A Green Capacitated Vehicle Routing Problem with Fuel Consumption Optimization Model

1,İlker Küçükoğlu , 2,Seval Ene, 3,Aslı Aksoy*, 4,Nursel Öztürk

[1,2,34,] Uludag University Faculty of Engineering and Architecture Industrial Engineering Department
Gorukle Campus, 16059, Bursa, Turkey

*Corresponding author asliaksoy@uludag.edu.tr

ABSTRACT:

In recent years, since environmental issues and regulations impact strategic and operational decisions of companies, green logistics has a critical and gaining value for researchers and companies. In this context green capacitated vehicle routing problem (G-CVRP) under minimizing fuel consumption, which is rarely encountered in real life systems, is studied in this paper. A mixed integer linear programming model is proposed for solving G-CVRP. In the G-CVRP optimization model, fuel consumption is computed considering the vehicle technical specifications, vehicle load and the distance. Fuel consumption equation is integrated to the model through a regression equation *proportional to the distance and vehicle load. G-CVRP optimization model is validated by various instances with different number of customers. Achieved results show that G-CVRP model provides important reductions in fuel consumption.*

KEYWORDS: *Fuel consumption model, green vehicle routing, mixed integer linear programming.*

I. INTRODUCTION

In recent years, with the growing environmental concerns, industrial organizations have to take into account environmental factors in order to enhance the competitive aspect. In this global world, logistics stays at the center of the modern transportation systems. As the competition increases, the technology rapidly changes, product life-cycle becomes shorter and the expectation of the customer increases. In high competitive environment, industrial organizations have to investigate logistics strategies and adopt green supply chain applications to consider accurate use of natural resources as a social responsibility. The importance of environmental issues is continuously translated into regulations, which potentially have a tangible impact on supply chain management. As a consequence, there has been an increasing amount of research on the intersection between logistics and environmental factors [1].

The scope of this study is, developing a mathematical model which minimizes the fuel consumption for green capacitated vehicle routing problem (G-CVRP). The study has two dependent components. The first component includes a graphical user interface to calculate fuel consumption for a given load and distance. The second component includes a mixed integer linear optimization model, to determine the route that minimizes fuel consumption. The optimization model works with the data obtained from fuel consumption calculation model. The remainder of this paper is organized as follows. Section 2 presents a literature review. Section 3 explains the fuel consumption calculation. Mixed integer linear programming model is presented in Section 4. Application examples and results are provided in Section 5. Finally, conclusions are presented in Section 6.

II. LITERATURE REVIEW

Laporte described the vehicle routing problem (VRP) as a problem of designing optimal delivery or collection routes from one or several depots to a number of geographically scattered cities or customers, subject to side constraints [2]. VRP is a classic combinatorial optimization problem involved in many applications. The VRP plays a vital role in distribution and logistics. Since its introduction by Dantzig and Ramser, VRP has been extensively studied. By considering additional requirements and various constraints on route construction, different VRPs have been formulated, such as, pick-up and delivery VRP, capacitated VRP, multiple depots VRP, VRP with time windows. Although there are different forms of VRPs, most of them minimize the cost by minimizing the total distance without considering the fuel consumption rate. In fact, statistics show that fuel cost is a significant part of total transportation cost [3].

There are many studies in the literature concerning with vehicle routing problem (VRP) [2, 4, 5, 6, 7 etc.]. In recent years, the interest in practical applications and literature about green logistics has increased; but, according to our knowledge, not much research has been conducted on the VRP under minimizing fuel consumption and emissions. Apaydın and Gönüllü studied the VRP model on reducing the CO_2 emissions vehicles used for waste collection operations in a city [8]. A shortest path model was used in order to optimize solid waste collection. Kuo, proposed a model for calculating total fuel consumption for the time-dependent vehicle routing problem (TDVRP) where speed and travel times are assumed to depend on the time of travel when planning vehicle routing. Simulated annealing (SA) algorithm is proposed for finding the vehicle routing with the lowest total fuel consumption [9]. Bektaş and Laporte described an approach to reduce energy requirements of vehicle routing based on a comprehensive emissions model that takes into account load and speed. The authors present a comprehensive formulation of the problem and solve moderately sized instances [10]. Suzuki, presented a model indicates that a significant saving in fuel consumption and CO_2 emissions may be realized by delivering heavy items in the early segments of a tour while delivering light items in the later segments, so that the distance a vehicle travels with heavy pay-loads can be minimized [11]. Wygonik and Goodchild developed a VRP with time windows to minimize emissions for urban pick-up and delivery system [12]. Xiao et al. proposed a mathematical model which minimizes fuel consumption of the vehicles. They developed a simulated annealing based algorithm to solve the model [3]. Erdogan and Hooks, considered fuel consumption of the vehicles and seek a set of vehicle tours that minimize total distance traveled to serve a set of customers while incorporating stops at alternative fueling stations in route plans [13].

III. FUEL CONSUMPTION CALCULATION

In this research, the proposed model aims to calculate total fuel consumption. Apaydın and Gönüllü used fixed values for fuel consumption [8]. Kuo; calculated fuel consumption with respect to vehicle load and vehicle speed [9]. Jabali et al. modeled fuel consumption as a function of speed [1]. Bektaş and Laporte calculated fuel consumption based on average speed and vehicle load [10]. Suzuki; considered vehicle load, average speed based on the road gradient and average amount of fuel consumed per hour while a vehicle is waiting at customer sites [11]. Ubeda considered vehicle load [14]. Xiao et al. formulated a linear function dependent on vehicle load for fuel consumption [3]. Huang et al. modeled linear regression for fuel consumption as a function of vehicle load and distance [7]. Demir et al. [15, 16] and Liu and Helfand used mathematical equation to calculate fuel consumption [17]. Table 1 indicates the summary of selected literature regarding criteria for fuel consumption and calculation methods.

Table 1. The summary of selected literature regarding criteria for fuel consumption and calculation methods

Reference	**Criteria**						**Calculation Method**		
	Fixed	Speed	Load	Road Gradient	Acceleration Rate		Regression	Table	Equation
Apaydın, Gönüllü [8]	+							+	
Jabali et al. [1]		+							+
Kuo [9]		+	+						+
Suzuki [11]		+	+	+					+
Ubeda et al. [14]			+					+	
Xiao et al. [3]			+				+		
Bektaş, Laporte[10]		+	+						+
Huang et al. [7]			+				+		
Demir et al. [15,16]		+	+		+				+
Liu, Helfand [17]	+								+

According to our knowledge, the previous studies ignored the acceleration rate and/or did not directly consider the vehicle load while calculating the fuel consumption. In this study, the fuel consumption is calculated considering the vehicle technical specifications, vehicle load and the route distance. This way of calculation yields more realistic approach. The vehicle needs energy to move with constant speed or acceleration in real time. This energy is equal to resistance forces. The resistance force includes rolling resistance (F_{Ro}) aerodynamic resistance (F_{Ae}), grade resistance (F_G) and acceleration resistance (F_{Acc}) [18]. The total force (F_T) at the wheels for a given acceleration and grade is

$$F_T = F_{Ro} + F_{Ae} + F_{Acc} \tag{1}$$

Assume that, P_T is the total power at the wheels and calculated as

$$P_T = F_T \, v \tag{2}$$

where v is the speed. To convert the total force to the fuel consumption, consumption value (b) multiplicities with the total power (P_T) as:

$$consumption = b\, P_T \tag{3}$$

To determine the time - speed graphic of a vehicle from node i to node j, it is assumed that, the vehicle accelerates from 0 to t_1, the speed is constant at speed limit (v_{max}) from t_1 to t_2, and decelerates from t_2 to t_3. Figure 1 shows the time - speed graphic which has three discrete areas: A_1 represents the area for the acceleration time, A_2 represents the area for the time with the constant speed and A_3 represents the area for the deceleration time. In this study, it is accepted that the acceleration rate of the vehicle is equal to the deceleration rate of the vehicle ($a_{acc} = a_{dec}$), thereby, the speed of the vehicle in real-time can be determined.

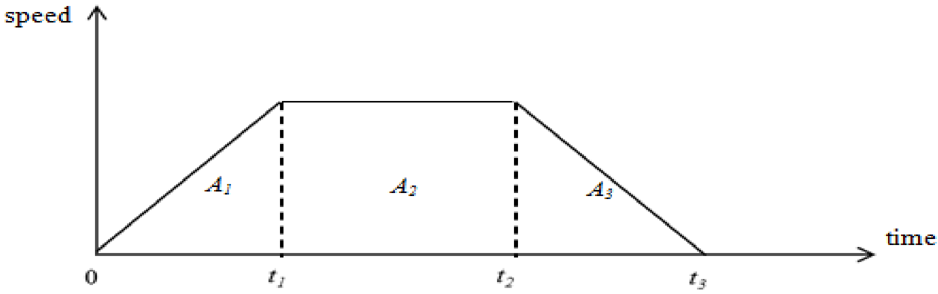

Figure 1. The speed vs. time graphic to calculate fuel consumption

In order to calculate the total fuel consumption of a vehicle in a specified distance, total power and consumption formulations are developed as a function of time with the following equations:

$$v(t) = \begin{cases} a_{acc}\, t, & if \;\; 0 \le t < t_1 \\ v_{\max}, & if \;\; t_1 \le t < t_2 \\ v_{\max} - a_{dec}(t - t_2), & if \;\; t_2 \le t < t_3 \\ 0, & otherwise \end{cases} \tag{4}$$

$$F_T(t) = F_{Ro}(t) + F_{Ae}(t) + F_{Acc}(t) \tag{5}$$

$$F_T(t) = (fmg) + (c_d A \rho \frac{v^2(t)}{2})(\lambda ma) \tag{6}$$

$$P_T(t) = F_T(t) v(t) \tag{7}$$

$$Consumption = bP_T(t) \tag{8}$$

$$Total\ consumption = \int_0^{t_3} bP_T(t)\, d_t \tag{9}$$

where f is the coefficient of rolling resistance, m is the vehicle weight (empty plus carried load) and g is the gravitational constant, c_d is the aerodynamic coefficient, A is the frontal surface area, ρ is the air density and v is the speed, λ is the transmission variable (λ =1.04+0.0025*i^2, i is the overall gear ratio), a is the acceleration rate, α is the coefficient related to gradient of the road.

In this study, because of the complexity of the fuel consumption calculation (in equation 9), the parameters, except load and distance are assumed to be constant similar to Huang et al. [7]. So, the fuel consumption equation is proportional to the distance and vehicle load and regression analysis is achieved by using Minitab to obtain a mathematical equation between load, distance and fuel consumption, as shown:

$$Fuel\ consumption = a_1\ distance + a_2\ vehicle\ load + b \tag{10}$$

The data set used to achieve regression analysis, includes 2020 examples for different distances and vehicle loads in the range of 1300 – 1400 kg for vehicle weight and 0 – 100 km for distance. After the analysis, the calculated value of a_1, a_2 and b are; 0.0589, 0.000991, -1.5, respectively. The regression equation analysis is also performed in Minitab. The *R-square* value of the proposed regression model is 95.7%, indicating that the model fits the data extremely well.

IV. LINEAR INTEGER PROGRAMMING FORMULATION

The capacitated vehicle routing problem (CVRP) is an interesting combinatorial optimization task, which occurs frequently in real-world applications [19]. The CVRP may be described as the following graph problem. Let $G=(V,A)$ be a complete directed graph, where $V=\{0,1,\ldots,n\}$ is the vertex set and A is the arc set. Vertices $i=1,\ldots,n$ correspond to the customers, each with a known non-negative demand d_i and vertex 0 corresponds to the depot. A nonnegative cost c_{ij} is assigned to each arc $(i,j) \in A$, and it represents the cost of travelling from vertex i to vertex j. A set of K identical vehicles, each with the capacity C, is available at the depot. The CVRP consists of finding a collection of elementary cycles less than K in G with minimum total cost, such that:

- each cycle visits the depot vertex 0,
- each customer vertex $i=1,\ldots,n$ is visited by exactly one cycle,
- the sum of the demand d_i of the vertices visited by a cycle does not exceed the vehicle capacity C.

In this paper, a mixed-integer linear integer programming model is considered to construct routes for a set of vehicles to meet the demand of all customers, k homogenous vehicles depart from and return to the depot node after serving all the customers, each customer is visited at once and vehicles cannot carry load more than capacity. The overall objective is to minimize the travelling distance and fuel consumption. The green-capacitated vehicle routing problem (G-CVRP) optimization model, proposed in this research, can be developed as follows:

Indices

$i, j = 0,1,\ldots N$	Set of customers
$k = 1,\ldots K$	Set of vehicles

Parameters

c_{ij}	Traveling distance between customer *i* and customer *j*
d_i	Demand of customer *i*
Q	Capacity of vehicles
a_1	Coefficient of regression equation
a_2	Coefficient of regression equation
b	Coefficient of regression equation
M	Big number
Min_load	Unladen weight of the vehicle
Max_load	*Min_load+Q*

Decision Variables

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ travels from customer } i \text{ to customer } j \quad (i \neq j) \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \text{load of vehicle at customer } i$$

Model

$$Min\ z = \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{K} a_1 c_{ij} x_{ijk} + Ka_2 y_0 + \sum_{i=1}^{N}\sum_{k=1}^{K} a_2 x_{ijk} y_i + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{K} bx_{ijk} \qquad (11)$$

Subject to

$$\sum_{i=0}^{N}\sum_{k=1}^{K} x_{ijk} = 1 \qquad for \;\; i \neq j \text{ and } j \in \{1,2,\ldots, N\} \tag{12}$$

$$\sum_{j=0}^{N}\sum_{k=1}^{K} x_{ijk} = 1 \qquad for \;\; i \neq j \text{ and } i \in \{1,2,\ldots, N\} \tag{13}$$

$$\sum_{j=1}^{N} x_{0jk} = 1 \qquad for \;\; k \in \{1,2,\ldots, K\} \tag{14}$$

$$\sum_{i=1}^{N} x_{i0k} = 1 \qquad for \;\; k \in \{1,2,\ldots, K\} \tag{15}$$

$$\sum_{i=0}^{N} x_{ijk} = \sum_{i=0}^{N} x_{jik} \qquad for \;\; k \in \{1,2,\ldots, K\} \text{ and } j \in \{1,2,\ldots, N\} \tag{16}$$

$$y_0 = Min\_load \tag{17}$$

$$y_i - y_j \geq d_i - M(1 - x_{ijk}) \qquad for \;\; \mathrm{i} \in \{1,\ldots, N\},\; j \in \{0,\ldots, N\} \,and\; k \in \{1,\ldots, K\} \tag{18}$$

$$Min\_load \leq y_i \leq Max\_load \qquad for \;\; i \in \{0,\ldots, N\} \tag{19}$$

$$y_i \geq 0 \tag{20}$$

$$x_{ijk} \in \{0,1\} \tag{21}$$

The objective function (11) seeks to minimize fuel consumption related to distance and vehicle load. Constraints (12) and (13) ensure that each customer is visited exactly at once. Constraints (14) and (15) ensure the depart from and return to the depot node. Constraint (16) ensures flow balance for each node. Constraint (17) indicates, vehicles are empty at the depot node. Constraints (18) and (19) restrict the load of vehicles at the visited customer and also eliminate the sub tours. Constraints (20) ensure y_i are nonnegative and constraints (21) define the binary variables x_{ijk} . The objective function (11) is bilinear. In this study, it is observed that, y_i is nonnegative only if $x_{ijk} = 1$. Therefore, the bilinear objective function is converted into the equivalent linear form (22).

$$Min\ z = \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{K} a_1 c_{ij} x_{ijk} + Ka_2 y_0 + \sum_{i=1}^{N} a_2 y_i + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{K} bx_{ijk} \tag{22}$$

V. RESULTS AND DISCUSSION

This section presents the results of computatitional applications performed to assess the performance of G-CVRP model. To test the proposed G-CVRP model 15 different instances were generated randomly. The size of the instances generated in this study ranges from 5 to 25 nodes. To better explain the 15 test instances, the charactersitics of the instances, such as; customer number, average load, averagre distance and distance range can be seen in Table 2. The customer number specifies the number of nodes for a given route, the seed number is the code for random number generator, the average load refers the average demand of the customer nodes, the average distance refers to average distance of c_{ij} matrix, and the distance range refers to the difference between max distance and minimum distance of the given route.

Table 2. The characteristics of the 15 instances

Instances	Number of Customers	Average Demand	Average Distance	Distance Range
1	5	50.00	42.66	78.99
2	8	65.63	38.71	64.66
3	10	52.50	51.62	98.61
4	12	62.08	51.05	110.81
5	13	54.62	49.21	89.87
6	15	59.33	43.23	107.24
7	18	57.22	52.02	115.91
8	16	62.81	50.48	99.81
9	15	60.00	51.92	98.77
10	16	74.06	51.11	93.21
11	20	71.75	58.38	125.99
12	22	63.41	53.20	114.82
13	25	54.80	51.58	127.88
14	20	52.75	50.30	94.86
15	21	46.90	52.00	102.32

Proposed G-CVRP model is coded in MPL (Mathematical Programming Language) and solved with GUROBI 5.1.0 solver. Each instance is performed with both CVRP [5] and G-CVRP models and optimum results are obtained. Comparative results of the CVRP and G-CVRP model as to total distance, fuel consumption and vehicle number can be seen in Table 3.There can be differences in vehicle numbers for CVRP and G-CVRP because of the different objective functions of each model. For example the instance set with 10 customer nodes reaches the optimal solution with one vehicle for CVRP and with two vehicles with G-CVRP. It can be concluded from the computational results in Table 3, although the average distance obtained from CVRP (353.81 km) is smaller than average distance obtained from G-CVRP (378.13 km), the average fuel consumption in G-CVRP (25.61 *l*) is less than CVRP (26.79 *l*). In G-CVRP vehicles should serve the customer with large demand earlier in the route. With possible increase in total distance, this approach, reduces the distance weighted average load of vehicles and therefore reduces fuel consumption. This outcome can be clearly seen from the last column of Table 3, such as,

Table 3. Computational Results

Instances	Number of Customers	CVRP			GCVRP			%GAP	
		Total Distance	Fuel Consumption	Number of Vehicles	Total Distance	Fuel Consumption	Number of Vehicles	Distance	Fuel Consumption
1	5	211.93	13.4042	1	211.93	13.4042	1	0.00	0.00
2	8	233.81	15.8502	1	233.81	15.8502	1	0.00	0.00
3	10	299.09	20.8082	1	327.57	20.3957	2	9.52	-1.98
4	12	288.49	21.8091	1	298.69	19.4501	2	3.54	-10.82
5	13	295.34	21.2212	1	295.34	21.2212	1	0.00	0.00
6	15	346.29	29.2608	1	374.88	25.2197	2	8.26	-13.81
7	18	428.51	32.8033	2	438.13	29.8403	2	2.24	-9.03
8	16	369.37	29.3391	2	390.76	26.5671	2	5.79	-9.09
9	15	340.04	26.4800	1	369.77	25.3692	2	8.74	-4.20
10	16	376.43	26.8922	2	376.43	26.8922	2	0.00	0.00
11	20	441.57	33.1009	2	442.41	33.0952	2	0.19	-0.02
12	22	448.64	34.3727	2	460.27	33.6299	2	2.59	-2.16

13	25	474.35	35.9817	2	557.46	34.5884	3	17.52	-3.87
14	20	370.98	29.3911	2	411.24	28.3289	3	10.85	-3.61
15	21	382.37	31.2762	1	483.28	30.3941	3	26.39	-2.82

G-CVRP provides average 4.09 % fuel consumption reduction. The percentage gap refers the percentage difference of CVRP and G-CVRP for each instance. In G-CVRP the model tries to serve the customers with heavier demand earlier and this obtains
the reduction in fuel consumption. It can be concluded from the results that G-CVRP model outperforms CVRP model according to fuel consumption results.

VI. CONCLUSIONS

In this paper G-CVRP, a variant of widely known VRP, is analyzed and formulated. Differently from classical VRP which tries to minimize total distance of the route, G-CVRP means to minimize total fuel consumption of the route, consequently CO_2 emission of the vehicles in the route. The contribution of this paper is multiple. Initially a computation for calculating fuel consumption considering the vehicle technical specifications, vehicle load and the distance is proposed. Secondly because of the complexity of the fuel consumption calculation, the parameters except load and distance are assumed to be constant and regression analysis is achieved by using Minitab to obtain a mathematical equation between load, distance and fuel consumption. Differently from previous studies in regression analysis both vehicle load and distance are considered in fuel consumption calculation. Finally a mixed integer linear programming model for the G-CVRP is presented and solved optimally with MPL-Gurobi solver. G-CVRP optimization model is validated by various instances. Achieved results show that G-CVRP model provides important reductions in fuel consumption. Results also show that distance minimization does not necessarily mean fuel consumption minimization. Future research can be studying the G-CVRP with larger number of customer instances and also developing heuristic algorithm for quick responses in real-life applications. Proposed fuel consumption equation methodology can be adapted to different VRP formulations, such as pick-up and delivery VRP, capacitated VRP, VRP with time windows and multiple depots VRP.

ACKNOWLEDGEMENT

This work was supported by The Commission of Scientific Research Projects of Uludag University, Project number KUAP(M)-2012/53.

REFERENCES

[1] O Jabali, T Van Woensel, AG Kok. Analysis of travel times and CO_2 emissions in time-dependent vehicle routing. Production and Operations Management, 21(6): 1060-1074, 2012.
[2] G Laporte. The Vehicle routing problem: An overview of exact and approximate algorithms. European Journal of Operations Research, 59: 345-358, 1992.
[3] Y Xiao, Q Zhao, I Kaku, Y Xu. Development of a fuel consumption optimization model for the capacitated vehicle routing problem. Computers & Operations Research, 39: 1419-1431, 2012.
[4] L D Bodin. Twenty years of routing and scheduling. Operations Research, 38(4): 571-579, 1990.
[5] P Toth, D Vigo. The vehicle routing problem. SIAM. Philadelphia: PA; 2002.
[6] A Hoff, H Andersson, M Christiansen, G Hasle, A Lokketangen. Industrial aspects and literature survey: Fleet composition and routing. Computers & Operations Research, 37: 2041-2061, 2010.
[7] Y Huang, C Shi, L Zhao, T Van Woensel. A study on carbon reduction in the vehicle routing problem with simultaneous pickups and deliveries. IEEE International Conference on Service Operations and Logistics and Informatics. 8-10 July 2012. Suzhou, China.
[8] Ö Apaydın, MT Gönüllü. Emission control with route optimization in solid waste collection process: A case study. Sadhana, 33(2): 71-82, 2008.
[9] Y Kuo. Using simulated annealing to minimize fuel consumption fort the time-dependent vehicle routing problem. Computers & Industrial Engineering, 59: 157-165, 2010.
[10] T Bektaş, G Laporte. The pollution-routing problem. Transportation Research Part B, 45: 1232-1250, 2011.
[11] Y Suzuki. A new truck-routing approach for reducing fuel consumption and pollutants emission. Transportation Research Part D, 16: 73-77, 2011.
[12] E Wygonik, A Goodchild. Evaluating CO_2 emissions, cost, and service quality trade-offs in an urban delivery system case study. IATTS Research, 35: 7-15, 2011.
[13] S Erdoğan, E Miller-Hooks. A green vehicle routing problem. Transportation Research Part E, 48: 100-114, 2012.
[14] S Ubeda, FJ Arcelus, J Faulin. Green logistics at Eroski: A case study. International Journal of Production Economics, 131: 44-51, 2011.
[15] E Demir, T Bektaş, G Laporte. A comparative analysis of several vehicle emission models for road freight transport. Transportation Research Part D, 16: 347-357, 2011.
[16] E Demir, T Bektaş, G Laporte. An adaptive large neighborhood search heuristic for the Pollution-Routing Problem. European Journal of Operations Research, 223: 346-359, 2012.
[17] Y Liu, GE Helfand. The alternative motor fuels act, alternative-fuel vehicles, and greenhouse gas emissions. Transportation Research Part A, 43: 755-764, 2009.

[18] HH Braess, U Seiffert. Handbook of Automotive Engineering. SAE International. Pennsylvania USA; 2005.
[19] P Toth, D Vigo. Exact algorithms for vehicle routing. In: Crainic T, Laporte G, editors. Fleet management and logistics, Boston: Kluwer Academic Publishers; p. 1–31, 1998.