Novel Encoding and Decoding Algorithm for Block Turbo Codes over Rayleigh Fading Channel

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Abstract

Proposed Error Correcting codes are widely used for error detecting and correcting which are present in during receiption and they are also widely used in compact disc and wireless and digital communications. Proposed paper mainly discuss about the Block Encode and Decoder architectures with less complexity in terms of encoding and decoding. There are Six types of block codes are mainly discussed i.e Block (7, 3), Block (15,11), Block (31,15), Block (31,27), Block (255, 165) and Block(255,205). Simulation results over a AWGN and Rayleigh fading Chanenel shows that Block Codes with higher block lengths are giving more gain compared to the lower block length and also error correcting capacity also higher for higher block lengths but complexity of encoding and decoding algorithms will increase and simulation results are shown for both voice and image and compared with existing algorithm. When I have used the single Block codes, error correction capacity is lower but it is doubled in case of block turbo codes and also performance is compared.

Keywords: Block codes, convolutional codes, forward error correctiong codes and decoding algorithms.

1. Review Of Work:

Block Codes(S,v) are mainly used in campact disks and mobile communications [1][2] and they are able to correct

$\dfrac{S-v}{2}$ of sequence of errors present in the received data. For example, if Block(31,15) is used then S =31 and v=15 then this code can correct errors upto 4 symbols. And its correction capacity $Q_c = \dfrac{31-15}{2} = 4$ symbols. This code able to correct upto 16 bits in sequence if at all 16 bits are corrupted. These block codes are mainly concentrated on symbols but not on bits. The Block codes are represented by the following characteristics.

Number of bits to represent one symbol	:**λ**
Code Length in symbols S	:$\mathbf{2^{\lambda}-1}$
Number of Information symbols v	: $\mathbf{2^{\lambda} - 1 - 2}\, Q_c$
Error correcting capability	: Q_c
Number of Check symbols	:**2 Qc**
Minimum distance	:$\mathbf{d = 2}\, Q_c\ \mathbf{+ 1}$

1.1 Previous Work:

Proposed Encoder and decoder of block codes are used in any communication system and these block codes are classified as a forward error correcting codes in alll aspects. Block codes (S, v) encoder architecture has been designed to produce the code word in terms of symbols rather than bits. One more type of error correcting code is called convolutional coding and which will produce the code word in terms of bits[3]. In order to transmit the data from source to the destinaiton, every communication system has to satisfy the condition of Shannon Capacity theorem[4][5].A lot of research is going on error correcting codes, which are linear Block codes, BCH codes, cyclic codes and convolutional codes. But each code having its own limitations. Out of all these codes, convolutional codes are very flexible in design and they will give better performance in terms of BER and SNR through the channel of AWGN compared to all other codes. But convolutional codes can not be used alone to correct sequence of errors are present in the received data. In order to avoid this problem, convolutional codes are connected in series[7] and parallel through interleaver is called product turbo codes[6]. Recently research is going on turbo codes, Soft out Veterbi Algorithm (SOVA) and which are very powerful error correcting codes because it has a iterative decoding algorithm. They have excellent Bit Error Rate performance against the Signal to Noise Ratio but still have some problems. First of all, their error performance tails of, or exhibits an error "floor" at high signal-to-noise ratio (SNR)[8]. The complexity of the required soft-input, soft-output (SISO) decoder is such that a cost-efficient decoder was unavailable for most commercial applications. Block Codes are give better performance in terms of Bit Error Rate and in terms of sequence of Error Correction capability that's why they are still used in Voice and Video applicationsThis paper is organized as follows. In Section I, explained about *Block Encoder Descriptive Example*, while Section II summarizes with Block Decoder Descriptive example. Section III summarizes the simulation result of Block Codes for a random generated data with different modulaiton techniques over an AWGN Channel. Section IV briefly discuss about voice signal encoding and decoding using modulation techniques and section V gives a simulation result for image transmission through a AWGN channel. Section VI concludes my research work.

1.2 Proposed Work:

I have designed a new class of error correcting code called block code and denoted by Block (S,v). This Block code is tested for random generated data, voice signal and for image transmission. There are different types of Block codes are designed in my proposed work which are Block(7,3), Block(31,15), Block (255,205) and Block(255,225). All Block codes are simulated with different lengths of S and with different modulation techniques such as BPSK, QPSK AND QAM and transmitted from source to destination through the AWGN channel. From fig1. For proposed Block codes architecture, how the error rate has calculated as shown. I extended this work for block turbo codes. When i use the single Block codes, error correction capacity is lower but for error correction capacity becomes doubled in case of block turbo codes. For example (7,3) block code, error correction capacity is 2 while in case of block turbo codes it becomes 4.

2. Block Encoder Descriptive Example

Let S = 2^{λ} — **1** be the block length of a Block code of designed distance *d* in $GF(2^{m\lambda})$. The number of λ-bit message symbols *is v* = ***S* — *d* + *I.***

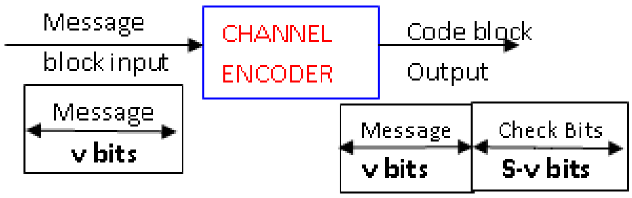


Fig:1 Block Encoder

First we define the generator polynomial to encode the v information symbols into length of S = $2^{\lambda} - 1$ symbol Block code word is

$\pi(y) = \prod_{i=1}^{2Q_c}(y-\beta^i)$ where **π (y)** is the generator polynomial. Example Generator polynomial for Block (255,249) is $\pi(y) = (y-\beta^1)(y-\beta^2)(y-\beta^3)(y-\beta^4)(y-\beta^5)(y-\beta^6)$ or $\pi(y) = y^6 + \pi_5 y^5 + \pi_4 y^4 + \pi_3 y^2 + \pi^2 y^1 + \pi_1$. Let S = **15** and consider a primitive, narrow sense, three-error correcting code over **GF (2^4),** where the field is constructed modulo the primitive polynomial $y^4 + y + 1$. let **β** be a primitive element in the field and this generator has six roots of $\beta, \beta^1 \ldots \beta^{2Qc}$. i. **e** $2Q_C = S - v$. **roots are**

$\beta^1, \beta^2, \beta^3, \beta^4, \beta^5, \beta^6$

The generator of the **(15 , 9)** code is

$\Pi(y) = (y-\beta^1)(y-\beta^2)(y-\beta^3)(y-\beta^4)(y-\beta^5)(y-\beta^6)$ **or**

$\Pi(y) = \beta^6 + \beta^9 y + \beta^6 y^2 + \beta^4 y^3 + \beta^{14} y^4 + \beta^{10} y^5 + y^6$

Block codes may be encoded just as any other cyclic code. Given a input message vector polynomial $a(y) = a_0$, $a_1, a_2, \ldots\ldots\ldots\ldots a_{v-1}$ where each $a_i \in GF(2^{\lambda})$. And systematic encoding process is

$$H(y) = m(y)\, y^{S-v} + rem(y) = q(y) \cdot \Pi(y)$$

Let us assume that input message as 010 110 111 convert this into 3 bit data in decimal as **A = [2,6,7] for a Block code of [7,3]** and corresponding message polynomial is written as $A(y) = \beta^1 + \beta^3 y + \beta^5 y^2$ and $y^4(\beta^1 + \beta^3 y + \beta^5 y^2)$

is $A(y) = \beta^1 y^4 + \beta^3 y^5 + \beta^5 y^6$ Using vector to power conversion, we will divide the message polynomial by the generator polynomial as $\beta^3 + \beta^1 y + \beta^0 y^2 + \beta^3 y^3 + y^4$ And reminder the above division is gives by $p(y) = \beta^0 + \beta^2 y + \beta^4 y^2 + \beta^6 y^3$ and resulting codeword polynomial is written as $H(y) = p(y) + y^{s-v} m(y)$ **is** $\beta^0 + \beta^2 y + \beta^4 y^2 + \beta^6 y^3 + \beta^1 y^4 + \beta^3 y^5 + \beta^5 y^6$ In Encoder each symbol (**β**) is made with 3 bits of input data and which is convenient for modems for processing. The input to the encoder is 7 symbol of information word. The conventional method to achieving this uses an arrangement containing multipliers and **is** expensive to implement in hardware. The implementation of block encoders based on an Linear shift register, which implements the polynomials division over the finite field [8]. The Block encoder architecture had slice blocks containing a constant multiplier, an adder, and a register . The number of slices to implement for an Block **(S, v)** code is s-v**..** The additions and multiplications are performed on **GF(2^λ)** and g $_i$ are the coefficients of the generator polynomial **π(y).** Finally output codeword polynomial is written as

H(y)= $\beta^0 + \beta^2 y + \beta^4 y^2 + \beta^6 y^3 + \beta^1 y^4 + \beta^3 y^5 + \beta^5 y^6$

Or H(y) = (100)+(001)Y + (011) Y^2 + (101) Y^3 + (010)Y^4 + (110) Y^5 + (111)Y^6

3. **Block Decoder**

The algebraic decoding of block codes has the following general steps:

1. Computation of the syndrome
2. Determination of an error locator polynomial, whose roots provide an indication of where the errors are.
3. Finding the roots of the error locator polynomial. This is usually done using the novel search, which is an exhaustive search over all the elements in the field.
4. For block codes, the error values must also be determined. This is typically accomplished using proposed method search.

Suppose that received vector has v errors at locations of **i_1, i_2, i_3……i_v** with corresponding error values in these locations $e_{ij} \neq 0$.

Error Locator polynomial is written as $W(Y) = \sum_{n=0}^{6} W_s Y^S$

W(Y) = (000) +(000)Y +(000) Y^2 + (001) Y^3 + (111)Y^4 + (000) Y^5 + (000) Y^6. This Error polynomial is considered as one error at data location and other error is at parity symbol location.

Now received polynomial is written as D(Y) = H(Y)+W(Y)

D(Y) = (100)+(001)Y + (011) Y^2 + (101) Y^3 + (010)Y^4 + (110) Y^5 + (111)Y^6 + (000) +(000) Y +(000) Y^2 + (001) Y^3 + (111)Y^4 + (000) Y^5 + (000) Y^6

D(Y) = (100)+(001)Y + (011) Y^2 + (100) Y^3 + (101)Y^4 + (110) Y^5 + (111)Y^6 and above expression in polynomial form as

D(Y) = $\alpha^0 + \alpha^2 x + \alpha^4 x^2 + \alpha^0 x^3 + \alpha^6 x^4 + \alpha^3 x^5 + \alpha^5 x^6$ in order to detect the errors and correct the errors at the decoder, we need to calculate the syndrome and which is given by

$$O_i = D(Y)/_{Y=\beta^i} = D(\beta^i) \qquad i = 1.......S - v$$

Then we can find the syndromes in the following way.

$O_1 = \beta^3$, $O_2 = \beta^5$, $O_3 = \beta^6$, $O_4 = 0$ Suppose in the received codeword, there are n number of errors are received then we have to find the Error locator polynomial in the form of Matrix as. Above Syndrome equations are written in the form Matrix form as

$$\begin{bmatrix} S_1 & S_2 \\ S_2 & S_3 \end{bmatrix} \begin{bmatrix} \sigma_2 \\ \sigma_1 \end{bmatrix} = \begin{bmatrix} S_3 \\ S_4 \end{bmatrix}$$

Substituting the syndrome values in matrix form as **$\sigma_2 = \alpha^0$ and $\sigma_1 = \alpha^6$** Above equation are used to find the locations of errors in the received polynomial is given by

$\sigma(x) = \alpha^0 + \sigma_1 x + \sigma_2 x^2$

$\sigma(x) = \alpha^0 + \alpha^6 x + \alpha^0 x^2$

$\sigma(\alpha^0) = \alpha^6$, $\sigma(\alpha^1) = \alpha^2$, $\sigma(\alpha^2) = \alpha^6$, **$\sigma(\alpha^3) = 0$ (Error)**,

$\sigma(\alpha^4) = 0$ (Error) , $\sigma(\alpha^5) = \alpha^2$, **$\sigma(\alpha^6) = \alpha^0$**

$S_1 = r(\alpha) = e_1 \beta_1 + e_2 \beta_2$

$S_2 = r(\alpha^2) = e_1 \beta_1^2 + e_2 \beta_2^2$

$\sigma(\alpha^3) = 0 \leftrightarrow 1/\ \beta_1 = \alpha^3 \leftrightarrow \beta_1 = \alpha^{-3} = \alpha^4 = \beta_1$

$\sigma(\alpha^4) = 0 \leftrightarrow 1/\ \beta_1 = \alpha^4 \leftrightarrow \beta_1 = \alpha^{-4} = \alpha^3 = \beta_1$

In order to calculate the Errors present in the location are given by following matrix form as.

$$\begin{bmatrix} \alpha^3 & \alpha^4 \\ \alpha^6 & \alpha^8 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} \alpha^3 \\ \alpha^5 \end{bmatrix}$$

And two errrors are given by e_1 = α^2, **e_2**= α^5.And Error Polynomial is given by.,

$e(x) = e_1 x^{j1} + e_2 X^{j2} = \alpha^2 x^3 + \alpha^5 x^4$

u (x) = r (x) + e (x)

$u(x) = 4 + x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 1x^3 + 7x^4$

u (x) = 4 + x + 3 x ^{2} + 5 x ^{3} + 2 x ^{4} + 6 x^{5} +7 x ^{6}

From the above equation of U(x) has parity symbols and message symbols. Message polynomial is **2 x ^{4} + 6 x^{5} +7 x ^{6}** and in binary form as 010 110 111 which is same as given input message.

4 .Rayleigh Slow Fading Channels:

The Rayleigh fading model is one of the most widely used fading channel model which assumes that there exist no direct line of sight path between the transmitter and the receiver and all the arriving signals at the receiver are due to reflected waves. This assumption is a typical characteristic of mobile communication scenario in urban areas. The normalized Rayleigh distribution, its mean and variance are as given below

$$p(r) = \begin{pmatrix} \frac{r}{\sigma^2} e^{\left(-\frac{r^2}{2\sigma^2}\right)} & r \geq 0 \\ 0 & otherwise \end{pmatrix}$$

With mean $m_r = 0.8862$ and variance is $\sigma^2 = 0.2146$

Channels dispersive in frequency are often referred to as time selective. More common term is frequency flat or simply flat fading channels. The term flat fading comes from the fact that ALL frequencies of transmitted signal are modulated by the same function. Frequency dispersion B often referred to as the Doppler spread or bandwidth. If the doppler spread is small compared to the reciprocal of the symbol rate T_s, the fading process is considered slow. For slow fading processes, the channel gain can be assumed constant over the symbol duration.

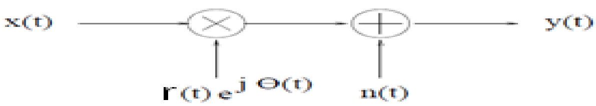


Fig2. Figure represents the Fading channel.

A sufficient and acceptable model is to consider the input-output relation of the digital channel of the form:

$$\mathbf{y_k = r_k . x_k + n_k}$$

where x_k and y_k are the transmitted and received data for the time slot k, respectively; the parameter r_k is a random value, fluctuations from symbol to symbol (fast fading) or from block to block (block fading). Its distribution determines the channel type: Rayleigh, Rice or Nakagami. The input sequence $\{x_k\}$ is binary, random, in NRZ bipolarformat,

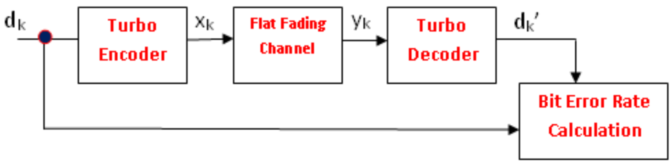


Fig 3. Turbo coded data transmission through different fading channels.

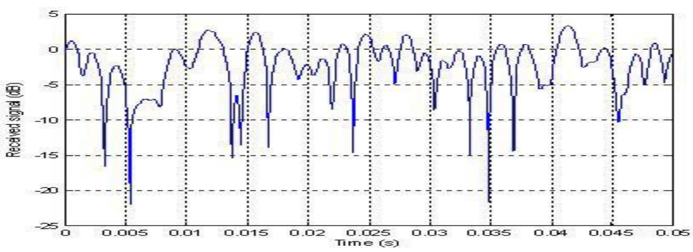


Fig4. Fading channel received signal amplitude characteristics.

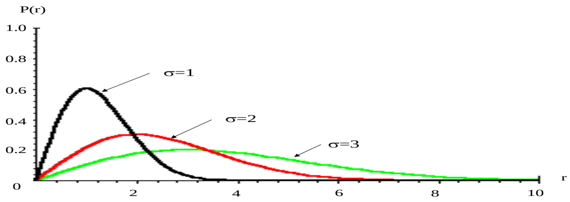


Fig.5 The pdf of the envelope variation

In order to generate the transmitted sequence *{xk}* another sequence denoted *{uk}* is generated by random numbers with uniform distribution in the interval *[0,1)*. The targeted required output is obtained after the following transformation:

$$X_k = 2.[2U_k] - 1$$

In above equation, [] represents the truncated information of integer part, and $X_k^2 = 1$.

r_k represents rice districution and rk2 =1 and effective signal to noise ratio for transmission is defind as

$$\frac{E_b}{N_o} = \frac{1}{2}.\frac{2E_b}{N_o} = \frac{1}{2}\frac{\overline{r_k^2}.\overline{x_k^2}}{\overline{w_k^2}} = \frac{1}{2}\frac{1}{\overline{w_k^2}}$$

Where Eb/no is SNR in dB and variance of additive noise can be expressed as

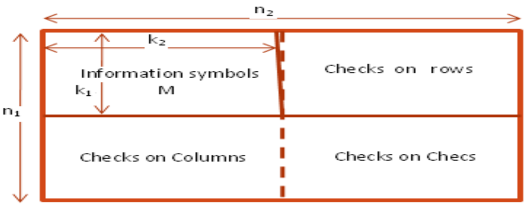
$$\overline{w_k^2} = \frac{1}{2.10^{\frac{SNR}{10}}}$$

5. Block Turbo Codes:

Let us consider the two linear block codes B_1 and B_2 having parameters B_1 of (n_1, k_1, δ_1) and B_2 having parameters $((n_2, k_2, \delta_2)$. The product code $P = B_1 \otimes B_2$

Block Turbo Code obtained by

1. Placing $(k_1 \text{ X } k_2)$ information symbols in an array of k_1 rows and k_2 columns.
2. Coding the k_1 rows using code B_2
3. Coding the n_2 columns using Code B_2 as shown in fig 1

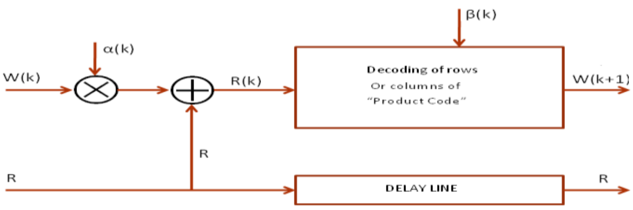


(n1- k1) last rows of the matrix are code words of B_2 and $(n_2\text{-}k_2)$last columns of matrix are code words of B_1 by construction. Resulting Product Code P is

$$n = n_1 \text{ x } n_2 \quad k = k_1 \text{ x } k_2 \quad \delta = \delta_1 \text{ x } \delta_2$$

Fig6. Formation of data and check bits

5.1 Decoding Algorithm for Block Turbo Codes



Scaling factors:

α (k) = { 0, 0.2,0.4,0.43, 0.5, 0.56, 0.6 , 0.7 }

β (k) = { 0.64, 0.64, 0.64, 0.64, 0.7, 0.74, 0.8, 0.8}

R = received data matrix

Fig7. Decoidng of blcok Turbo turbo codes block diagram

6. Simulation Results:

Observation#1. MATLAB is one of the high performance programming language in easy to use environment where problems and solutions are expressed in a frequent mathematical notation. A complete block diagram with Encoder and decoder and encoded data was modulated with Frequency shift keying as shown in fig1. Results with block codes with equal code rate of 0.7 approximatelly as shown in fig 4. The folloiwng Block codes with Block(15,11), Block(31,23), Block(63,47) and Block(127,95) for simulation in AWGN Channel. It is observed that Block codes are more efficient in large code size and it gives less noise effect compared to the lower codeword size of block codes and at the same time increases the complexity in the implementation of the transmission.For a Block Code (127,95) simulation, it is observed that for a BER of 10^{-2} respected SNR is 3dB and which is lowest noise effect compared to other block codes.

Block (S,v)	Code Rate CR = (v/S)
Block(15,11)	0.738
Block(31,23)	0.742
Block(63,47)	0.746
Block(127, 95)	0.748

Table1. code rate for different Block lengths.

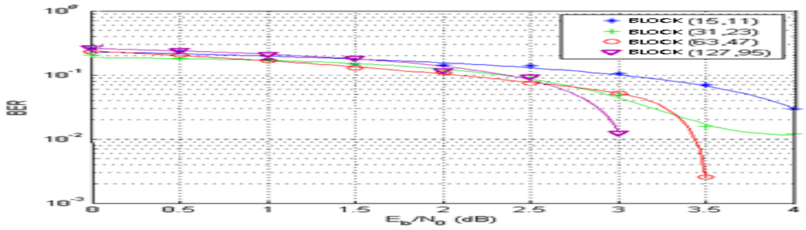


Fig 8. Block code performance in terms of Codeword Size at constant code rate

6.1 Observation. Now for next observation, I have performed the simulations for Block codes for different code rates as well as higher block lengths as shown in fig6. For a Block(255,245), Block(255,225), Block(255,205), Block(255,165) corresponding code rates are 0.96, 0.88, 0.80 and 0.647. As code rate decreasing from 0.96 to 0.647, noise effect also decreasing for a particular BER 10^{-2}. We can see that the absolute BER performance is approx. 2dB better for 0.647 code rate than 0.96. Another point i need to emphasize here that, for constant Block length with same error correcting capability, BER performance improved as shown in fig 6.

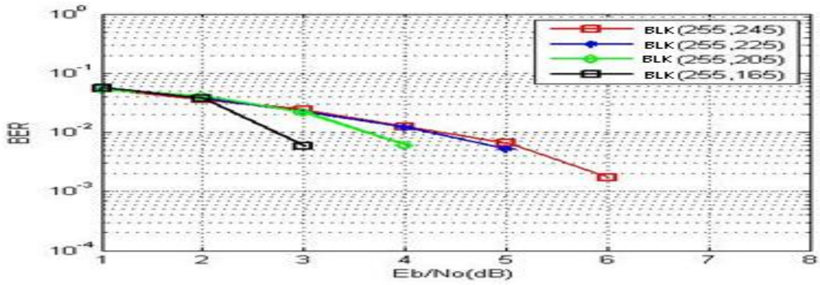


Fig 9. The BER performance comparison of Block codes for different code rates and fixed block length of 255

6.2 Observation. Previous simulation results are observed for 0nly FSK and now block codes are simulated with different lentgths and different code rates are modulated with different modulatin sechemes. BPSK and QPSK with CR =0.5 has very less noise effect on signal compared to all other modulation schemes as shown in fig 8.

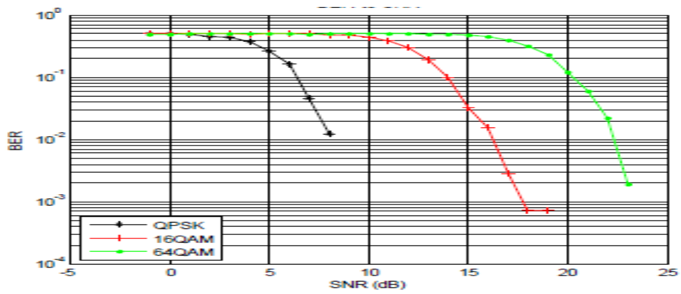


Fig 10. Performance of Block codes for code rate of ½(CR=0.5)

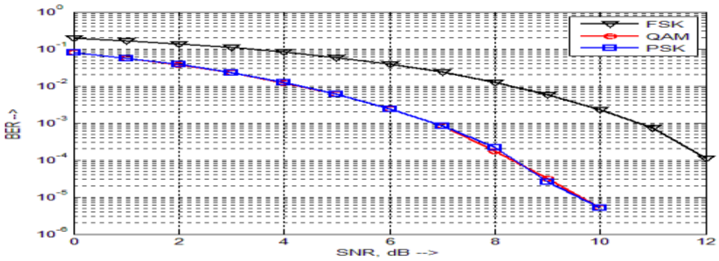


Fig 11. Performance of Block codes for code rate of ¾(CR=0.7)

Block Codes Rate	QPSK at 10^{-2}	16QAM at 10^{-2}	64QAM at 10^{-2}
½	~ 8dB	~ 16.7dB	~ 23 dB
¾	~ 13.5 dB	~ 18.7 dB	27.2 dB

Table 2. Block Codes with different code rates with QAM.

7. Result For Voice Signal.

Fig 9 is a original voice signal and which has recorded. This original voice signal is encoded and modulated with FSK, PSK and QAM and plotted its bit error rate with respect to signal to noise ratio. Block codes (127,123) has simulated with PSK and QAM has given lesser noise effect on signal compared to the FSK. Also observed that BER performance on signal is directly proportional to the Modulation order of M if it is modullated with M-ary Modulation schemes.

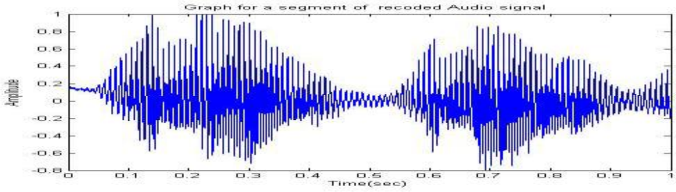


Fig 12. Original recorded audio signal

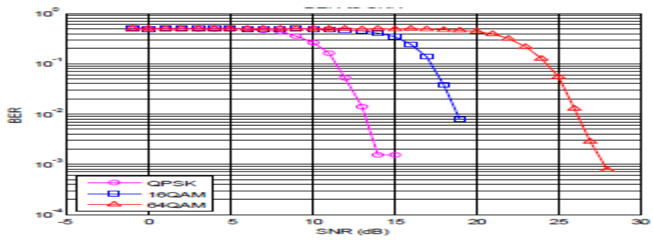


Fig 13. Block codes(127,123) for FSK,PSK and QAM

7. 1 Result for Image Signal.

In order to use any forward error correcting code in digital communication it is required to transmit images through that FEC codes[7]. Simulation results shows the image quality degrades as the code rate increasing from lower to higher as shown in table 5. In order to transmitt the image transmission using block codes, i suggest to use lower code rate compared to the higher order code rates where low Signal to Noise ratio is important.

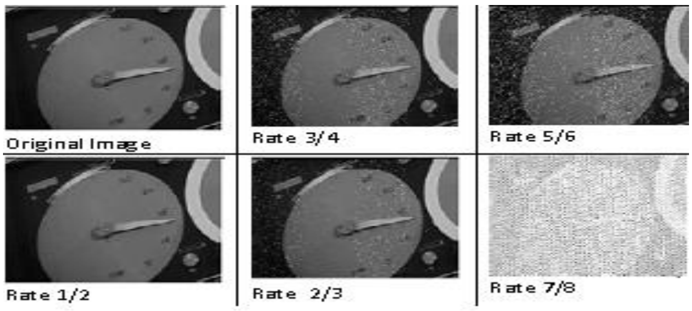


Fig 14. Recovered images with different rates with SNR=6dB

Modulation Scheme	BER at 10^{-2}
BPSK – ½	3 dB
QPSK - ½	7 dB
16 QAM – ½	15 dB
64 QAM – 2/3	17 dB
QPSK – ¾	9 dB
16 QAM - ¾	15 dB
64 QAM – ¾	20 dB

Table.4 Illustrates the modulation schemes are observed at BER 10^{-3}

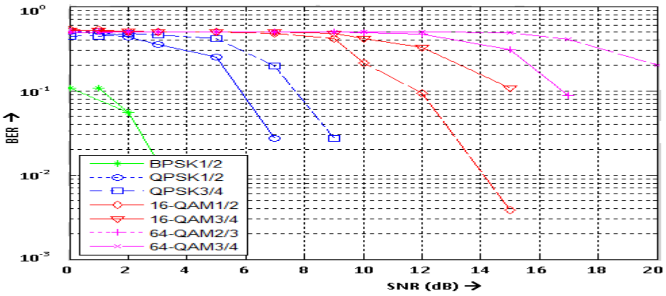


Fig 12. Image encoding for Block Codes for BPSK,QPSK AND QAM

Eb/No(db)	0	9	20
16 QAM (Theiratical Symbol Error ate)	0.9821	0.3919	0.0001721
16 QAM (Practical Symbol Error Rate)	0.7397	0.3540	0.0001652

Table.5. Block (15,11) codes are simulated over AWGN with 16 QAM Modulation.

Eb/No(db)	0	9	20
16 QAM (Theiratical Symbol Error ate)	0.9821	0.4751	0.013475
16 QAM (Practical Symbol Error Rate)	0.7428	0.4195	0.007829

Table 6. Block (15,11) codes are simulated over Rayleigh fading Channel with 16 QAM Modulation.

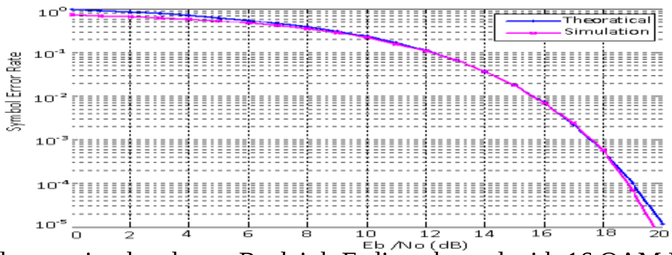


Fig. 13. Block (15,11) codes are simulated over Rayleigh Fading channel with 16 QAM Modulation.

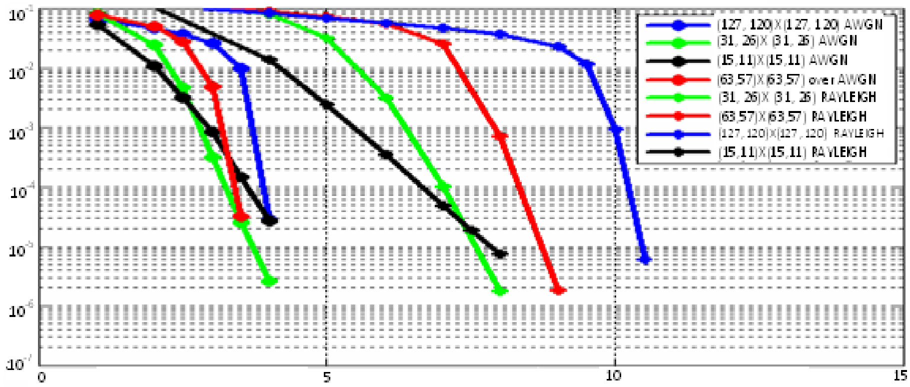


Fig. Performance of block turbo codes after 4 iterations through AWGN and Rayleigh fading channels using BPSK Modulation.

It has been shown that some of these high rate codes perform only at 1.2dB from the Shannon limit [15]. From above graph it was observe that decoding is easier with out errors overa an AWGN Channel compared to rayleigh fading channel. If we coampare the (31,26) and (63,57) will give the better performance compared survey of [15].

Block Turbo Code	SNR at 10^{-5} AWGN	SNR at 10^{-5} Rayleigh	Difference between channels(dB).
$(15,11,4)^2$	4.10	10.0	5.90
$(31,26,4)^2$	4.42	8.04	3.62
$(63,57,6)^2$	4.52	11.14	6.62
$(127,120,4)^2$	4.75	13.10	8.35

Table7. Performance of Block Turbo codes at fourth iteration over a Gaussian and a rayleigh Channels.

8. Conclusion:

In this paper, block codes are simulated with different lengths and different code rates for data, voice and image signals. From simulation results, it is reveald that, highest length of block codes has given the lowest noise effect than lower length of block codes, if input is a data is modulated with QPSK modulation. And if the input is voice signal then highest length of block codes has given lowest SNR for FSK and QAM. For voice signal, BPSK and QPSK has given lowest SNR for a particualr BER as shown in fig 12 and proposed method had given better results compared with [5] and [6]. The block turbo codes simulations have shown very good results better than those obtained so far with other algorithms[9][10]. We have obtained BER curves for block turbo codes with identical slopes for Gaussian and Rayleigh channels. For a BER of 10^{-5}, the coding gain obtained is upto 5.5 dB and the difference with Shannon's limit is about 3.5 dB after four iteration on a Gaussian Channel. From table5 and 6 we can conclude that higher singnal ot noise ratio gives lower symbol Error rate.

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