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Observations on Icosagonal number

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ABSTRACT

We obtain different relations among Icosagonal number and other two, three and four dimensional figurate numbers.

Keyword: *Polygonal number, Pyramidal number, centered polygonal number, Centered pyramidal number, Special number*

I. INTRODUCTION

The numbers that can be represented by a regular arrangement of points are called the polygonal numbers (also known as two dimensional figurate numbers). The polygonal number series can be summed to form solid three dimensional figurate numbers called Pyramidal numbers that be illustrated by pyramids [1].Numbers have varieties of patterns [2-16] and varieties of range and richness. In this communication we deal with Icosagonal numbers given by $t_{20,n} = 9n^2 - 8n$ and various interesting relations among these numbers are exhibited by means of theorems involving the relations.

Notation

$t_{m,n}$ = Polygonal number of rank n with sides m

p_n^m = Pyramidal number of rank n with sides m

$F_{m,n,p}$ = m-dimensional figurate number of rank n where generated polygon is of p sides

jal_n = Jacobsthal Lucas number

$ct_{m,n}$ = Centered Polygonal number of rank n with sides m

cp_n^m = Centered Pyramidal number of rank n with sides m

g_n = Gnomonic number of rank n with sides m

p_n = Pronic number

$carl_n$ = Carol number

mer_n = Mersenne number, where n is prime

cul_n = Cullen number

Tha_n = Thabit ibn kurrah number

wo_n = Woodall number

II. INTERESTING RELATIONS

1. $\sum_{n=1}^{N} t_{20,n} = 9p_N^4 - 8t_{3,N}$

Proof

$$\sum_{n=1}^{N} t_{20,n} = \sum_{n=1}^{N} [9n^2 - 8n]$$

$$= 9\sum_{n=1}^{N} n^2 - 8\sum_{n=1}^{N} n$$

$$= 9\frac{N(N+1)(2N+1)}{2} - 8\frac{N(N+1)}{2}$$

$$\sum_{n=1}^{N} t_{20,n} = 9p_N^4 - 8t_{3,N}$$

2. $\left(t_{20,n} * t_{12,n}\right) - 4\left(t_{7,n} * t_{11,n}\right) + 25cp_n^6 = 22p_n^5$

Proof

$$\left(t_{20,n} * t_{12,n}\right) - 4\left(t_{7,n} * t_{11,n}\right) = -14n^3 + 11n^2$$

$$= -25n^3 + 11\left(n^3 + n^2\right)$$

$$\left(t_{20,n} * t_{12,n}\right) - 4\left(t_{7,n} * t_{11,n}\right) + 25cp_n^6 = 22p_n^5$$

3. $t_{20,n+1} - 2t_{11,n} - g_n - 15n = 2$

Proof

$$t_{20,n+1} = 9n^2 - 10n + 1$$

$$= 9n^2 - 7n + 2n - 1 + 15n + 2$$

$$= 2t_{11,n} + g_n + 15n + 2$$

$$t_{20,n+1} - 2t_{11,n} - g_n - 15n = 2$$

4. $t_{20,n+1} + t_{20,n-1} - n = t_{38,n} + 18$

Proof

$$t_{20,n+1} + t_{20,n-1} = 18n^2 - 16n + 18$$

$$= 18n^2 - 17n + n + 18$$

$$t_{20,n+1} + t_{20,n-1} - n = t_{38,n} + 18$$

5. The following triples are in arithmetic progression

a) $\left(t_{19,n}, t_{20,n}, t_{21,n}\right)$

Proof

$$t_{19,n} + t_{21,n} = \frac{1}{2}\left(17n^2 - 15n + 19n^2 - 17n\right)$$

$$= 2\left(9n^2 - 8n\right)$$

$$t_{19,n} + t_{21,n} = 2t_{20,n}$$

b) $\left(t_{20,n}, t_{21,n}, t_{22,n}\right)$

Proof

$$t_{20,n} + t_{22,n} = 9n^2 - 8n + 10n^2 - 9n$$

$$t_{20,n} + t_{22,n} = 2t_{21,n}$$

c) $\left(t_{10,n}, t_{20,n}, t_{30,n}\right)$

Proof

$$t_{10,n} + t_{30,n} = 4n^2 - 3n + 14n^2 - 13n$$

$$t_{10,n} + t_{30,n} = 2t_{20,n}$$

d) $\left(t_{18,n}, t_{20,n}, t_{22,n}\right)$

Proof

$$t_{18,n} + t_{22,n} = 8n^2 - 7n + 10n^2 - 9n$$

$$t_{18,n} + t_{22,n} = 2t_{20,n}$$

e) $\left(t_{16,n}, t_{20,n}, t_{24,n}\right)$

Proof

$$t_{16,n} + t_{24,n} = 7n^2 - 6n + 11n^2 - 10n$$

$$t_{16,n} + t_{24,n} = 2t_{20,n}$$

6. $nt_{20,n+1} = 6p_n^{10} + 2p_n^5 + 12t_{3,n}$

Proof

$$nt_{20,n+1} = 9n^3 + 10n^2 + n$$

$$= 8n^3 + 3n^2 - 5n + n^3 + n^2 + 6n^2 + 6n$$

$$nt_{20,n+1} = 6p_n^{10} + 2p_n^5 + 12t_{3,n}$$

7. $t_{20,2^n} + 17 = 9ky_n - 26mer_n$

Proof

$$t_{20,2^n} = 9\left(2^{2n}\right) - 8\left(2^n\right)$$

$$= 9\left(2^{2n} + 2\left(2^n\right) - 1\right) - 26\left(2^n - 1\right) - 17$$

$$t_{20,2^n} + 17 = 9ky_n - 26mer_n$$

8. $t_{20,2^n} - t_{16,2^n} = mer_{2n} + carl_n + 2$

Proof

$$t_{20,2^n} - t_{16,2^n} = 2\left(2^{2n}\right) - 2\left(2^n\right)$$

$$= \left(2^{2n} - 1\right) + \left(2^{2n} - 2^{n+1} - 1\right) + 2$$

$$t_{20,2^n} - t_{16,2^n} = mer_{2n} + carl_n + 2$$

9. $n\left(3t_{8,2^n} - t_{20,2^n}\right) = cul_n + wo_n$

Proof

$$n\left(3t_{8,2^n} - t_{20,2^n}\right) = n\left(2^n\right) + 1 + n\left(2^n\right) - 1$$

$$n\left(3t_{8,2^n} - t_{20,2^n}\right) = cul_n + wo_n$$

10. $3t_{8,2^n} - t_{20,2^n} = Tha_{2n} + 1$

Proof

$$3t_{8,2^n} - t_{20,2^n} = 3\left(2^{2n}\right)$$

$$= Tha_{2n} + 1$$

11. $4\left(2t_{12,2^n} - t_{20,2^n}\right) = jal_{2n+2} - 1$

Proof

$$4\left(2t_{12,2^n} - t_{20,2^n}\right) = 4\left(2^{2n}\right)$$

$$4\left(2t_{12,2^n} - t_{20,2^n}\right) = jal_{2n+2} - 1$$

12. The following is a Nasty number

a) $t_{20,n} - t_{8,n} + 6n$

Proof

$$t_{20,n} - t_{8,n} = 9n^2 - 8n - 3n^2 + 2n$$

$$= 6n^2 - 6n$$

b) $6\left(t_{58,n} - 3t_{20,n} - 3n\right)$

Proof

$$t_{58,n} - 3t_{20,n} = 28n^2 - 27n - 27n^2 + 24n$$

$$t_{58,n} - 3t_{20,n} - 3n = n^2$$

c) $6\left(\dfrac{t_{20,n} - t_{16,n} + n}{t_{6,n}}\right)$

Proof

$$\frac{t_{20,n} - t_{16,n} + n}{t_{6,n}} = \frac{2n^2 - n}{2n^2 - n}$$

d) $6\left(t_{8,n} - t_{24,n} + t_{20,n}\right)$

Proof

$$t_{8,n} - t_{24,n} + t_{20,n} = 3n^2 - 2n - 2n^2 + 2n$$

$$= n^2$$

e) $t_{20,n} - 3ct_{3,n} + 15n - 2g_n$

Proof

$$t_{20,n} - 3ct_{3,n} = 6n^2 - 15n + 4n - 2$$

$$= 6n^2 - 15n + 2(2n-1)$$

$$t_{20,n} - 3ct_{3,n} + 15n - 2g_n = 6n^2$$

f) $\left(t_{20,n} * 2t_{3,n}\right) - 9_{n^2} - 2p_n^5 + 24n^2$

Proof

$$t_{20,n} * 2t_{3,n} = 9n^4 + n^3 - 8n^2$$

$$= 9\left(\left(n^2\right)^2 + n^2\right) + \left(n^2 + n^3\right) - 18n^2$$

$$\left(t_{20,n} * 2t_{3,n}\right) - 9_{n^2} - 2p_n^5 + 24n^2 = 6n^2$$

13. $\left(t_{20,n} * t_{6,n}\right) + 7n = 36t_{3,n^2} - 30p_n^7 + 2t_{7,n}$

Proof

$$\left(t_{20,n} * t_{6,n}\right) = 18n^4 - 25n^3 + 8n^2$$

$$= 18\left(n^4 + n^2\right) - 5\left(5n^3 + 3n^2 - 2n\right) + \left(5n^2 - 3n\right) - 7n$$

$$\left(t_{20,n} * t_{6,n}\right) + 7n = 36t_{3,n^2} - 30p_n^7 + 2t_{7,n}$$

14. $n\left(t_{20,n} + 4g_n\right) = 3cp_n^{14} + 2cp_n^6$

Proof

$$n\left(t_{20,n} + 4g_n\right) = 9n^3 - 4n$$

$$= 7n^3 - 4n + 2n^3$$

$$n\left(t_{20,n} + 4g_n\right) = 3cp_n^{14} + 2cp_n^6$$

15. $t_{20,n} - t_{14,n} = t_{24,n} - t_{18,n}$

Proof

$$t_{20,n} - t_{14,n} = 3n^2 - 3n$$

$$t_{24,n} - t_{18,n} = 3n^2 - 3n$$

$$t_{20,n} - t_{14,n} = t_{24,n} - t_{18,n}$$

16. $t_{20,n} + 1 = s_n + t_{8,n}$

Proof

$$t_{20,n} = 9n^2 - 8n$$

$$= \left(6n^2 - 6n + 1\right) + \left(3n^2 - 2n\right) - 1$$

$$t_{20,n} + 1 = s_n + t_{8,n}$$

17. $n\left(t_{20,n} + 1\right) + t_{24,n} = 6p_n^{10} + 3cp_n^{10} - 6cp_n^4$

Proof

$$nt_{20,n} + t_{24,n} = 9n^3 + 2n^2 - 10n$$

$$= \left(8n^3 + 3n^2 - 5n\right) + \left(5n^3 - 2n\right) - 2\left(2n^3 + n\right) - n$$

$$n\left(t_{20,n} + 1\right) + t_{24,n} = 6p_n^{10} + 3cp_n^{10} - 6cp_n^4$$

18. $n\left(18p_n^5 - t_{20,n}\right) - 17n^2 = 108F_{4,n,4}$

Proof

$$n\left(18p_n^5 - t_{20,n}\right) = 9n^4 + 8n^2$$

$$= 9\left(n^4 - n^2\right) + 17n^2$$

$$n\left(18p_n^5 - t_{20,n}\right) - 17n^2 = 108F_{4,n,4}$$

19. $t_{20,n} + p_n^{14} + 3n = 6p_n^6 + t_{34,n}$

Proof

$$t_{20,n} + p_n^{14} = 9n^2 - 8n + \frac{4n^3 + n^2 - 3n}{2}$$

$$= \left(4n^3 + 3n^2 - n\right) + \left(16n^2 - 15n\right) - 3n$$

$$t_{20,n} + p_n^{14} + 3n = 6p_n^6 + t_{34,n}$$

20. $4t_{20,n} = t_{38,n} + t_{18,n} + t_{22,n} + n$

Proof

$$2t_{20,n} = t_{38,n} + n \qquad (1)$$

$$2t_{20,n} = t_{18,n} + t_{22,n} \qquad (2)$$

Add (1) and (2), we get

$$4t_{20,n} = t_{38,n} + t_{18,n} + t_{22,n} + n$$

21. $t_{58,n} - t_{40,n} + n = t_{20,n}$

Proof

$$t_{58,n} - t_{40,n} = 9n^2 - 9n$$

$$= t_{20,n} - n$$

$$t_{58,n} - t_{40,n} + n = t_{20,n}$$

22. $\left(t_{20,n} * g_n\right) + 4n = 6cp_n^{18} - t_{52,n}$

Proof

$$\left(t_{20,n} * g_n\right) = 18n^3 - 25n^2 + 8n$$

$$= 6\left(3n^3 - 4n\right) - \left(25n^2 - 24n\right) - 4n$$

$$\left(t_{20,n} * g_n\right) + 4n = 6cp_n^{18} - t_{52,n}$$

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