

# ANFIS Based Terminal Sliding Mode Control Design for Nonlinear Systems

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## Abstract

In this paper, an adaptive network based fuzzy inference system (ANFIS) terminal sliding (TS) mode (ANFISTS) controller is developed to deal with the disturbed nonlinear system. This ANFISTS control system is composed of an ANFIS controller and a TS controller. The ANFIS controller is designed to do as a main controller and the TS controller is designed to cope with disturbances and uncertainties. In simulations, the nonlinear inverted pendulum system is illustrated to verify this proposed ANFISTS control methodology. The results possess better performances and robustness by comparisons with fuzzy control.

**Keywords:** Nonlinear system; adaptive network based fuzzy inference system (ANFIS) control; terminal sliding mode control

## I. INTRODUCTION

The neural network control scheme has already been proposed as a design method for control of dynamic systems in recent years. The most useful property of neural networks is their ability to approximate linear or nonlinear mapping through learning. With this property, the neural network-based controllers have been developed to compensate the effects of nonlinearities and system uncertainties, so that the stability, convergence and robustness of the control system can be improved. The concept of fuzzy-neural control incorporating fuzzy logic into a neural network has recently grown into popular research topics [1]. The fuzzy neural network possesses advantages both of fuzzy systems and neural networks since it combines the fuzzy reasoning capability and the neural network on-line learning capability. Since the adaptive network based fuzzy inference system (ANFIS) [2] has been embedded in Matlab™ software to become a tool of more easily using and implementations. Meanwhile, there are many applications of ANFIS have been developed which can be searched in web site such as Google Scholar. The ANFIS is a hybrid learning algorithm which is to construct a set of input-output pairs of fuzzy if-then rules with appropriate membership functions. The outputs of ANFIS can be generated as a fuzzy associated memory (FAM).

Sliding mode (SM) controls or switching controls have been proposed by many researchers, recently [3]. This control scheme is based on the variable structure system (VSS) methodology and its goal is to drive the trajectories of the system's states onto a specified sliding surface and maintain the trajectories on this sliding surface continuously to guarantee the stability of system. In recent years, there have been some new mechanisms developed for improving SM control performance, such as the terminal sliding (TS) control [4]. The TS scheme has ability to eliminate the singularity problems of conventional sliding mode control [4]. In this paper, an intelligent method called ANFIS based terminal sliding mode (ANFISTS) control is developed for the design of the pendulum system. This ANFISTS control system comprises an ANFIS controller and a TS controller. The ANFIS controller is used to approach an ideal controller, and the TS controller is designed to compensate for the approximation error between the ANFIS controller and the ideal controller. Finally, a simulation is demonstrated to compare the performances and effectiveness for inverted pendulum system design between the proposed ANFISTS control and the traditional fuzzy control.

## II. ANFIS BASED TERMINAL SLIDING MODE CONTROLLER DESIGN

Consider a nonlinear system includes disturbances:

$$\dot{\mathbf{x}} = \mathbf{f} + \mathbf{g}u + \mathbf{d} \quad (1)$$

where  $\mathbf{x}$  denotes the state vector,  $u$  denote the control input,  $\mathbf{d}$  denotes the unknown disturbances with a known upper bound and functions of  $\mathbf{f}$  and  $\mathbf{g}$  depend on  $\mathbf{x}$ .

Since the system parameters may be unknown or perturbed, the ideal controller  $u_{id}$  cannot be implemented. To overcome this, an ANFIS controller will be designed to approximate this ideal controller. In addition, a terminal sliding (TS) controller is designed to compensate for the approximation error between the ANFIS controller and the ideal controller of TS and to overcome the disturbances. Suppose that a first order terminal sliding function is defined as [4]

$$s = \dot{\mathbf{x}} + k\mathbf{x}^p \quad (2)$$

where  $k > 0$  and  $p > q$ , all are positive constants. Then

$$\dot{s} = \ddot{\mathbf{x}} + k\mathbf{x}^{p-1} \dot{\mathbf{x}} \quad (3)$$

The control law of ANFISTS control is defined as

$$u_i = u_{anfists} = u_{anfis} + u_{ts} \quad (4)$$

where  $u_{anfis}$  is the ANFIS controller and  $u_{ts}$  is the TS controller.

The inputs of the ANFIS controller are  $s$  and its derivative value of  $\dot{s}$ .  $u_{ts}$  is designed as  $h \operatorname{sgn}(s)$ , where  $h$  is a positive constant; and 'sgn' implies a sign function which can be replaced by a saturation function to avoid chattering effect.

The ANFIS is a hybrid learning algorithm which can generate Takagi-Sugeno (T-S) type fuzzy inference systems (FIS) to model a given training data. The principle of ANFIS is briefly described as follows [2].

$$\begin{aligned} R_i: & \text{If } x_1 \text{ is } A_{i1} \dots \text{and } x_n \text{ is } A_{in} \\ & \text{then } u_i = p_{i1}x_1 + \dots + p_{in}x_n + r_i \end{aligned} \quad (5)$$

where  $R_i$  denotes the  $i$ th fuzzy rules,  $i=1, 2, \dots, j$ ;  $A_{ik}$  is the fuzzy set in the antecedent associated with the  $k$ th input variable at the  $i$ th fuzzy rule; and  $p_{i1}, \dots, p_{in}, r_i$  are the fuzzy consequent parameters.

Based on the ANFIS, the output  $u$  can be got as

$$\begin{aligned} u &= \frac{w_1}{w_1 + \dots + w_j} u_1 + \dots + \frac{w_j}{w_1 + \dots + w_j} u_j \\ &= \bar{w}_1 u_1 + \dots + \bar{w}_j u_j \end{aligned} \quad (6)$$

where  $\bar{w}_1 = \frac{w_1}{w_1 + \dots + w_j}, \dots, \bar{w}_j = \frac{w_j}{w_1 + \dots + w_j}$ .

Because the fuzzy inference system is a T-S type, i.e.  $u_i = p_{i1}x_1 + \dots + p_{in}x_n + r_i$ , equation (6) can be expressed as

$$\begin{aligned} u &= \bar{w}_1 u_1 + \dots + \bar{w}_j u_j \\ &= (\bar{w}_1 x_1) p_{11} + \dots + (\bar{w}_1 x_n) p_{1n} + (\bar{w}_1) r_1 \\ &\quad + \\ &\quad \vdots \\ &\quad + (\bar{w}_j x_1) p_{j1} + \dots + (\bar{w}_j x_n) p_{jn} + (\bar{w}_j) r_j. \end{aligned} \quad (7)$$

The neural network learning algorithm developed in [2] can be applied to (7) directly. A neural network structure of ANFIS is shown in Fig. 1.

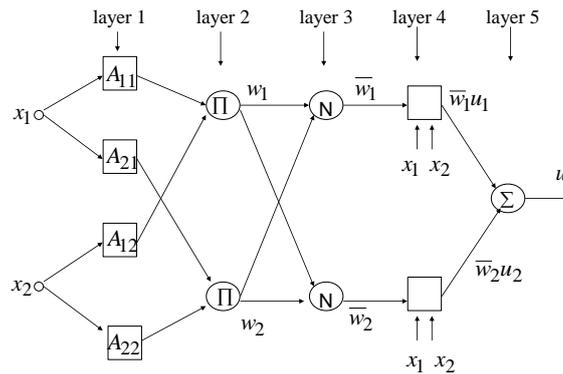


Fig. 1. Diagram of ANFIS architecture [2]

### III. SIMULATION RESULTS

For nonlinear system, the equation as following famous pendulum system is considered: [5]

$$\begin{aligned}\dot{x}_1 &= x_2 + w_1 \\ \dot{x}_2 &= \frac{g \sin(x_1) - amlx_2^2 \sin(2x_1)/2 - a \cos(x_1)u}{4l/3 - aml \cos^2(x_1)} + w_2\end{aligned}\quad (8)$$

where parameters are  $\mathbf{w} = [w_1 \ w_2]^T = [0.05 \cos(0.05t) \ 0.05 \sin(0.05t)]^T$ , the initial states are  $\pi/6, 0$ . The  $x_1$  denotes the angle of the pendulum from the vertical and  $x_2$  is the angular velocity and  $g = 9.8 \text{ m/sec}^2$  is the gravity constant,  $m$  is the mass of the pendulum,  $M$  is the mass of the cart,  $2l$  is the length of the pendulum, and  $u$  is the forced applied to the cart.  $A = 1/(m + M)$ . The  $m = 2.0 \text{ kg}$ ,  $M = 8.0 \text{ kg}$  and  $2l = 1 \text{ m}$ . In these initial fuzzy rules, membership functions of  $x_1$  and  $x_2$  are given as  $\pm \pi/4$ . The learning rate is set as 0.1,  $p = 3$  and  $q = 1$ . The designed FAM of ANFIS is shown in Fig. 2. By using this designed FAM of ANFIS controller and TS controller, the inverted pendulum can be controlled to be stable and robust. The simulation results are shown in Fig. 3 which is compared with fuzzy control. These simulation results demonstrate better performances and robustness of ANFISTS.

### IV. CONCLUSION

An adaptive networks based fuzzy inference system (ANFIS) terminal sliding (TS) mode control denoted as ANFISTS controller for inverted pendulum system is developed in this paper; and is compared with fuzzy control system. In the simulations, the better performances and effectiveness of ANFISTS is possessed by comparison with that of the traditional fuzzy control.

### V. ACKNOWLEDGMENT

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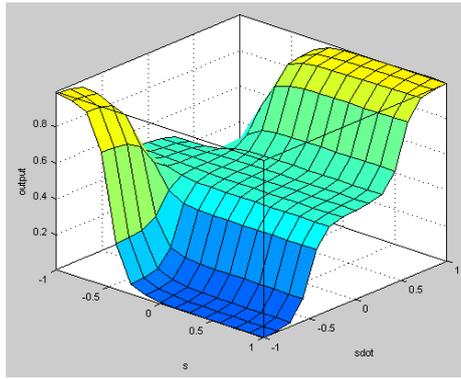


Fig. 2. Diagram of ANFIS FAM surface

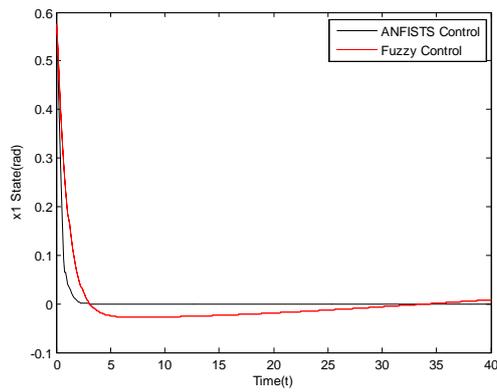


Fig. 3 (a). States of x1 trajectories

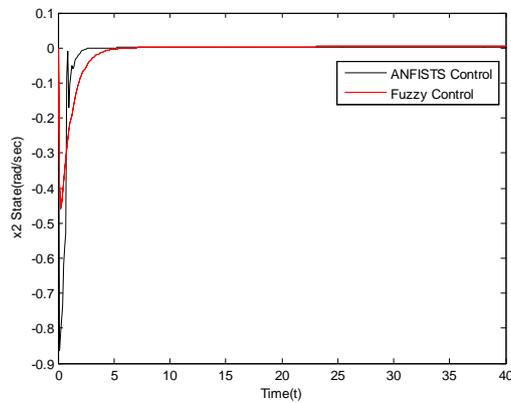


Fig. 3 (b). States of x2 trajectories

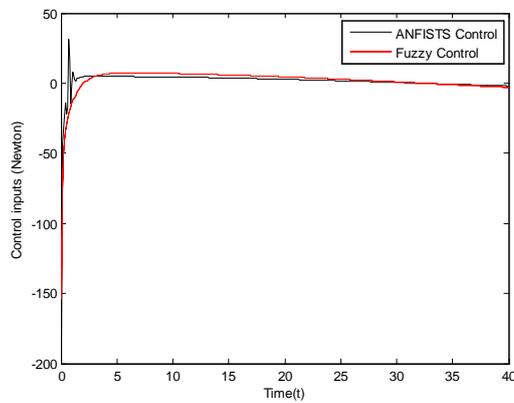


Fig. 3 (c). Control inputs