Model Order Reduction By MixedMathematical Methods

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ABSTRACT

In this paper, a mixed method mathematical technique is proposed for finding reduced order model of Single-Input Single-Output (SISO) system. The model reduction of large scale systems is achieved by using Padé approximation and utilizing the characteristics of the higher order system which is maintained in the reduced model. The denominator polynomial of the reduced model is determined by the characteristic equation which retains the basic characteristics of the original system, and the coefficients of the numerator polynomial are obtained by using the Padé approximation technique.

Index Terms: *Integral Square Error, Padé Approximation, Single-Input Single-Output*

I. INTRODUCTION

The approximation of high-order complex systems to low-order simple models has been the concern of many researchers during the last two decades. The low-order simple model can replace the original one to facilitate the design and online implementation of the feedback controllers and digital filters. The ease with which the analysis can be performed depends mainly upon the complexity of the model. Simple lower order models are very desirable in the design of control systems, as the analysis of higher order systems is usually cumbersome and mathematically intensive. The purpose of obtaining reduced model is to provide a good approximant for the higher order systems, maintaining the characteristics of the latter as much as possible. Nowadays, systems have become complex and the interrelationship of many controlled variables need to be considered in the design process.

Various model reduction methods have been proposed. Two approaches have attracted much attention in this field and are worth mentioning. These are Moore’s balanced realization [1] and optimal Hankel norm approximation [2]. During the past three decades, several more model order reduction techniques have been developed [6-9]. Each of them has their own merits and applications. In recent times, mixed mathematical techniques [10-11] have gained much importance in model order reduction of large scale Single-Input Single-Output (SISO) systems. Extensions of these SISO techniques for MIMO systems are not trivial. Shamash [3] has proposed a multivariable system reduction using Padé Approximation and dominant eigen values. This method assumes that the dominant pole of the higher order system are known, and hence suffer from the drawback of its applicability to systems with no dominant poles, or where the dominant poles are difficult to identify. Pal [4] has developed a system reduction methodology using the continued fraction approach and Routh Hurwitz Array, in which the initial transient response of the reduced order model might not match with the higher order system, as only the first few time moments are considered depending upon the order of the reduced model. The viability and limitations of similar methods have been discussed by Shamash [5].

Recently, mixed methods are getting greater attention in the model order reduction of MIMO systems. In these methods, the common denominator of the transfer function of the reduced order model is fixed by using a stability preserving algebraic method, while the numerators are obtained using one of the available model reduction methods.

II. PROPOSED METHOD

In this proposed method the advantages of mixed method of model order reduction is used for Single-Input Single-Output (SISO), where the numerator is reduced by Padé Approximation [17] and second order reduced denominator is derived by utilizing the basic characteristics [19] of higher order original system. These characteristics are undamped natural frequency of oscillations (ω_n), damping ratio (ξ), settling time (T_s), peak amplitude (P_a) and peak time (T_p). the reduced second order approximant maintains the characteristics of the original system. This method tries to minimize the Integral Square Error (ISE) and the reduced second order model behaviour to the input signal almost matches with the original system behaviour.

Padé Approximation:
An asymptotic expansion or a Taylor expansion can often be accelerated quite dramatically by being re-arranged into a ratio of two such expansions.

A Padé approximation

$$G(s) = \frac{\sum_{k=0}^{M} a_k s^k}{\sum_{k=0}^{N} b_k s^k} \qquad \text{(i)}$$

(normalized by $b_0 = 1$) generalizes the Taylor expansion with the same total number of coefficients:

$$T_{M+N}(s) = \sum_{k=0}^{M+N} c_k s^k \qquad \text{(ii)}$$

(the two expansions being the same in the special case of M = 0). From a truncated Taylor expansion (ii), one determines the corresponding Padé coefficients by requiring that if (i) is Taylor expanded; the result shall match all the terms given in (ii).
Such that:

$$G(s) = \frac{\sum_{k=0}^{M} a_k s^k}{\sum_{k=0}^{N} b_k s^k} = c_0 + c_1 s + c_2 s^2 + \ldots..$$

The coefficients of the power series can be calculated as:

$c_0 = a_0$
$c_k = 1/b_0[a_k - \sum_{j=1}^{k} b_j c_{k-j}]$; $k > 0$
$a_k = 0$ $\qquad k > n-1$

Hence, Padé Approximants can be given as:

$a_0 = b_0 c_0$
$a_1 = b_0 c_1 + b_1 c_0$
.
.
. $\qquad a_{k-1} = b_0 c_{k-1} + b_1 c_{k-2} + \ldots + b_{k-1} c_1 + b_k c_0$

The reduced model's transfer function can be formed by using Padé Approximants in the numerator and the denominator is derived by the basic characteristics of higher order systems as:

$$G(s) = \frac{a_0 + a_1 s}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

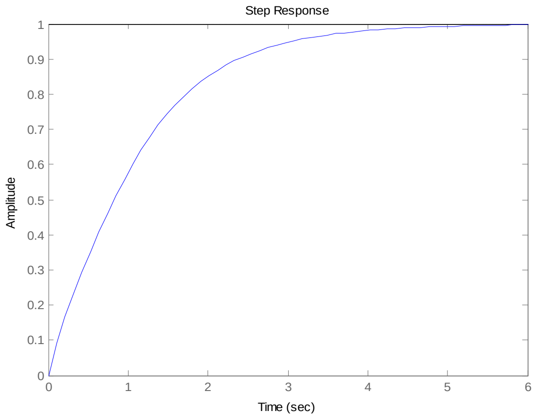
III. NUMERICAL EXAMPLE

Consider a fourth order system:

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

Characteristics:
Rise Time: 2.2609
Settling Time: 3.9312
Settling Min: 0.9062
Settling Max: 1.0000
Overshoot: 0
Undershoot: 0
Peak: 1.0000
Peak Time: 10.4907

The unit step response can be shown as:



Reducing the denominator by utilising the characteristics of the system, like damping ratio (ξ), undamped natural frequency of oscillations (ω_n) etc.

For an aperiodic or almost periodic system, $\xi = 0.99$, number of oscillations before the system settles = 1

Since, $\omega_n = 4/\xi * T_s$

Therefore, $\omega_n = 4/(0.99*3.93) = 1.0281$

Reduced denominator:

$D_2(s) = s^2 + 2\xi\omega_n s + \omega_n^2$

Therefore, $D_2(s) = s^2 + 2.0356s + 1.0569$

Now, reducing numerator by Padé Approximation:

$b_0 = 1.0569$
$b_1 = 2.0356$
$c_0 = 1$
$c_1 = - 1.08333$
Now, reduced model representation:

$$G(s) = \frac{a_0 + a_1 s}{s^2 + 2.0356s + 1.0569}$$

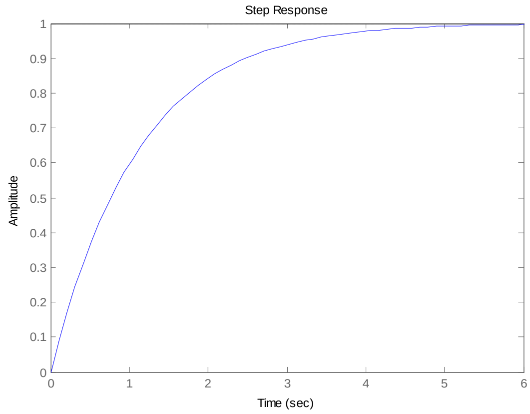
$a_0 = b_0 c_0 = 1.0569$
$a_1 = b_0 c_1 + b_1 c_0 = 0.891$

Hence, final reduced model:

$$G(s) = \frac{1.0569 + 0.891s}{s^2 + 2.0356s + 1.0569}$$

Its characteristics can be given as:
Rise Time: 2.3589
Settling Time: 4.1169
Settling Min: 0.9021
Settling Max: 1.0000
Overshoot: 0
Undershoot: 0
Peak: 1.0000
Peak Time: 10.3072

The unit time step response can be shown as:



Integral Square Error, ISE = 5.1018×10^{-4}

IV. CONCLUSION

In this work a mixed mathematical technique has been proposed for model reduction. It can be clearly seen from the step response that the reduced model is in close approximant with the original model and tries to minimize the error. If the original model is stable, the reduced model must also be stable. The proposed technique is evaluated for Linear Time Invariant Single-Input Single-Output (SISO) system, and the result proves to be better than the past proposed methods. This method utilises the basic characteristics of the higher order system and forms second order approximant that resembles the second order characteristics equation. The method is computationally simple and reduced model stability is assured if the original system is stable.

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