematical Models, Plant Species Interactions, Competition, Mutualism.*

I. INTRODUCTION

Following Our Previous Mathematical Modelling Of Lotka-Volterra-Like Competi- Tion Models ([22], [26]), We Have Selected The Following Parameters In The Analysis Of The Problem We Propose To Study In This Paper: The Intraspecific Coefficient Values Of 0.00165764; The Interspecific Coefficient Values Of 0.0016 For The First Variety Of Sorghum And 0.0015 For The Second Variety Of Sorghum; The Daily Intrinsic Growth Rate Of 0.15 For The Second Variety Of Sorghum And The Daily Intrinsic Growth Rate Of 0.16 For The First Variety Of Sorghum. Without A Detailed Mathematical Analysis That Leads To The Next Background Vital Ecological Information, Using This Set Of Model Parameters Provides Us With Four Possible Steady-State Solutions Namely The Trivial Steady-State (0, 0) Where The Two Varieties Of Sorghum Will Go Extinct Followed By Two Other Steady-State Solutions (0, 90.49) And (96.52, 0) When Either Of The Varieties Will Tend To Survive At Its Carrying Capacity. These Two Varieties Of Sorghum Will Coexist Together When The Biomass Of The First Variety Is 72.53 And The Biomass Of The Second Variety If 24.86. Since The Inhibiting Effect Of The Second Variety On The Growth Of Variety 1 Is 0.96 [Obtained By Dividing The Interspefic Coefficient Of 0.0016 Of The First Variety By Its Intraspecific Coefficient Of 0.00165764] ([22]) Is Less Than The Ratio Of The Carrying Capacity Of The First Variety To The Carrying Capacity Of The Second Variety [That Is Dividing 96.5228 By 90.4901], It Follows That The First Variety Of Sorghum Will Survive Under This Simplifying Tested Formula. The Second Variety Of Sorghum Will Also Survive Because The Inhibiting Effect Of The First Variety On The Growth Of The Second Variety Of Sorghum Is 0.905 Which Is Less Than 0.9375 Being The Ratio Of The Carrying Capacity Of The Second Variety Of Sorghum To The Carrying Capacity Of The First Variety Of Sorghum.The Topic Of This Research Study Will Tackle A Challenging Interdisciplinary Prob- Lem By Using The Tool Of Mathematical Modelling, Environmental And Applied Physics, Computational Science, And Numerical Simulation Of Plant Species Interactions In A Harsh Climate. As A Matter Of Fact, According To The Declarations Of The 1992 Earth Summit, Interdisciplinarity Was Cited Repeatedly As One Of The Means For In- Creasing Our Understanding Of And Developing Solutions To Pressing Environmental Issues Such As Sustainable Resource Development, Climate Change, Ecosystem Rehabi-

Tation To Mention A Few ([68]). Interesting Enough, Interdisciplinary Approaches Have Moved On To Consider Issues Other Than Broad Global Issues. In This Context, Interdisciplinarity Has Facilitated Research On Subjects Which Are More Narrow In Scope. For Example, Mutualism Has Been Suggested As An Important Factor Of Com- Munity Stability In General ([35], [44], [54], [44], [10], [63]). On The Other Hand, We Know From These Authors That Population Dynamics Of Mutualistic Interactions Are Rarely Described Except In The Case Of Positive-Density.In This Work, We Shall Attempt To Adapt Numerical Methods To Solving This Novel Ecological Problem With The Expectation Of Providing Further Insights And Contribut- Ing New Knowledge. Driving This Motivation Is Our Recognition Of The Complexity Of Inhospitable Arctic Environments And The Complex Links Between Ecological And Dynamical Systems.One Of The Well Known Ecological Interpretations Of Understanding The Interaction Between Plant Species Is Through The Process Of Competition. But In A Harsh Climate Where It Takes A Longer Time To Understand If The Process Of Competition Is Taking Place Which Is Very Rare, We Choose To Assume A Summer Growing Season Where Competition Takes Place Along With A Winter Season Where Occasional Frequency Of Storms May Affect The Biomass.

This Research Study Will Attempt To Tackle The Following Issues:First, We Would Consider Issues Relating To Global Warming, Lengthening Summer And Shortening Winter. This Would Be Followed With A Brief Introduction To TheConcepts Of Mathematical Modelling And Numerical Simulation. Then, We Would Consider The Central Purpose Of This Thesis, A Few Observations Of Ecologists That Directly Relate To Our Investigation. This Will Be Followed With A List Of Objectives That This Thesis Expects To Achieve.Second, We Shall Consider The Main Methodology Which We Have Used In The Analysis Of Our Summer-Winter Model.Third, We Shall Define And Discuss The Key Ecological Hypotheses And Other Re- Search Questions On Which This Thesis Is Designed. It Is Very Important To Define And Discuss In Detail Other Important Factors And Issues That Affect The Growth Of Plant And Plant Species Interactions. For Example, We Need To Understand The Concept Of The Kinetics Of Plant Growth, Competitive Exclusion And Species Coexistence Among Other Related Ecological Concepts That Would Provide Insights To Understanding The Process Of Plant Growth And The Dynamics Of Plant Species Interactions.In This Talk, We Would Also Consider Five Types Of Plant Species Interactions On Which Our Subsequent Mathematical Analysis And Simulations Would Be Based. This Introductory Chapter Ends With A Conclusion That Points Out What We Would Expect The Next Chapters To Achieve.

1.1. Global Warming. One Of The Effects Of A Climate Change Will Take The Form Of A Significant Global Warming. This Change Is Expected To Be Most Pronounced At Polar Latitudes ([4]). As A Result Of This, Plant Species Are Predicted To Change In Response To Changing Climates ([4], [28], [46], [61], [25], [20], [62], [64], [60], [57], [40], [37]). In Particular, [62] Have Shown That A Warmer Climate Could Lead To New Competitive Relationships Between Plant Species That Will Consequently Diminish The Reproductive Capacity Of Plant Species.

1.2. Lengthening Summer And Shortening Winter. Plants Require Specific Grow- Ing Season Lengths To Complete Their Life Cycles. These Requirements Are Said To Vary Significantly With Different Species ([36]). For Example, Red Raspberries Which Are Produced In Scotland Requires A Short, Cool Growing Season While In The Tropical And Subtropical Regions, Sugarcane Requires Long, Hot, Humid Growing Seasons. On The Other Hand, Other Plant Species Can Grow And Perform Better Over A Wide Range Of Temperatures And Length Of Season.The Quantification Of Lengthening

A Summer Season And Shortening A Winter Sea- Son Has Been Reported In The Literature ([39]). According To These Researchers, The Summer Season Is Said To Be Lengthened Significantly By 11 Days Whereas The Winter Season Is Said To Be Contracted Or Shortened By 30 Days. These Climate Changes Could Alter The Complex Interactions Between Plant Species.According To A Global Warming Resource ([34]), It Was Reported That Summer Days Without Snow Cover Have Increased From Fewer Than 80 In The 1950′S To More Than 100 In The 1990′S. In The Same Context, A Group Of Other Researchers Have Reviewed The Evidence That Global Warming Has Affected The Growth Period Of Plants And Also Reported That The Lengthening Of The Growing Season Can Contribute To The Global Carbon Fixation ([55]). Hence, The Lengthening Of A Summer Growing Season Is More Likely To Enhance The Process Of Competition Than Facilitation.

1.3. Other Factors Of A Benign Environment. The Initial Biomass Is An Im- Portant Benign Factor That Can Play A Key Role In The Shift Between Positive And Negative Interactions Along Environmental Gradients ([30], [32], [41], [13]). Another Important Factor Of A Benign Environment Is The Intensity Of Species Interactions ([17], [13]).

1.4. Regional Variation Of Frequency Of Storms In The Arctic. Just As Chronic Wind Is An Important Ecological Parameter, So Is The Impact Of Fierce Storms OnThe Biomass ([24]). According To The Arctic Data Source, It Was Reported That The Frequency Of Storms Was Greatest During The Months Of June, July, And August With An Average Of Two Or Three Per Month ([1]). This Occurence Of Storms Enables Us To Choose An Annual Average Of Storms To Be Between 6 And In This Thesis, We Propose To Use The Poisson Distribution To Approximate The Mean Number Of Storms Over A Period Of 10 Years Whereas We Propose To Use The Gamma Distribution To Approximate The Intensity Of Storms. These Distributions Are The Two Most Popular Models Of Studying The Occurrence Of Events In An Interval And The Increasing Intensity Of Storms In Particular.

1.5. Impact Of Temperature And Other Stresses On The Growth Of Plant Species. The Growth Of Plant Species Can Be Affected By A Range Of Abiotic Stresses Such The Temperature Stress, Soil Stress, And Ph Stress To Mention A Few ([56]). In This Thesis, We Can Investigate The Impact Of Temperature Stress On The Type Of Plant Species Interactions Indirectly By Changing Either The Daily Intrinsic Growth Rate Or The Intra-Specific Coefficient Of An Appropriate Competition Model In A Benign Cli- Mate. This Would Indirectly Provide Some Important Ecological Qualitative Insights From Our Expected Numerical Simulation In This Thesis.

1.6. Mathematical Modelling. Mathematical Modelling Is An Integral Part Of Attempting To Understand The Dynamics Of A Given Scientific Problem Which Is Fa- Miliar In The Mathematical Literatures (([51], [29], [5], [6], [9], [33]). In General, A Mathematical Description Of A System Serves To Put Our Knowledge Of That System Into A Rigorous Quantitative Form That Is Subject To Rigorous Testing. In This Sense, We Would Mention That A Mathematical Model Serves As An Embodiment Of A Hy- Pothesis About How A System Is Constructed Or How It Functions. We Also Think That The Model Forces One To Focus Thinking And Make Inexact Ideas More Precise.

In The Context Of This Thesis, We Intend To Use Only A System Of First Order Coupled Differential Equations To Study The Interaction Dynamics Between Two Competiting Plant Species. Other Appropriate Models Involving Partial Differential Equations, Dif- Ference Equations, Delay Equations, And Other Types Of Functional Differential Equa- Tions Can Be Extended By Another Researcher To Model The Interaction Dynamics Between Two Competiting Plant Species.

1.7. Numerical Simulation. A Numerical Simulation Is A Satisfactory Method Of Tackling A Mathematical Model Which Has Complex Characteristics And Does Not Have A Closed-Form Solution ([5], [6], [9]). It Is An Important Component Of Developing A Mathematical Model. This Viewpoint Is Consistent With The General Consensus That As Fields Of Science Develop, Dissemination Of Knowledge Seems To Evolve In Theory From Analytic To Numerical Solutions ([27]).

We Learn From This Author That, As Soon As A Theoretical Formulation Is Well Defined And Validated For Simple Test Equations, The Next Stage Of Analysis Would In- Volve The Application Of The Theory To Understanding More Complex Systems. When The System To Be Solved Becomes Very Complex, That Is, When The Model Equations That Describe The Phenomena Being Considered Consist Of Many Many Parameters, Familiar Analytic Mathematical Techniques Will In Most Scenaria Fail To Provide Pre- Cise Solutions. It Is At This Point That Numerical Simulation Or Computational Science Becomes An Important Mathematical Technique.For Example, To Study The Mathematical Modelling Of Plant Species Interactions In A Harsh Climate Which Is Motivated By A System Of Complex Model Equations, The Application Of A Numerical Simulation Is Inevitable In Order To Draw Useful Ecological Insights ([26]).1.8. Purpose Of This Talk. Our Primary Goal In This Study Is To Use The Tool Of Numerical Simulation To Investigate The Effect Of Climate Change On The Extent Of Obtaining Cases Of Mutualism And Facilitation From A Combination Of Our Summer Competition Model And Our Stochastic Winter Model Which Are Consistent With Widely Accepted Ecological Theories. Our Other Secondary Goals Are To

- Find Out How Sensitive The Environment Is To Particular Model ParametersThat Can Be Affected By Climate Change.
- Find Those Model Parameters,Which When Varied, Have The Biggest Effect On The Approximate Solution Of A System Of Nonlinear Deterministic ModelEquations Of Competition Interaction.
- Find Those Winter And Summer Parameters Which When Varied Will Lead ToChanges In The Interaction Behaviour.

1.9. Observations Of Ecologists. The Idea That Interactions Between Plant Species Are Affected By Some Environmental Conditions Such As Changes In Weather Condi- Tions In Which The Species Grow Is Well Established([66],[19], [21], [15]) And Several Other References Which Are Cited By These Authors. According To These Authors, The Prediction Of Ways That Changes In The Environment Will Affect Biodiversity Is Of Particular Concern.
Nevertheless, These Authors Have Reported That, In Delicate Ecosystems, The Pres- Ence Of Research Scientists May Pose A Major Influence On The Environment And On The Expected Scientific Results That Would Be Obtained.

1.10. ObJectives Of Research. The Key Objectives Of This Study Are To
- Develop A Model That Will Accept As Input Data Details Of The EnvironmentalFactors And The Distribution Of Different Plant Species.
- Develop A Model That Will Provide Predictions Of Future Distributions Of TheInteracting Plants Over Time, Taking Account Of Various Hypotheses Regarding Climate Variations.
- Find Which Model Parameters When Varied Have The Biggest Effect On TheSolutions.
- Decide On A Method Of Calculating The Effect Of Summer And Winter Param- Eters On The Biomass.
- Investigate The Possibility Of Using An Ecological Simulation To Obtain Mutu- Alism And Facilitation From A Combination Of A Summer Competition Model And A Stochastic Winter Model Due To A Variation Of Winter Model Parame- Ters.
- Find Out The Critical Environmental Factors That Can Cause Mutualism And Facilitation To Change To Other Patterns Of Plant Species Interactions.

II. RESEARCH METHODOLOGY

A Research Methodology Is An Important Part Of Developing A Mathematical Model ([53], [5], [6]). Our Research Methodology Consists Of Three Main Phases Namely The Modelling Phase, The Simulation Phase, And The Review/Revisit Phase.

2.1. Modelling Phase And Its Challenges: The Modelling Phase Of Our Research Methodology Considers Three Main Issues Namely
(1) Issues About Species Interactions. (2) Issues About Data Availability.
(3) Issues About Parameter Estimation Problem

2.1.1. Issues About Species Interactions. In Terms Of Species Interactions, We Would Only Consider The Competition $(-,-)$ Interaction Between Two Competing Plant Species For Resources In Combination With A Stochastic Winter Model. Our Com- Petition Model Is Characterised By A Set Of Defining Parameters Such As The Intrinsic Growth Rates For The Two Plant Species, The Self Or Intraspecific Interaction Coeffi- Cients For The Two Plant Species, The Interspecific Interaction Coefficients For The Two Plant Species And The Starting Biomasses Over A Long Time Interval.

2.1.2. Issues About Data Availability. In Terms Of Data Availability, We Have Only Analysed The Given Plant Growth Data Provided By ([12]) Because The Results Which We Obtain By Analysing These Data Provide Useful Ecological Insights Which Are Consistent With The Key Objective Of This Thesis.
Moreover, We Could Not Find A Set Of Plant Growth Data Because Of The Constraint Of The Inhospitability Of The Arctic Climate And Lack Of Funding. In The Literature, We Are Yet To See Any Other Analysis Of These Data Using Our Method Of Analysis. Despite The Problem Of Data Paucity Which Is Characteristic Of Most Interdisciplinary Studies, Our Analysis Of These Available Data Forms A Background For Other Further Analyses.

2.1.3. Issues About Parameter Estimation Problem. The Problem Of Parameter Es- Timation To Be Considered In This Thesis Is Described By A System Of M Nonlinear Ordinary Differential Equations Of First Order

(2.1)Dx = F (T, X, P)

Dt That Depend On A Set Of Parameters $P \varepsilon \Re^p$ Where $X \varepsilon \Re^m$ And $T \varepsilon [0, T]$. The Initial Values X_0 = X(0) Are Usually Treated As Additional Unknown Parame- Ters And These Are Included In The Parameter Set P ([9]). We Consider The Observed Quantity Y_i As A Function Of The System State X Which Are Sampled At Discrete Times

T_i Such That

(2.2) $Y_i = G(X(T_i, P))$

For I = 1, 2, ..., N .

If X(T, P) Is The Approximate Solution Of The Above Equation For A Given Set Of Parameters P. The Objective Function Φ(P) Is Defined As The Sum Of Squared Residues Between The Data And The Model Such That

(2.3) $\Phi(P) = \Sigma^N_{i=1}$

$| Y_i - G(X(T_i, P)) |^2$

In This Talk, Our Approach Is To Choose An Error Function Which Is Also Called The Penalty Function That Measures The Agreement Between The Data And The Model. The Parameters Are Then Slightly Varied To Achieve A Minimum In The 2-Norm Penalty Function Which Will Yield The "Best-Fit" Parameters. With Nonlinear Dependencies, However, The Minimization Must Proceed Incrementally/Iteratively, That Is, Given Trial Values For The Parameters, We Develop A Procedure That Improves The Trial Solution. Our Procedure Is Then Repeated Until Φ(P) Stops Decreasing And Starts Increasing Again, Hence Indicating The Property Of A Monotone Sequence.When The Measurement Points Are Good, Our Scheme Correctly Identifies The Min- Imum Point And Hence The Best Fit Parameters Are Chosen Subject To A Relative Error Tolerance Of 0.1 Percent.

We Know That The Construction Of A Mathematical Model Is Not A Simple Task For Several Reasons. According To [42], It Is Impossibe To Identify A Single Model Structure For A Natural System Since Such A System Is Never Closed And More Than One

Model Would Appropriately Provide Reliable Realistic Result. In Some Circumstances, The Modeller Is Compelled To Use One Single Reliable Model Which Best Describes The Phenomenon Under Investigation As Long As The Construction Of This Single Model Can Be Justified With An Appropriate Numerical Scheme.Next, Models Are Built Under Uncertainties In The Values Of The Defining Param- Eters, In The Parameterization Of The System And In The Choice Of Equations That Describe Dynamics ([59], [11]).Lastly, Uncertainty Can Also Be Related To An Inherent Stochasticity Of The Model Where The Dynamics Includes A Random Term. Issues Of Parsimony In Model Identi- Fication Are Discussed In Great Depth By ([71], [5]).In An Interaction Between Two Dis-Similar Plant Species, A Parameter Which Is Numerically Characterised As Less Important Could Become An Important Parameter When An Interaction Between Two Similar Plant Species Is Considered. To Avoid This Type Of Contradiction And Inconsistency In The Interpretation Of Our Analysis, It Would Be A Good Idea To Simply Differentiate Those Parameters Which Have The Biggest Effect On The Solutions As Important Parameters And Those Which Have The Smallest Effect As Less Important Parameters.

2.2. Simulation Phase. Our Simulation Phase Is Characterised By Two Distinct Components Comprising Of The Numerical Simulation Of Our Summer Competition Model Using Fourth Order Runge-Kutta Methods And The Assumptions Leading To The Stochastic Winter Model.

2.2.1. Numerical Simulation Of Summer Competition Model. For Our Summer Season Prototype Model, We Consider

(2.4)

$Dn_1Dt=$ —— $F(A, B, C, N_1, N_2, N_1(0), N_2(0))$

WhereDn_2

Dt
$= G(D, E, F, N_1, N_2, N_1(0), N_2(0))$

(1) A Denotes The Intrinsic Growth Rate For The First Species N_1 In The Absence Of Interaction With N_2.
(2) B Denotes The Self Or Intraspecific Interaction Coefficient For The First Species N_1.
(3) C Denotes The Interspecific Interaction Coefficient Of The Second Species With The First Species Inhibiting The Growth Of The First Species.
(4) D Denotes The Intrinsic Growth Rate For The Second Species N_2 In The Absence Of Interaction With N_1.
(5) E Denotes The Interspecific Interaction Coefficient Of The First Species With The Second Species Inhibiting The Growth Of The Second Species.
(6) F Denotes The Self Or Intraspecific Interaction Coefficient For The Second Species N_2.
(7) N_1 And N_2 Are The Given Biomasses For The First And Second Plant Species.
(8) $N_1(0)$ And $N_2(0)$ Are The Given Starting Biomasses For The First And Second

Plant Species.Our Summer Competition Model Is Characterised By Two Continuous And Differ-Entiable Interaction Functions In Terms Of The Defining Model Parameters Which We Have Talked About In The Early Section Of Our Research Methodology. These Two Interaction Functions Are Solved Numerically By The Following Explicit Fourth Order Runge-Kutta Method ([38], [43]). This Numerical Method Which Is Well Established For Solving An Initial Value Problem And Also For Solving A System Of Equations Is A Procedure That Produces Approximate Solutions At Particular Points.
For A Standard System Of Two Equations, We Consider(2.6) $\frac{Dx}{Dt} = F(X, Y)$

(2.7) With Initial Conditions $\frac{Dy}{Dt} = G(X, Y)$

(2.8) $X(0) = X_0$

(2.9) $Y(0) = Y_0$

We Know That To Achieve A Higher Order Of Accuracy When Applying The Taylor Series, One Is Expected To Find Various Higher Order Derivatives. This Approach Involves Tedious Algebraic Manipulations. However, If The Derivatives Are Replaced By Evaluating $F(X, Y)$ And $G(X, Y)$ At Intermediate Points, It Becomes Possible To Achieve The Same Desired Accuracy. The Methods That Are Derived In This Way Are Called Runge-Kutta Methods But There Are Numerous Variations Of These Method. The Version Which We Have Used In This Study Is The One Proposed By ([43]).Our System Of Model Competition Equations Are Analysed Using A Fourth Order Runge-Kutta Scheme With Which The Starting Biomasses Before The Start Of Our Winter Season Can Be Calculated Under Our Assumption That In The Summer Season The Growing Conditions Are Reasonably Favourable And Species Will Compete For Resources.

2.2.2. Numerical Simulation Of Stochastic Winter Model. The Arctic Climate Is Also Characterised By A Dormant Season Called Winter. We Assume That In The Winter Season There Will Be No Further Growth And The Plant Populations Will Instead Be Subjected To Various Weather Events Such As Storms Which Lead To Destruction Of Some Or All Of The Biomass. The Simplifying Assumptions That Lead Us To Set Up Our Winter Model Will Be Considered In Detail In Chapter Five Of This Thesis. Some Ecological Questions Such As How Do We Approximate The Number And Intensity Of Storms Can Be Answered By Simulating The Poisson Probability Distribition And The Gamma Distribution In Order To Obtain Estimates For The Number And Intensity Of Storms. Detailed Definition And Analysis Can Be Found In Chapter Five Of This Thesis.

2.3. Review And Revisit Phase Of Our Summer-Winter Model. In This Section Of Our Research Methodology, We Used Our Summer Simulation Program To Obtain Solution Trajectories Over A Longer Time Interval For Other Variations Of The Length Of Summer Growing Season. This Confirms That Our Program Is Working Correctly.

2.3.1. Summer Season Prototype Model. For Our Summer Season Prototype Model, We Use Our Matlab Coded Runge-Kutta Program To Calculate Maximum Biomass For Each Plant Species. These Maximum Biomasses For The First And Second Species Form The Values At The Start Of Winter Dormant Season. Then, We Would Stop Our Simulation Of This Summer Growing Season.

2.3.2. Illustrating Our Winter Dormant Season. For Our Winter Season, We Follow The Following Steps In Our Research Methodology.

(1) Use Gamma Distribution To Model Storm Intensity.
(2) Calculate The Proportion Of The Biomass That Remains After Storm 1, After Storm 2, After Storm 3, Etc As Generated By The Poisson Probability Distri- Bution For The First Year Winter 1 For The First And Second Species.
(3) At The End Of The First Year Winter 1, Use The Biomass That Remains For Each Species To Form The Starting Biomass At The Start Of The Second Year Summer Season And Winter Season.
(4) Continue The Process For The Second Year Winter 2 For The First And Second Species.
(5) At The End Of The Second Year Winter 2, Use The Biomass That Remains For Each Species To Form The Starting Biomass At The Start Of The Third Year Summer Season And Winter Season.
(6) The Above Steps Are Repeated For Winter 3, Then Summer 4, Winter 4, Etc For 10 Summer Growing Seasons And 10 Dormant Winter Seasons.

In This Talk, We Have Used A Matlab Program To Simulate Our Summer-Winter Model.

2.4. Application Of Our Research Methodology. We Have Successfully Updated Our Summer-Winter Program And Decided On A Method

(1) For Calculating The Minimum Biomass For Each Plant Species Over A 10 Year Period Of One Example Trajectory Instead Of Exact Solutions.
(2) To Simulate 1000 Ten Year Periods With The Same Starting Values With Which We Can Calculate Our Experimental Probability Of Extinction Of Each Plant Species.
(3) To Allow Our Program To Reflect Shortened Winter And Lengtheining Summer Based On Ecological Literature Idea.
(4) To Obtain Cases Of Mutualism, Commensalism, Parasitism, Competition, And Facilitation If Possible From A Combination Of Our Summer Model And Stochas- Tic Winter Model Which Are Consistent With Dominant/Mainstream Ecological Theory.

Our Next Task Is To Discuss A Few Types Of Species Interactions Which Would Form The Background To This Study.

III. TYPES OF PLANT SPECIES INTERACTIONS

From Our Discussions So Far, We Know That When Two Species In An Ecosystem Have Some Common Activities Or Requirements, They May Interact To Some Degree. The Principal Types Of Species Interactions Are Interspecific Competition, Mutualism, Commensalism, Parasitism And Predation ([26]).

IV. OTHER RESEARCH QUESTIONS

The Hypotheses Being Considered Above Have Both Ecological And Mathematical Components. Hence, We Will Need To Rely On Some Reliable Mathematical Techniques To Answer The Related Ecological Questions.

4.1. Ecological Questions. In This Study, We Shall Attempt To Focus On A Few Important Ecological Questions. These Questions Are Not Exhaustive. As Far As We Know, These Are The Ones That Relate To Our Present Analysis.

(1) Ecologists Know How To Measure Plant Interactions Experimentally ([2]) But They Want To Know How To Measure Some Performance Variables Usually

Biomass Between Individual Plants Interacting Together And In Isolation By A Simulation Technique In The Absence Of Actual Experimentation.

(2) Ecologists Want To Find Out The Effect Of Varying The Length Of Summer Growing Season And Its Senstivity On The Probability Of Extinction Of Plant Species Over A Longer Time Interval.

(3) In The Winter Season Characterised By Occasional Fierce Storms, Ecologists Will Like To Know If Shortening The Length Of Winter Leads To Some Degree Of Extinction Of Plant Species Over A Longer Time Interval.

(4) Ecologists Also Want To Know If Global Warming Could Trigger Either The Persistence Or Extinction Of Two Interacting Plant Species.

(5) Ecologists Want To Know If They Can Use An Alternative Mathematical Method Different From Their Classical Experimental Approach To Determine Mutualism From Competition From A Combination Of A Summer Competition Model And A Stochastic Winter Model.

4.2. Mathematical Questions. In This Talk, We Would Specify The Main Questions That Mathematicians Are Interested To Tackle:

- How Do We Set Up The Summer And Winter Models?
- How Do We Approximate The Number Of Storms That Occur In The Winter Season?
- For Each Storm, How Do We Approximate The Intensity Of Storm In The Winter Season?
- How Do We Approximate The Quantity Of Biomass That Remains At The End Of Each Storm?
- To Find Out How To Select Model Parameters Of Summer Model Only.

4.3. Modelling Assumptions. This Thesis Will Consider The Following Assump- Tions Which Are Based On A Few Insights About The Arctic Climate ([58], [26])

(1) The Arctic Climate Can Be Characterised By A Growing Season Called Summer And A Dormant Season Called Winter.

(2) In The Summer Season Growing Conditions Are Reasonably Favourable And Species Are More Likely To Compete For Plentiful Resources.

(3) In The Winter Season There Would Be No Further Growth And The Plant Popu- Lations Would Instead Be Subjected To Fierce Weather Events Such As Storms Which Is More Likely To Lead To The Destruction Of Some Or All Of The Biomass.

Question: Under These Assumptions, Is It Possible To Find Those Changes In The Environment That Might Cause Mutualism From Competition (See Section 1.9.1) To Change?

4.4. Review Of Related Literatures. Related Literatures To This Study Can Be Found In

(1) Facilitation In Plant Communities: The Past, The Present, And The Future ([15]).

(2) Inclusion Of Facilitation Into Ecological Theory ([18]).

(3) The Importance Of Complexity In Model Selection ([50]).

(4) Computational And Mathematical Modelling Of Plant Species Interactions In A Harsh Climate

(5) Mathematical Modelling Of Plant Species Interactions In A Harsh Climate

(6) Modeling Mutualism Of Sorghum Species Interactions In A Context Of Climate Change: A South African Case Study ([23]).

V. NUMERICAL METHODOLOGY

ur Task In This Important Section Is To Define, Illustrate, And Discuss The Numer- Ical Methodology Which We Have Utilized In This Chapter.5.1. **Preamble.** Following The Methodology Embodied In The Work Of ([22]), We Would Start To Define The Relationship Between The Length Of A Growing Season And The Daily Intrinsic Growth Rate Hereby Denoted By The Notation R. Without Delving Into A Detailed Analysis Of This Method, On The Assumption Of The Well Established Theory Of Exponential Growth For A Plant Species, We Know That If A Plant Species Doubles Its Biomass And We Know The Length Of Its Growing Season, We Can Calculate Its Daily Intrinsic Growth Rate. For Example, If The Length Of A Growing Season Is5 Days, Our Calculated Daily Intrinsic Growth Rate Is 0.13863 Approximately. We Remark Here That These Calculations Are Only Estimations Which Are Correct To The Given Number Of Decimal Places. Next, If The Lengths Of

The Growing Season Are 10 And 20 Days, Our Calculated Daily Intrinsic Growths Are 0.069 And 0.034 Respectively. In The Same Manner, If The Lengths Of The Growing Season Are 30 And 40 Days, Our Calculated Daily Intrinsic Growth Rates Are 0.023 And 0.017 Respectively. Similarly If The Length Of The Growing Season Is Denoted By LS And The Daily Intrinsic Growth Rate Is Denoted By I GR, Then The Data Points In Each Set For These Variables Can Be De- Fined By LS = (50, 60, 70, 80, 100) And IGR = (0.014, 0.012, 0.0099, 0.0086, 0.0069). In This Scenario, We Can Deduce That As The Length Of The Growing Season Is Slightly Increased, The Daily Intrinsic Growth Steadily Decreases Because Over A Given Interval The Growth Of The Plant Species Is More Likely To Be Limited By Other Factors Within The Environment. We Observed A Similar Pattern In The Decreasing Values Of The Daily Intrinsic Growth Rate When The Plant Species Either Triples Or Quadruples, This Dimension Of Calculations Ia Not Presented In This Work Because Of Our Observation. In Summary, The Consequence Of These Calculations Are Also Consistent With A Similar Idea Of Mathematical Modelling ([8]; [22]). Hence, The Exponential Growth Assump- Tion Would Become Unrealistic And The Logistic Growth Assumption In This Situation Can Be Applied.An Interesting Insight Can Be Gained By A Further Analysis Of The Data Which Have Been Provided By ([70]). Our Further Numerical Simulations Of These Data Indicate The Following Observations: For A Growing Season Of Six Weeks, The Growth Data Of Pea For A Starting Value Of 4 Grams Are P = (4, 34.474, 66.6827, 100.7238, 136.7002, 174.7202). In This Scenario, The Weekly Growth Rate Can Be Estimated By Dividing The Beginning Of The Second

Week Data Point Which Is 34.474 By The Beginning Of The First Week Data Point Whose Value Is 4 And Taking The Logarithm Of This New Value, That Is The Logarithm Of 8.6185 Which Would Give Us The Value Of 2.153911056 For The Weekly Growth Rate. By Dividing The Weekly Growth Rate By 7, We Obtain The Daily Intrinsic Growth Rate Of 0.30770 Grams. Despite This Calculation, It Is Important To Observe The Increase Of The Starting Biomass For A Period Of Six Weeks. From These Data, The Beginning Of The Sixth Week Biomass Is 174.7202 Grams In Comparison With The First Week Starting Biomass Which Is 4 Grams. Hence, Our Calculated Percentage Increase In Biomass Over The Growing Season Of Six Weeks Is 43.68. Following ([22]), The Daily Intrinsic Growth Rate Over A Six Week Period Of Growth Is 0.089 Which We Can Obtain By Dividing The Logarithm Of The Percentage Increase In Biomass (Or 43.68) By 42 Days Being The Equivalent Of 6 Weeks. Similarly, Our Calculated Intrinsic Growth Rate Per Week Is 0.629 Which We Can Obtain By Dividing The Logarithm Of The Percentage Increase In Biomass (Or 43.68) By 6 Weeks. In Summary, From These Weekly Growth Data Of Pea, We Can Report That The Weekly Growth Rate Is Seven Times In Value Of The Daily Growth Rate. Under A Different Changing Starting Values For The WeeklGrowth Data Of Maize And Winter Wheat Crop ([70]),We Have Made A Similar Obser- Vation Which We Would Not Present In This Work. Similar Methods For Calculating Both Weekly And Daily Growth Rates For The Growth Of Agricultural Crops In Africa Will Be Applied In Our Subsequent Analysis In This Work.

5.2. Best Fit Logistic Model Parameters. Following ([22]), We Shall Find Those Logistic Model Parameters That Minimize The 2-Norm. Our Calculations Are Presented Below. What Do We Want To Find Out? We Are Interested To Find A List Of Best Fit Model Parameters Of Our Logistic Model That Minimise The Agreement Between The Provided Model And Our Simulated Model. Our Calculations Are Presented In The Table Below:

Parameter	Calculated Values Of The 2-Norm Penalty Function			
N	H	G	Gh	2 − Norm
1	0.0014215	97.50	0.1386	5.4828
2	0.001418	97.75	0.1386	5.4324
3	0.0014143	98	0.1386	5.3902
4	0.0014106	98.25	0.1386	5.3564
5	0.001407	98.50	0.1386	5.3312
6	0.0014035	98.75	0.1386	5.3145
7	0.0014	99	0.1386	5.3065
8	0.0013965	99.25	0.1386	5.3071
9	0.0013929	99.50	0.1386	5.3164
10	0.0013895	99.75	0.1386	5.3342
11	0.001386	100	0.1386	5.3604
12	0.0013825	100.25	0.1386	5.3949
13	0.001379	100.50	0.1386	5.4376
14	0.001375	100.75	0.1386	5.4880

Table 1. The Calculation Of Our 2-Norm Penalty Function From The Measured Data And Our Simulated Data

What Do We Learn From This Table Of Values For The 2-Norm Penalty Function? From The Last Column Of This Table, It Is Now Clear That The Value Of The 2-Norm Penalty Function That Minimizes This Sequence Of Values Is 5.3065 Which Corresponds To When The Value Of The Carrying Capacity Is 99 Grams Per Area Of Land Required To Grow The Sorghum Species. Our Estimated Value Of The Intraspecific Coefficient Is 0.0014. Following The Methods Of ([26]) And ([22]), We Propose To Grid Further Around The Carrying Capacity Value Of 99 Grams So As To Find If We Can Find A Further Minimum Value Of The 2-Norm Penalty Function. By Using This Same Numerical Method, We Found A Minimum Value Of The 2-Norm Penalty Function To Be 5.3057 When We Considered The Carrying Capacity Interval (C C I) Where C C I = (98.96, 99.25). In This Scenario, Our Best Fit Model Parameters Are The Daily Intrinsic Growth Rate A = 0.1386, The Intraspecific Coefficient B = 0.001398, The Carrying Capacity K = 99.09, The Starting Value Of 4 Grams, And The Growing Summer Season Of 42 Days.

VI. MATHEMATICAL FORMULATION

Hence, Our Best Candidate Nonlinear Model Among A List Of Other 13 Similar Mod- Els Which We Have Selected Using The Method Of The Penalty Function Is Formulated As.1)

$$\frac{Dn}{Dt} = N(0.1386 - 0.001398N)$$

Where The Starting Biomass Is 4 Grams Per Area.

Since We Are Interested To Construct A Competition Model Between Two Interacting Species Of Sorghum, Following ([45]), ([26]), And ([22]), We Propose The Following Nonlinear Coupled System Of First Order Ordinary Differential Model Equations Of The Form

6.2)

(6.3)

$$\frac{Dn_1}{Dt} = N_1(0.1386 - 0.001398N_1 - 0.0005N_2)$$

$$\frac{Dn_2}{Dt} = N_2(0.002 - 0.00002N_1 - 0.000015N_2)$$

Where The Starting Biomasses Are 4 Grams And 10 Grams Respectively.

From The Concept Of Doubling Time For Each Sorghum Species, The First Species Will Double Its

Biomass In 5 Days Whereas The Second Species Will Take About 346 Days To Double Its Biomass. This Is One Of Our Reasons Why The First Species Is Growing Fastly Than A Slowly Growing Second Species. Two Other Modified Versions Of These Model Equations Are

(6.4)

(6.5)

Dt

Dn2

Dt= N1 (0.15 − 0.0016N1 − 0.0012N2)

= N2 (0.12 − 0.0012N1 − 0.0016N2)

(6.6)

(6.7) Dn1

Dt

Dn2

Dt

= N2 (0.002 − 0.00002N1 − 0.000015N2)

Where The Starting Biomasses Are 4 Grams And 10 Grams Respectively.

Remark 6.1. We Remark That We Can Use Any Of These Models To Discuss Our Later Aim Of Attempting To Find The Extent Of Obtaining Mutualism From A Combination Of Our Summer Model And Stochastic Winter Model. In This Project, We Consider The Model Parameters A = 0.15, B = 0.00165764, C = 0.0005, D = 0.002, E = 0.00002, F =
0.000015. Here With A Daily Intrinsic Growth Rate Of A = 0.15 Grams, The Population Of Species 1 Can Be Expected To Double In Biomass In Around 4 Days While For A Daily Intrinsic Growth Rate Of 0.002 Grams, The Population Of Species 2 Can Be Expected To Double In Biomass In A Longer Time Frame Which Qualifies The Second Species As A Slow Growing Species. The Solution Trajectories Over 100 Days, 365 Days And 1825 Days Can Be Graphically Represented For The Model Parameters A = 0.15, B = 0.0106, C =
0.008, D = 0.148, E = 0.008, F = 0.0108. These Graphs Are Not Presented In This Paper.

Remark 6.2. In This Paper, We Will Use Our Numerical Simulation Matlab Program To Analyse And Answer The Following Questions

How Do We Approximate The Number Of Storms?

For Each Storm, How Do We Approximate The Intensity Of The Storm?

For Each Storm, How Do We Approximate How Much Biomass Remains At The End Of The Storm?

Decide On A Method For Calculating The Minimum Biomass For Each Plant Species Over The 10 Year Period Of One Trajectory.

Write A Program To Simulate 1000 Ten Year Periods With The Starting Values And Calculate Experimental Probabilities Of Extinction Of Each Species.

How Do We Upgrade Our Program To Reflect The Concepts Of Shortened Winter And Lengthening Summer And Use These To Calculate Experimental Probabil- Ities Of Extinction Of Each Species?

Can We Use Our Summer-Winter Model To Produce A Situation In Which Mu- Tualism Can Be Observed Based On A Summer Competition Model With Winter Storms?

VII. METHODOLOGY OF THE SUMMER SEASON

In This Paper, We Shall Merge The Usual Seasons Of Spring, Autumn, And Summer Into One Growing Season Called The Summer Model. In Order To Analyse This Model Subsequently, We Shall Make The Following Realistic Assumptions:

[1] In A Summer Season Which Is Characterised By A Mild Climate, We Assume A Continuous Growth Of Two Plant Species.
[2] We Assume The Possibility Of Two Plant Species N1 And N2 That Live Together And Compete With Each Other For The Same Limited Resource.
[3] We Assume That Each Population Of Plant Species Is Inhibited Not Only By Members Of Its Own Species But Also By Those Of The Other Population.
[4] We Assume Linear Growth Rates And Intra-Specfic Parameters Are The Logistic Parameters For Species N1 And N2 If They Were Living Alone.
[5] Our Deterministic Summer Model Rests On The Assumption That The Envi-
[6] Ronmental Parameters Involved With Our Model System Are All Constants Irrespective To Time And Environmental Fluctuations.

VIII. METHODOLOGY OF THE STOCHASTIC WINTER SEASON

In This Paper, We Will Define, Characterise, And Discuss The Features Of The Sto- Chastic Winter Season.

8.1. Stochastic Winter Model. The Winter Season Is Characterised By An Occa- Sional Frequency Of Storms Which Does Not Promote The Growth Of Plant Species. According To The Analysis Of Arctic Climatology, The Number Of Storms Varies Within The Arctic Region ([1]). The Occurrence Of 2 Or 3 Storms Every Three Months Pre- Supposes That We Would Expect To Have An Annual Mean Number Of Storms To Be Between 6 Storms And 9 Storms. Since The Enviornment Is So Uncertain, We Might Consider Figures Below This Range In Our Further Analysis.

8.2. Poisson Distribution. We Are Motivated To Use The Poisson Distribution Because It Is An Important Discrete Distribution Frequently Used In Engineering To Evaluate The Risk Of Damage. By Assuming That All Possible Number Of Storms In The Winter Model Occur Only One At A Time, That All Such Events Occur Independently, And That The Probability Of A Storm Occuring Is Constant Per Unit Time, We Can Describe Our Winter Model As A Poisson Process, Where The Mean Number Of Storms Is Distributed Exponentially ([48], [49]).
Hence, A Discrete Poisson Probability Density Function (Pdf) Is Defined By (8.1)

$$F(X) = E^{-\Lambda} \frac{\Lambda^{X}}{X!}$$

For $X = 0, 1, 2, \ldots$ Where $E(X) = \Lambda$, $V Ar(X) = \Lambda$.

8.3. **Gamma Distribution.** Another Concern Is That Of Measuring The Intensity Of Each Storm On The Biomass At The Start Of A Winter Season. In This Chapter, We Propose To Use The Gamma Distribution To Determine The Intensity Of Storm Under Some Chosen Shape And Scale Parameters ([48], [49]). The Gamma Distribution Is An Extension Of The Exponential Distribution Which Is Characterised By A Scale Parameter Which Describes The Spread Of The Exponential Distribution.
Hence, A Gamma Distribution Is A Two-Parameter Family Of Continuous Probability Distributions Characterized By A Scale Parameter And A Shape Parameter.
There Are Several Applications Of Gamma Distribution In Several Books Of Math- Ematical Statistics. The Probability Density Function Of A Gamma Distribution Is Defined By

$$F(X) = \begin{cases} \frac{\Lambda}{(R-1)!} (\Lambda x)^{R-1} E^{-\Lambda x} & X > 0 \\ 0 & \text{Elsewhere} \end{cases} \tag{8.2}$$

According To [52], The Structure Of A Plant Is Characterised By Its Shape And Size. We Know That Two Plant Species Can Take Several Shapes Such As Spherical, Square, Rectangular, Triangular And So On. For Example, If The Shape Of A Plant Is Spherical, It Has A Base And The Effect Of Any External Force On The Plant Can Be Studied As The Impact Of This Force Is Distributed In Terms Of Its Shape And Base Or Scale.
The Gamma Distribution Model Is Defined In Terms Of Several Values Of The Shape And Scale Parameters. But The Shape And Scale Parameters That Could Fairly Model The Physical Structure Of A Plant Species Do Not Have Precise Values. Since The Geometries Of Plant Species Differ

([52]), We Have Followed The Idea In Our Paper To Choose The Values Of $R = 5$ And $\Lambda = 1$ ([26]). Why Do We Propose To Use A Probability Distribution? Having Mentioned In Chapter 1 Of This Thesis That Actual Experimentation In A Harsh Climate Is More Costly, The Simple Relations Between The Mean Number Of Storms And Its Frequency Of Occurrence May Not Be Realised. In This Situation, We Would Think That The Best Description We Can Provide To Model The Occurrence Of Occasional Storms In A Harsh Climate Is In Terms Of A Probability Distribution.

XI. BUCKLING

Buckling Of A Column Occurs When The Euler Critical Load Is Exceeded ([67], [69]). The Euler Load Is Defined By The Formula

(9.1) $$P_E = \frac{\pi^2 EI}{L^2}.$$

Where P_E Is The Euler Buckling Load, E Is The Young's Modulus For The Material, I Is The Least Second Moment Of Area Of Cross Section, L Is The Length Of The Strut Between The Pinned Ends.The Young's Modulus Is A Measure Of The Amount Of Stress That A Plant Species Can Take Before Buckling. We Assume That One End Of A Plant Species Structure Is Fixed In The Direction Of Wind And The Above Ground Section Is Free. Let The Length Of Plant Stem Above Soil Surface Be L Units And Effective Length Be 2L Units.According To ([52]), The Euler Buckling Load For The Plant Stem For The Case L = 2L In A Wind Direction Can Be Similarly Defined By

(9.2) $$P_E = \frac{\pi^2 EI}{4L^2}.$$

A Detailed Mathematical Analysis And Proof Of Euler Bukling Formula Can Be Found In The Works Of ([67], [69]).Other Researchers Have Examined The Mechanical Effect Of Wind On The Growth Of Plants ([7], [3]). We Are Interested To Tackle The Effect Of Storm On The Ecology Of Plant Species In A Severe Arctic Region Where Growth Of Plants Is Not A Common Process.Next, We Will Explain How The Euler Buckling Load Will Be Used With Wind Speed In The Model To Determine How Much Biomass Is Destroyed In A Storm.Assume That The Force At The Base Of A Plant Of Height H Caused By Wind Speed V Is Proportional To V^2H^3, That Is, $F = Cv^2H^3$. Any Force Acting On A Material Can Be Described As Producing A Stress. The Unit For Stress Is The Pascal (P A), Which Is The Force Per Unit Area. For The Above Ground Plant Species, Assume That $P_E = F$. Then

(9.3) $$\frac{\pi^2 EI}{4L^2} = Cv^2H^3$$

From This Equation, We Can Simply Solve For C To Obtain

(9.4) $$C = \frac{\pi^2 E_{plantstem} I}{4V^2H^3L^2}$$

In The Theory Of Elasticity ([67]), The Young's Modulus Is Defined Mathematically As The Slope Of The Stress-Strain Relationship

(9.5) $$E = \frac{\Sigma}{\varrho}$$

Where The Symbol Σ Represents The Stress In The Material While The Symbol ϱ Represents The Strain In The Material.

The Amount Of Stress E That A Plant Species Can Take Before Buckling Can Be De- Termined By Dividing The Stress Exacted On The Plant By Any Change In The Dimension Of The Plant Component ([72], [52]).

For Example, If The Force Exerted On The Biomass Due To Increasing Storm Intensity Is 2 Newtons In A Patch Of Plant Species Of A 10m By 10m Dimension, Then The Stress $\sigma = \frac{2N}{100N/M^2}$ Which Is $0.02N/M^2$. We Know That The Strain Is A Dimensionless Quantity.

Since The Value Of The Strain Does Not Have A Precise Value In The Work Of Zebrowski (1991), Let Us Consider A Situation When The Strain $Q = 0.0474$.
Consider When The Length Of The Plant Species Before Winter Storm Is $\ell_1 = 2m$. What Do We Want To Find? We Want To Define And Discuss How To Calculate The Amount Of Stress That A Plant Species Can Take Before Buckling.

The Effect Of Storm On The Length Of The Stem Is Modelled By $\frac{\ell_1 - \ell_2}{\ell_1}$. Since, The Strain Is Modelled By Dividing The Change In The Length Of The Stem By The Original Length Before The Winter Storms, In This Case

(9.6) $\ell_1 - \ell_2 = 0.0474.$
From This Equation, We Know That

(9.7) $\ell_1 - \ell_2 = 0.0948.$
By Substituting For The Value Of ℓ_1, $\ell_2 = 2 - 0.0948 = 1.9052$. Therefore, The Amount Of Stress E That A Plant Species Can Take Before Buckling9.8) $E = \frac{0.02}{0.0474}$

$= 0.4219N/M^2$. What Are We Trying To Find Out? We Want To Find If A Variation Of The Young's Modulus For The Grass Species Would Have Any Impact On Our Later Calculation Of The Minimum Biomass After Each Storm And Its Implication For Approximating The Experimental Probability Of Extinction Of Each Plant Species.12

Similarly, Since The Height Of A Plant Is Approximated By $H = B^{\frac{1}{3}}$, $H^2 = B^{\frac{2}{3}}$ And $H^3 = B$. Assume That $L = H$, Then $L^2 = H^2$ And $4L^2 = 4B^{\frac{2}{3}}$. By Substituting For These Expressions In The Above Formula, We Would Obtain

(9.9) $$C = \frac{\Pi^2 EI}{4V^2 B^{\frac{5}{3}}}$$

For A Given Calculated Wind Speed And A Calculated Value For Biomass, We Can Use The Above Formula To Measure The Effect Of Fierce Storm On The Biomass Or The Effect Of Storm Intensity On The Biomass.
In Summary, Since The Young's Modulus Is Defined In Terms Of The Stress And Strain, Strain Is Dimensionless (Extension Of Material Divided By The Original Length Of Material) And Stress Is Defined As Load Per Unit Area, We Would Expect The Sectional Area To Vary From One Grass Species To Another. Therefore The Stress Is More Likely To Vary And So One Can Expect The Young Modulus E To Vary Also.

IX. ANALYSIS OF STOCHASTIC WINTER MODEL

In This Section, Our Task Is To Attempt To Analyse A Few Important Questions About The Stochastic Winter Model.

10.1. Assumptions Leading To Stochastic Winter Model. From The Literature, Taller Plant Species Are Generally Subjected To Greater Mechanical Stress Because Wind Speed Is Said To Increase With Height Above The Ground Surface ([47]). Hence, The Relationship Between Wind Speed Or Velocity V And Height Of Plant's Biomass H Can Be Defined By

(10.1) $$V = Bh^2$$

Where B Is A Positive Constant.
In Order To Construct A Meaningful Winter Model, We Shall Assume That

(1) Force At Base Of Plant Of Height H Caused By Wind Of Velocity Is Proportional To $(V^2 H^3)$.
(2) The Impact Of This Force On The Old Biomass (Or Biomass At The Start Of Winter) Causes Some Parts Of The Old Biomass To Be Destroyed.
(3) In The Winter Season There Will Be No Further Growth And The Plant Pop- Ulations Will Instead Be Subjected To Various Weather Events (Storms Etc.) Which Lead To Destruction Of Some Or All Of The Biomass ([26]).

10.2. Old Biomass And New Biomass. The Relationship Between New Biomass And Old Biomass Is Defined By New Biomass = (1-Proportion Destroyed)Times Old Biomass Where The Proportion Of Plant Species Destroyed Is Directly Proportional To The Force At Base Of Plant Of Height H, That Is, The Proportion Destroyed, Denoted By P_d , Is

(10.2) $$P_d = Cv^2 H^3$$

Where C Is A Positive Constant That Depends On A Range Of Wind Speeds, Range Of Plant Heights, Strength Of Stem, Buckling Effect, Etc In Such A Way That The Quantity $P_d < 1$ With $H^3 = B$ Where B Represents The Quantity Of Biomass.If The Value Of Force At Base Of Plant Per Unit Area Is 1 Pascal And A Positive Constant ϙ Is Assumed To Control The Error Of Computing The Values Of C, By Using

(10.3) $$F = Cv^2 H^3$$

We Shall Obtain

1 (10.4) $$C_1 = \frac{}{V^2 H^3_1 + ϙ}$$

For Species 1 And

1 (10.5) $$C_2 = \frac{}{V^2 H^3_2 + ϙ}$$

For Species 2

Here The Parameter V Measures The Effect Of Fierce Storm On The Biomass, C_1 Measures The Individual Intensity Of Storm On Species 1, C_2 Measures The Individual Intensity Of Storm On Species 2, H^3_1 Measures The Maximum Biomass At The Start Of Winter For Plant Species 1 And H^3_2 Measures The Maximum Biomass At The Start Of Winter For Plant Species 2. The Two Values Of C Are Only Calculated Once.We Have Used A Numerical Method Of Fourth Order Runge-Kutta To Simulate The Summer Only Model From Which The Maximum Biomass At The Start Of Winter Can Be Calculated.Under The Winter Model, We Are Interested To Tackle Three Important Questions: (1) How Do We Approximate The Number Of Storms? (2) For Each Storm, How Do We Approximate The Intensity Of The Storm?(3) For Each Storm, How Do We Approximate How Much "Grass" Species Remains At The End Of The Storm?

10.3. How Do We Approximate The Number Of Storms? Ecologists Are Inter- Ested About How To Determine The Number Of Storms Experimentally. But Mathe- Maticians Approximate The Number Of Storms.Having Mentioned That The Mean Number Of Storms Can Be Determined Using The Poisson Distribution, We Shall Focus On Illustrating This Idea With A Simple Example.For A 10 Year Ecological Simulation, Each Simulation Run Will Produce A Sample Of 10 Data Points Representing A Random List Of Mean Number Of Storms In The Arctic. For Example, A Possible Matlab Random Sample If Mean Number Of Storms Is 2 Is 4, 2, 3, 3, 2, 2, 2, 0, 3, 2. What These Data Mean Is That In Year 1, We Would Expect To Have 4 Storms, Followed By 2 Storms In Year 2, Then 3 Storms In Year 3 And So On. Then We Will Have No Storm In Year 8 And 2 Storms In Year 10. The Number Of Storms Varies For A 10 Year Simulation.

10.4. For Each Storm, How Do We Approximate The Intensity Of The Storm? We Have Used The Gamma Distribution To Simulate The Intensity Of 1000 Storms On The Biomass During A Ten Year Simulation Period Of One Trajectory.Given That The Other Parameters Are Positive Constants With Varying Storm In- Tensity On The Biomass And The Size Of The Biomass Before The Start Of Winter, The Storm Intensity In Our Analyis Can Be Determined By Using The Formula

(10.6) $$C = \frac{\Pi^2 E_{plantstem} I}{5 \cdot 4V^2 B^3 + ϙ}$$

Where The Parameter V Measures The Wind Speed, B Measures The Biomass, And Q Is A Small Positive Constant That Takes Account Of The Error In The Calculation. By Substituting For Parameters $E_{plantstem}$ And I As 0.4219 Pa And 1.2586, We Would Obtain

(10.7) $C_1 = 1.311253146 \frac{}{V^2 B^{\frac{5}{3}} + Q}$

(10.8) $C_2 = 1.311253146 \frac{}{V^2 B^{\frac{5}{3}} + Q}$

The Square Of The Storm Speed Constitutes A Huge Set Of Data In 1000 Storm Simulations. We Have Used A Matlab Function To Order The Data Generated By This Simulation In Terms Of Their Fierceness. The Topmost Value In This Sequence Of The Square Of The Wind Speed For 1000 Storms Represents How Fierce The Storm Would Be On The Biomass At The Start Of Winter During A Period Of Ten Years. The Next Values In The List After The Worst Effect Of Storm Represent The Individual Intensity Of Each Storm On Each Plant Species.

10.5. How Do We Approximate How Much Plant Species Remains At The End Of Each Storm? Both Analytically And Computationally, We Used The Following Formula To Approximate How Much Plant Species Remains At The End Of Each Storm: (10.9) $NB = (1 - P_d)OB$ Where NB, P_d And OB Represent New Biomass, Proportion Of Old Biomass That Is Destroyed And Old Biomass. In A Combined Summer-Winter Model, The Detail Of Our Numerical Approach Is Briefly Defined: (1) Use The Most Popular Version Of Fourth Order Runge-Kutta Method To Sim- Ulate The Summer Only Model With The Chosen Starting Values (2) Next, From This Simulation, We Calculated The Maximum Biomass For Each Species At Start Of Winter For The Start Of First Year Winter, Our Summer Model Is Simulated Only Once With Which The Maximum Biomass For Each Species Before The Impact Of Storm Is Calculated. At The End Of First Year Winter Season, The Biomass That Remains Becomes The Starting Values To Run The Summer Model For The Second Year From Which We Calcu- Lated The Maximum Biomass For Each Plant Species At Start Of Winter For The Second Year. This Procedure Is Repeated For The Ten Year Period.

10.6. Example: Determining How Much Biomass Is Destroyed During The Frequency Of Storms By An Analytical Method. Under A Different Starting Value Of Our Summer Competition Model, The Species Biomass Before The Start Of Winter Are 79.7979 Grams And 0.0541 Grams For Species N_1 And N_2. We Choose The Force At Base Of Height Of Biomass To Be 1 Pascal Whereas $V = 227.5616$ Metres Per Second Is The Worst Storm Effect In A 1000 Simulations. We Used The Poisson Distribution To Determine The Number Of Storms For A Period Of 10 Years Which Gave Us A Random Sample Of 4 Storms In The First Year, 1 Storm In The Second Year, No Storms In The Third And Fourth Years, 2 Storms In The Fifth Year, 3 Storms In The Sixth Year, 3 Storms In The Seventh Year, 2 Storms In The Eight Year, 2 Storms In The Nineth Year, And 2 Storms In The Tenth Year. We Used The Gamma Distribution To Simulate 1000 Storms Subject To Scale And Shape Parameters And Observed A Sample Of A Fierce Storm Having A Speed Of 227.5616m/S, Followed By The Next Levels Of The Velocity Of Storm With 196.0322m/S, 162.9354m/S, 156.715m/S, And 151.2619m/S.To Calculate The Effect Of Storm On Species 1 For The First Year, The Value Of C_1 Can Be Calculated Using The Formula

(10.10) $$C_1 = \frac{1}{(227.5616)(79.7979) + 0.2)}$$

When This Formula Is Simplified, $C_1 = 0.00005506869385 < 1$ Which Measures The Effect Of Storm Intensity On Species 1. The Old Biomass OB For Species 1 Is 79.7979 Grammes. After Storm 1, The Proportion Of Species 1 Destroyed P_d Can Be Calculated By Using The Formula

(10.11) $P_d = 196.0322 c_1 OB$

Hence, $P_d = 0.861437259 < 1$.

The New Biomass NB After Storm 1 Can Be Calculated From The Formula

(10.12) $NB = OB(1 - P_d)$
By Substituting For The Old Biomass OB And The Proportion Destroyed P_d, The Calculated New Biomass NB Is 11.057 Grams. After The End Of The First Storm, The Old Biomass For The Start Of Storm 2 Is 11.057 Grams. Similarly, After Storm 2,

(10.13) $P_d = 162.9354c_1(11.057)$ Hence, $P_d = 0.099210476 < 1$. Our New Biomass NB Is

(10.14) $$NB = OB(1 - P_d)$$
Where OB = 11.057 Grams. In This Case, The New Biomass NB Is 9.96 Grams. After Storm 2, The Old Biomass Is Now 9.96 Grams For The Start Of Storm 3.
Next, After Storm 3,

(10.15) $P_d = 156.715c_1 OB$
Where OB = 9.96 Grams And $P_d = 0.085955699 < 1$. Our New Biomass NB Is Equal To $9.96(1 - P_d)$ Which Is Approximately 9.10388 Grams. At The End Of Storm3, The Old Biomass Before The Start Of Storm 4 Is 9.10388 Grams.Our Poisson Random Sample Of The Number Of Storms When The Mean Number Of Storms Per Year Is 2 Specifies That The Number Of Storms For The First Year Is 4. That Means, We Would Stop Our First Year Calculation After Storm 4. In This Scenario,

(10.16) $P_d = 151.2619c_1 OB$
Where OB = 9.10388 Grams And $P_d = 0.075833456 < 1$. Our Calculated New Biomass NB Is 8.4135 Approximately. Hence, At The End Of Storm 4, The Old Biomass Before The Start Of The Second Year Winter Season Is 8.4135 Grams.This Example Illustrates How We Have Calculated The Minimum Biomass At The End Of Storm 4 In The First Year For The First Species Which Has A Starting Biomass Of 79.7979 Grams Per M^2 Before The Start Of First Year Winter. When The Starting Values Are $N_1(0) = 0.04g/M^2$ And $N_2(0) = 0.045g/M^2$, Our Calculated Biomasses Without Winter Storms Are $N_1 = 83.1887g/M^2$ And $N_2 = 0.0533g/M^2$ When The Two Plant Species Are Interacting Together Whereas Our Calcu- Lated Biomasses Without Winter Storms Are $N_{1i} = 83.2013g/M^2$ And $N_2 = 0.0787g/M^2$ When The Two Plant Species Are Interacting Separately.

By Using The Poisson Distribution To Obtain A Sequence Of Storms For A Period Of 10 Years When The Annual Number Of Storms Is 6, We Would Obtain (4, 10, 9, 9, 4, 4, 7, 5, 7, 9). That Is, We Would Expect To Have 4 Storms In The First Year And 9 Storms In The Tenth Year. We Also Use The Gamma Distribution To Measure The Extent Of The Fierceness Of Winter Storm For A Simulation Of 1000 Storms. The First Five Cases Of Storm Intensity Are (179.6793, 174.8924, 167.9871, 153.7375, 141.5542). The Next Five Cases Of Storm Intensity Are (135.5836, 129.0163, 126.8739, 124.3238, 123.7429).
In This Example, We Would Simply Present Our Final Calculations For The Minimum Biomass For The First Year Winter Season In Table 5.1.

Plant Species	Analytical Calculation				
Type	Start Of Winter	After St1	After St2	After St3	After St4
N_1	83.1887	2.21734	2.162085	2.11400585	2.071684
N_2	0.0533	0.00248	0.002374352	0.002285	0.002209

Table 2. Calculations Of The Minimum Biomass For The First Year Winter Season

Where St(I) Denotes Storm I. For The Second Year Winter, The Starting Biomasses Will Be $2.072g/M^2$ For The First Plant Species And $0.0022g/M^2$ For The Second Plant Species.
To Avoid Lengthy Algebraic Calculations Which May Incur Approximation Errors, We Propose To Simulate Our Summer-Winter Model In Order To Calculate The Minimum Biomass After Each Storm. In Our Numerical Simulation, We Propose To Use A Fourth Order Runge-Kutta Numerical Method To Simulate Our Combination Of A Summer Model And A Winter Model In Only One Matlab Program And Hence Calculate The Minimum Biomass.

11. Other Simplifying Assumptions

Following One Of Our Assumptions ([26]), The Destructive Events During The Winter Season Can Be Modelled Based On An Annual Number Of Storms That Can Be Mod- Elled Using A Poisson Process With Mean 4.5. Each Storm Has An Intensity That Can Be Modelled Using A Gamma Distribution And The Destructive Effect Of The Storm Is Proportional To The Intensity. Following The Principle Of Buckling, A Storm Of Given Intensity Will Destroy A Proportion Of The Biomass In Which The Proportion De- Stroyed Depends Linearly On The Biomass At The Start Of The Storm. In This Project, We Have Focused On The Simulation Of Our Summer-Winter Model Of Sorghum Species Interactions Which Has Enabled Us To Decide On A Method For Calculating The Min- Imum Biomass For Each Species Over A Ten Year Period Of One Trajectory. We Have Also Decided On How We Should Allow Our Program To Reflect Shortened Winter And Lengthened Summer For Calculating The Minimum Biomass For Each Species Over A Ten Year Period Of One Trajectory. We Use An Example To Investigate The Extent Of Obtaining Mutualism From Our Summer Competition Model Due To A Variation Of The Length Of Summer Season In Days.

12. Interpreting The Numerical Simulation Results

Following Our Previous Idea ([26]), We Would Present A Systematic Method Of Interpreting The Key Results Which We Have Observed In This Work With The Hope Of Providing Further Ecological Insights. Here, We Have Utilized The Following Numerical Estimation Technique Which Is Consistent With Our Previous Publication ([26]): We Use The Results Of 40 Simulation Runs In Which Each Run Covers A 10-Year Period Of One Trajectory, The Probability Of Reaching A Zero Biomass For One Or The Other Slow Growing Sorghum Species. In Order To Illustrate This Technique, We Start By Considering The Minimum Biomass Of Each Species Of Sorghum Over The 10-Year With Repeated Simulated Trajectories For Three Different Climate Scenarios. In Our First Climate Scenario When The Number Of Storms Is 4 Over 40 Runs, We Observe That Species 1 Reached A Zero Biomass On Five Occasions While Species 2 Survived On 38 Occasions. Hence, When The Two Species Are Growing Together, We Can Conclude That The Simulation Predicts An Experimental Probability Of $\frac{5}{40} = 0.125$ That Species 1 Does Not Survive And An Experimental Probability Of $\frac{2}{40} = 0.05$ That Species

2 Does Not Survive. For This Scenario And Using The Same Model Especially When Each Of Species 1 And 2 Is Growing Separately Without The Effect Of Competition With Other Species, We Observe Over 40 Repeated Simulated Trajectories That Species 1 (Alone) Reached A Zero Biomass On 26 Runs Out Of 40, Whereas Species 2 (Alone) Reached Zero Biomass On 25 Runs Out Of 40. What Do We Learn From These Observations? We Learn That When The Species Are Growing Separately, Species 1 Has An Experimental Probability Of 0.65 Of Reaching A Zero Biomass, And Species 2 Has An Experimental Probability Of 0.625 Of Reaching A Zero Biomass Based On Our Present Numerical Simulation Results. Therefore, Our Results Indicate That Both Species Of Sorghum Are More Likley To Survive When They Grow Together Than When Growing Separately. In Summary, The Two Species Can Be Said To Be Behaving Mutualistically. Our Present Observation Is Consistent With The Current Viewpoint ([26]). For The Purpose Of Clarity In The Context Of Experimental Probability Of Species Survival, We Would Present Our Present Overview Below Which We Have Not Seen Elsewhere With The View Of Contributing To This Evolving Interdisciplinary Research:

Parameter	Calculated Values Of Experimental Probabilities				
N	Sp1	Sp2	Sp1i	Sp2i	Ns
Epzerobio	0.125	0.05	0.65	0.625	4
Epsurvival	0.875	0.95	0.35	0.375	4
Epzerobio	0.2	0.15	0.7	0.7	8
Epsurvival	0.8	0.85	0.3	0.3	8
Epzerobio	0.25	0.3	0.75	0.75	10
Epsurvival	0.75	0.7	0.25	0.25	10

Table 3. The Calculation Of Experimental Probability Of Species Survival

Here, The Meaning Of The Following Notations Is

(1) Epzerobio Stands For The Experimental Probability Of Zero Biomass For Each Species.
(2) Epsurvival Stands For The Experimental Probability Of Survival For Each Species.

These Calculations Of Experimental Probability Of Sorghum Species Emphasize Some Aspects Of The Negative Aspects Of This Problem. We Would Expect These Experimental Numerical Simulations To Have Beneficial Effects In Terms Of Public Awareness Of The Impact Of Global Warming On Sorghum Production In The Eastern Cape Province. Despite This Observation, Assessing The Vulnerability Of Sorghum In Eastern Cape To Climate Change Would Require A Further Dedicated Collaborative Research.What Do We Learn From These Information? In This Prototype Simulation, We Ob- Serve A Systematic Instance Of Mutualism And The Senstivity Of Experimental Prob- Abilty Of Reaching Zero Biomasss For Each Of The Species. At This Point, We Can Use The Model To Manipulate The Environment And Deduce Some Simple Conclusions About Climate Change Scenarios In Easter Cape Provionce Of South Africa. In Our Experimental Results That Are Graphically Presented Below, We Consider The Following Feasible Simplifying Examples Of Climate Change Scenarios:(1) Extending The Length Of The Growing Season. (2) Increasing The Number Of Winter Storms.Considering The Experimental Variability Of The Stochastic Model, We Repeated A 10-Year Simulation Ten Times And In This Work Recorded Our Observations Which Are Consistent In The Main With The Current Mathematical Ecological Modelling ([26]). Taking On Board The Assumption For The Existence Of Mutualism And Competition As Two Dominant Types Of Interactions, We Have Interpreted The Outcomes Of Our Simula- Tion Analysis So As To Represent Mutualism, Commensalism, Competition, And So On. The Graphs Presented Below Show The Incidence Of Each Type Of Interaction Which We Have Observed Over The Ten Repeated Simulations For Each Set Of Parameter Values. Precisely, Increasing The Length Of The Growing Season Will Reduce The Incidence Of Mutualism And Increases The Incidence Of Competition Whereas Increasing The Num- Ber Of Storms Tends To Increase The Incidence Of Mutualism And Tends To Reduce The Incidence Of Competition. Other Changing Patterns Of Mutualism And Compe- Tition Under Slightly Changing Climate Variations Are Also Presented Next. Under Other Simplifying Assumptions On The Climate Change Scenarios Over A Wider Range Of Parameter Variation, It Is Possible To Consider More Than Ten Repeated Simula- Tions Which We Have Implemented In This Work. Having Observed In A Few Instances The Decaying Patterns Of Mutualism, It Would Require A Further Dedicated Stochastic D-Bifurcation Scheme ([53]) Before We Can Detect Where Mutualism Will Completely Disappear. In The Meantime, Our Simulation Predicts A Dominant Competition To Be A Key Mechanism Behind The Loss Of Mutualism Or Facilitation Or Biodiversity. The Main Contribution Of Our Model Parameters Supports The Dominant Ecological Theory Of Mutualism. Therefore, The Concern That Climate Change Would Lead To A Loss Of Biodiversity Is Consistent With The Application Of This Model.
In This Work, We Have Achieved The Following Results Which We Have Obtained From A Combination Of Our Summer Competition Model And Our Stochastic Winter Model Over A 10 Period Of One Trajectory:

13. Key Results

In This Work, We Conducted Some Numerical Simulations On Sorghum Species Inter- Actions In A Defined Harsh Climate In Eastern Cape Province Over A 10-Year Period

Of One Trajectory. For The First Time, We Have Achieved The Following Specific Contri- Butions Which We Have Not Seen Elsewhere:

(1) A Variation In The Number Of Storms Due To Global Warming Predicts Mu- Tualism And Sunsequently Facilitation Qualitatively From A Combination Of Our Summer Model And Our Stochastic Winter Model. By Manipulating The Length Of The Summer Growing Season In Days, Both Mutualism And Facilita- Tion Are More Likely To Change To Competition.

(2) A Variation In The Number Of Storms Due To Global Warming Predicts Com- Petition And Sunsequently Facilitation Qualitatively From A Combination Of Our Summer Model And Our Stochastic Winter Model. By Manipulating The Length Of Summer Growing Season, Competition Is Seen As A Dominant Type Of Interaction While Mutualism Is A Less Dominant Type Of Interaction.

(3) A Variation In The Number Of Storms Due To Global Warming Predicts Mutu- Alism From A Combination Of Our Summer Model And Our Stochastic Winter Model. By Manipulating A Combination Of The Volume Of Precipitation And The Length Of The Summer Growing Season In Days, Both Mutualism And Facilitation Are More Likely To Change To Competition.

It Is Interesting To Report That Our Qualitative Numerical Simulation Results Are Consistent With The Dominant/Mainstream Plant Ecological Viewpoints Which Suggest That The Loss Of Mutualism And Facilitation Can Have An Impact On The Biodiversity.

Mathematical Modelling Of Sorghum Species Interactions In A Harsh Climate Sce- Nario Of Eastern Cape Province Of South Africa Presents Major Attractions: Con- Ventional Research Based On Substantial Data Collection Can Be Expensive During An Inhospitable Severe Climate Changes; Changes Happen Slowly Under Severe Climate Scenario And As A Result It Is May Not Be Possible To Initiate The Process Of Collec- Tiong Large Amounts Of Data; The Environmental Impact Of Large Amounts Of African Scientists Visiting The Locations Of Farms To Collect Data On The Minimum Biomass That Remains After The Effect Of Fierce Storm Events Can Change The Competition Co- Efficients. Hence, In This Situation, There Can Be Significant Environmental And Cost Advantages Of The Technique Of Mathematical Modelling And Numerical Simulation. We Would Hope That These Novel Contributions Will Provide Useful Insights Pending Some Policies Of Handling Ecological Problems Within The Eastern Cape Province.

14. Numerical Simulation And Policy Implications

The Results Of Our Present Study Suggest That The Eastern Cape Province In South Africa Will Suffer The Impact Of Climate Change. The Scale Of This Impact On Crop Growth And Production Is Yet To Be Empirically Verified. Our Numerical Simulation Approach Indicates The Possibility Of A Declining Biodiversity Which Is Capable Of Altering The Performance Of The Ecosystems Within This Region. The Results Of Our High Capacity Building Mathematical Modelling In Terms Of Predicting The Bifurcation Of Mutualism To Competition Have Implications For Appropriate Policies With Which To Mitigate This Inevitable Climate Change Effect On The Agricultural Sector And Livelihoods Of This Impoverished Dependent Populations. These Expected Policies If Well Designed And Applied Can Go A Long Way To Alleviating Poverty Within This Region. Achievable Adaptable Technologies Aimed At Improving And Sustaining The Agricultural Base Of This Region Would Have A Future Impact On Key Development Indicators.

15. Concluding Remarks And Further Research

Our Work As Presented In This Chapter Represents The First Step In Developing A Realistic Model Which Represents Harsh Climate Sorghum Species Interactions InSouth Africa. However, Our Results Demonstrate That It Is Possible To Obtain Out- Comes Which Can Be Consistent With Other Established Scientific Observations Using A Combination Of The Summer Model And A Stochastic Winter Model Which We Have Considered In This Unique Data Manipulation And Parameterization. In Order To Take This Interesting Project Forward, We Would Anticipate The Availabilty Of Data Which Would Enable Us To Refine, Calibrate, And Develop This Model For Its Application In Our Future Research. However, The Gap Between Applied Scientists And Their Understanding Of Problems In The Real World And Mathematicians Who Can Answer Questions In An Idealised And Simplified World Can Be Immense.Therefore, While The Possibility Of Knowing The Extent Of The Loss Of Biodiversity Is A Good Idea In Principle For African Agriculture, One Has To Be Realistic About How Long It Will Take Before The Outcomes Of The Research Are Ready To Make A Difference In Practice In The Fields (So To Speak). The Details Of Other Related Ecosystem Char- Acteristics In The Niger Delta Region Of Nigeria Will Be The Subject Of A Near Future Publication.

16. Acknowledgements

The Author Is Pleased To Acknowledge The Helpful Discussions With Professor Neville J. Ford And Dr. Patricia M. Lumb Of The University Of Chester And Dr. J. Potter Of Edge Hill University All In The United Kingdom Which Led To The Work Reported In This Paper And The Helpful Observations Of Professor Dan Chimere- Dan Of South Africa Research Organization And Professor Sunday Ndowa Ekoate Lale Of The Faculty Of Agriculture, University Of Port Harcourt In Port Harcourt Nigeria That Have Resulted In A Clearer Presentation Of The Ideas Which Are Presented In This Talk. We Would Like To Acknowledge The Kind Provision Of Data From The South Africa Weather Service Organization And Research Funding Received From The Gladstone Fellowship.

REFERENCES

[1] ARCTIC CLIMATOLOGY, Internet Downloaded Resource, The Link To This Information Is Google ARCTIC CLIMATOLOGY, It Will Come Up As A 34 Page PDF Word Document, 3 − 12. I Accessed This Material For The First Time On The 20th Of November 2006 And Again On The 23rd Of July 2009. The Author And Year Of Publication Of This Material Are Not Specified.

[2] C. Armas, R. Ordiales, And F.I. Pugnaire, Measuring Plant Interactions: A New Comparative Index, Ecology 85(10), (2004), 2682-2686.

[3] N.P.R. Anten, R.C. Garcia, And H. Nagashima, Effects Of Mechanical Stress And Plant Den- Sity On Mechanical

Characteristics, Growth, And Lifetime Reproduction Of Tobacco Plants, The American Naturalist 166, (2005), 6-10.

[4] Atmospheric Environment Service (AES), Past Climate Change In The Canadian Arctic, , Canadian Climate Centre Rep. 85-14, Downsview, Ontario 1985.

[5] C.T.H. Baker And F.A. Rihan, Sensitivity Analysis Of Parameter In Modelling With Delay- Differential Equations, MCCM Numerical Analysis Report, No. 349, Manchester University, 1999.

[6] C.T.H. Baker, G.A. Bocharov, J.M. Ford, P.M. Lumb, S.J. Norton, C.A.H. Paul, T. Junt, P. Krebs, And B. Ludewig, Computational Approaches To Parameter Estimation And Model Selection In Immunology, Journal Of Computational And Applied Mathematics 184, (2005), 50-76.

[7] C.J. Baker, The Development Of A Theoretical Model For The Windthrow Of Plants, Journal Of Theoretical Biology 175, (1995), 355-372.

[8] R.B. Banks, Growth And Diffusion Phenomena Mathematical Frameworks And Applications, Springer-Verlag, 1994.

[9] Y. Bard, Nonlinear Parameter Estimation, Academic Press, New York, 1974, 18-25.

[10] M.D. Bertness And G.H. Leonard, The Role Of Positive Interactions In Communities: Lessons From Intertidal Habitats, Ecology 78, (1997), 1979-1989.

[11] K. Beven, G. Aronica And B. Hanking, Uncertainty And Equifinality In Calibrating Distributed Roughness Coefficients In A Flood Propagation Model With Limited Data, Advanced Water Re- Sources 22, 4, (1998), 349-365.

[12] V.H.Blackman, The Compound Interest Law And Plant Growth, Annals Of Botany 33, 131, (1919), 353-360.

[13] R.W. Brooker, F.T. Maestre, R.M. Callaway, C.L. Lortie, L.A. Cavieres, G. Kunstler, P. Liancourt, K. Tielborger, J.M.J. Travis, F. Anthelme, C. Armas, L. Coll, E. Corcket, S. Delzon, E. Forey, Z. Kikvidze, J. Olofsson, F. Pugnaire, C. L. Quiroz, P. Saccone, K. Schiffers, M. Seifan, B. Touzard, And R. Michalet, Facilitation In Plant Communities: The Past, The Present, And The Future, Journal Of Ecology 96, (2008), 18-34.

[14] R.W. Brooker, F.T. Maestre, R.M. Callaway, C.L. Lortie Et Al., Facilitation In Plant Commu- Nities: The Past, The Present, And The Future, Journal Of Ecology 96, (2008), 18-34.

[15] R.W. Brooker And T.V.Callaghan, The Balance Between Positive And Negative Plant Interac- Tions And Its Relationship To Environmental Gradients: A Model Oikos 81(1), (1998), 196-207.

[16] J.F. Bruno, J.J. Stachowicz, And M.D. Bertness, Inclusion Of Facilitation Into Ecological The- Ory, Trends In Ecology And Evolution 18(3), (2003), 119-125.

[17] R. Brooker, Z. Kikvidze, F.I. Pugnaire, R.M. Callaway, P. Choler, C.J. Lortie And R. Michalet, The Importance Of Importance, Oikos 109 (2005), 63-70.

[18] J.F. Bruno, J.J. Stachowicz, M.D. Bertness, Inclusion Of Facilitation Into Ecological Theory, TRENDS In Ecology And Evolution 18, (2003), 119-125.

[19] R.M.Callaway, R.W. Brooker, P. Choler, Z. Kikvidze, C.J. Lortie, R. Michalet, L. Paolini, F.I. Pugnaire, B. Newingham, E.T. Aschehoug, C. Armas, And B.J. Cook, Positive Interactions Among Alpine Plants Increase With Stress, Nature 417 (2002), 844-848.

[20] C. Casty, C.R. Christoph, F.T. Stocker, W. Heinz And L. Jury, A European Patern Climatology 1766-2000, Climate Dynamics 29, (2007), 791-805.

[21] C. Dormann And R.W.Brooker, Facilitation And Competition In The High Arctic:The Impor- Tance Of The Experimental Approach, Acta Oecologia 23, (2002), 297-301.

[22] E.N. Ekaka-A, Computational And Mathematical Modelling Of Plant Species Interactions In A Harsh Climate, Phd Thesis, Department Of Mathematics, The University Of Liverpool And The University Of Chester, United Kingdom, 2009.

[23] E. Ekaka-A, O. Chimere-Dan, Modeling Mutualism Of Sorghum Species Interactions In A Con- Text Of Climate Change: A South African Case Study, Prepared For Chapter 10 Of Makiwane, M And Chimere-Dan, O. Eds Forthcoming, The State Of Population In The Eastern Cape, South Africa, Eastern Cape Provincial Department Of Social Development. Bisho, South Africa, 2010.

[24] A.R. Ennos, Wind As An Ecological Factor, Trends In Ecology And Evolution, Elsevier 12,3, (1997), 108-111.

[25] D.A. Etkin, Greenhouse Warming: Consequences For Arctic Climate, Journal Cold Reg. Eng. 4, (1990a), 54-66.

[26] N.J. Ford, P.M. Lumb, E. Ekaka-A, Mathematical Modelling Of Plant Species Interactions In A Harsh Climate, Journal Of Computational And Applied Mathematics 234, (2010), 2732-2744.

[27] A.J. Freeman, Materials By Design And The Exciting Role Of Quantum Computation/Simulation, Journal Of Computational And Applied Mathematics 149, (2002), 27-56.

[28] H.A. French Ed, Impact Of Climate Change On The Canadian Arctic, Atmospheric Enviornment Service, Canadian Climate Centre, Downsview, Ontario 1986.

[29] D.J. Gates And M. Westcott, Stability Of Plant Mixtures In The Absence Of Infection Or Pre- Dation, J.Theor.Biol. 131, (1988), 15-31.

[30] D.J. Gibson, J. Connolly, D.C. Hartnett, And J.D. Weidenhamer, Designs For Greenhouse Studies Of Interactions Between Plants, Journal Of Ecology 87, (1999), 1-16.

[31] S. Giliessman, Agroecology: Ecological Processes In Sustainable Agriculture, Sleeping Bear Press, MI, 1998.

[32] D.E. Goldberg, T. Rajaniemi, J. Gurevitch And A. Stewart-Oaten, Empirical Approaches To Quantifying Interaction Intensity: Competition And Facilitation Along Productivity Gradients, Ecology 80, (1999), 1118-1131.

[33] K.Gopalsamy, Stability And Oscillations In Delay Diffeential Equations Of Population Dy- Namics, Kluwer Academic Publishers, 1992, 17.

[34] Global Warming Resource, Arctic And Antarctic Warming-Link 69, Located At The Website Http://Www.Climatehotmap.Org/References.Htlm. I Accessed This Material For The First Time On The 20th Of November 2006 And Again On The 23rd Of July 2009. The Author And Year Of Publication Of This Material Are Not Specified.

[35] D.L. Gorlick, P.D. Atkins, G.S. Losey Cleaning Stations As Water Holes, Garbage Dumps, And Sites For The

Evolution Of Reciprocal Altruism, American Naturalist 112, (1978), 341-353. [36] H.T. Hartmann, W.J. Flocker, A.M. Kofranek, Plant-Science Growth, Development, AndUtilization Of Cultivated Plants, Englewood Cliffs, N.J; Prentice-Hall, Inc, 1981.

[37] R.W. Hofstetter, T.D.Dempsey, K.D. Klepzig, And M.P. Ayres, Temperature-Dependent Ef- ~ Fects On Mutualistic, Antagonistic, And Commensalistic Interactions Among Insects, Fungi And Mites, Community Ecology 8(1), (2007), 47-56.

[38] A. Iserles, A First Course In The Numerical Analysis Of Differential Equations, (First Edi- Tion) Cambrigde University Press, 1996.

[39] J.Jaagus And R.Ahas, Space-Time Variations Of Climate Seasons And Their Correlation With The Phenological Development Of Nature In Estonia, Climate Research 15, (2000), 207-219.

[40] J.A. Jentsch, J. Kreyling, C. Beierkuhnlein, A New Generation Of Climate-Change Experiments: Events, Not Trends, Frontiers In Ecology And The Environment 5(7), (2007), 365-374.

[41] Z. Kikvidze, C. Armas, And F.I. Pugnaire, The Effect Of Initial Biomass In Manupulative Experiments On Plants, Functional Ecology 20, (2006a), 1-3.

[42] L.F. Konikov And J.D. Bredehoeft, Groundwater Models Cannot Be Validated, Advanced Water Resources 15, 1, (1992), 75-83.

[43] J.D.Lambert, Computational Methods In Ordinary Differential Systems, (First Edition), John Wiley And Sons, 1973.

[44] R.M.May, Mutualistic Interactions Among Species, Nature 296, (1982), 803-804.

[45] R.M. May, Stability And Complexity In Model Ecosystems, Princeton University Press, Prince- Ton, New Jersey, USA, 1974.

[46] J.B. Maxwell And L.A. Barrie, Atmospheric And Climate Change In The Arctic And Antartic, Ambio 18, (1989), 42-49.

[47] J.L. Monteith, Principles Of Environmental Physics, Elsevier, New York, 1973.

[48] D.C. Montgomery And G.C. Runger, Applied Statistics And Probability For Engineers, ThirdEdition, John Wiley And Sons, Inc, 2003.

[49] D.C. Montgomery, G.C. Runger, N.F. Hubele, Engineering Statistics, Second Edition, John Wiley And Sons, Inc, 2001.

[50] I.J. Myung, The Importance Of Complexity In Model Selection, Journal Of Mathematical Psychology 44, (2000), 190-204.

[51] J.D.Murray, Mathematical Biology 1: An Introduction, Springer-Verlag, Third Edition, New York, 2002.

[52] K.J. Nicklas, Plant Biomechanics: An Engineering Approach To Plant Form And Function, The University Of Chicago Press, Chicago, 1992.

[53] N.J. Ford, S.J. Norton, Noise-Induced Changes To The Behaviour Of Semi-Implicit Euler Meth- Ods For Stochastic Delay Differential Equations Undergoing Bifurcation, Journal Of Computa- Tional And Applied Mathematics 229, (2009), 462-470.

[54] O. Pellmyr, C.J. Huth, Evolutionary Stability Of Mutualism Between Yuccas And Yucca Moths, Nature 372, (1994), 257-260.

[55] J. Penuelas And I. Filella, Phenology: Responses To A Warming World, Science 294, No. 5543, (2001), 793-795.

[56] M. Pessarakli, Handbook Of Plant And Crop Stress, Second Edition, Revised And Expanded, Edited By Mohammad Pessarakli. Marcel Dekker Inc (1999), 399.

[57] S.L. Piao, P. Friedlingstein, P. Ciais, V. Viovy And J. Demarty, Potential Impact Of The Growing Season On The Phenology Variability, Global Biogeochemical Cycles 21(3), (2007), 37-45.

[58] P.O.S.T, Arctic Changes, Parliamentary Office For Science And Technology, London, Postnote 334, 2009

[59] N. Oreskes, K. Shrader-Frechette, And K. Belitz, Verification, Validation And Confirmation Of Numerical Models In The Earth Sciences, Science 263, (1994), 641-646.

[60] L. Rogers-Bennett, Is Climate Change Contributing To Range Reductions And Localized Extinc- Tions In Northern (Haliotis Kamtschatkana) And Flat (Haliotis Walallensis) Abalones?, Bulletin Of Marine Science 81(2), (2007), 283-296.

[61] E.F. Roots, Climate Change: High-Latitude Regions, Climatic Change 15, (1997), 223- 253.

[62] A.R. Sherry, X. Zhou, S. Gu, A.J. Arnoneiii, S.D. Schimel, S.P. Verburg, L.L. Wallace, Y.Luo, Divergence Of Reproductive Phenology Under Climate Warming, Proceedings Of National Academy Of Sciences 104(1), (2007), 198-202.

[63] P. Stiling, Ecology: Theories And Applications, (Third Edition) Prentice Hall International, London, 1999.

[64] D. Straile And N.C. Stenseth, The North Atlantic Oscillation And Ecology: Links Between Historical Time-Series, And Lessons Regarding Future Climatic Warming, Climate Research 34(3), (2007), 259-262.

[65] P. Sullivan, Intercropping Principles And Production Practices, Appropriate Technology Transfer For Rural Areas (ATTRA), Fayetteville, AR, 1998.

[66] D.Tilman, Plant Strategies And The Dynamics And Structure Of Plant Communities, Princetown NJ, 1988.[67] S.P. Timoshenko And J.M. Gere, Menchanics Of Materials, Van Nostrand Reinhold, 1972. [68] United Nations, United Nations Conference On Environment And Development, Agenda 21: Program Of Action For Sustainable Development; Rio Declaration On Environment And Devel- Opment; Statement Of Forest Principles, New York: United Nations Department Of Public Information, 1993.

[69] R. Whitlow, Materials And Structures, 2nd Edition, Longman Scientific And Technical, 1991.

[70] X. Yin, J. Goudriaan, E.A. Lantinga, J. Vos, H.J. Spiertz, A Flexible Sigmoid Function Of Determinate Growth, Annals Of Botany 91 , (2003), 361-371.

[71] P. Young, S. Parkinson And M. Lees, Simplicity Out Of Complexity In Environmental Modelling, Journal Of Applied Statistics 23, 2/3, (1996), 165-210.

[72] J. Zebrowski, The Use Of Free Vibrations To Measure Peduncle Stiffness In Triticale, Jounal Of Experimental Botany 42(242), (1991), 1207-1212.