Convective Heat Transfer in Maxwell-Cattaneo Dielectric Fluids

S. Maruthamanikandan[1] and Smita S. Nagouda[2]
[1, 2]Department of Mathematics, Christ University, Bangalore-560 029, INDIA

Abstract:
The effect of second sound on the onset of Rayleigh-Bénard instability in a dielectric fluid subject to the simultaneous action of a vertical ac electric field and a vertical temperature gradient is investigated theoretically by means of the method of small perturbation. The horizontal layer of the fluid is cooled from the upper boundary, while an isothermal boundary condition is imposed at the lower boundary. The eigenvalues are obtained for free-free, isothermal boundary combinations and the influence of various parameters on the onset of electrothermal convection has been analyzed. Some of the known cases are derived as special cases. The Rayleigh-Bénard problem for a Maxwell-Cattaneo dielectric fluid is always less stable than that with classical heat conduction. It is shown that the destabilizing influence of the Cattaneo number is not attenuated by that of the dielectrophoretic force and vice versa, and that both second sound and electric forces change the aspect ratio of convection cells appreciably.

Keywords: AC electric field, Dielectric fluid, Electroconvection, Hyperbolic energy equation, Maxwell-Cattaneo heat flux, Roberts number, Stationary instability.

1. Introduction

The problems of natural convection in dielectric fluids have received widespread attention because they are representative of a variety of non-isothermal situations. The dielectric liquids are characterized by very slight electrical conductivity. Transformer oil (and most other organic substances) and distilled water are examples of such fluids. Compared to magnetic liquids, these liquids are easier to prepare. Under the influence of an external electric field such a fluid exhibits a large polarization and as soon as the field is removed, the fluid attains zero polarization state at once. Dielectric liquids can be controlled by electric forces and the relevant details were addressed by Melcher [1]. The general topic of electrically enhanced heat transfer in fluids and possible practical applications has been reviewed by Jones [2] and Chen *et al*. [3]. Electrically induced convection in dielectric liquids has been the subject of investigation for many decades right from the experimental work of Gross and Porter [4]. They observed that the convection pattern established by the electric field is quite similar to the familiar Bénard cells in normal convection. Turnbull [5] examined the effect of dielectrophoretic forces on the Bénard instability. The principle of exchange of stabilities is shown to hold for a certain set of boundary conditions. Approximate solutions for the critical temperature gradient as a function of the wavelength and the electric field are found using the variational principles and the Galerkin method.

The effect of uniform rotation on the onset of convective instability in a dielectric fluid under the simultaneous action of a vertical ac electric field and a vertical temperature gradient was considered by Takashima [6]. It is shown that the principle of exchange of stabilities is valid for most dielectric fluids. It is shown that, even when the electrical effects are taken into account, the coriolis force has an inhibiting effect on the onset of instability and as the speed of rotation increases the coupling between the two agencies causing instability (electrical and buoyancy force) becomes tighter. Stiles [7] investigated the problem of an electrically insulating liquid layer confined between horizontal conducting electrodes, the upper of which is warmer. It is found that the system becomes unstable with respect to the onset of steady convection when the electric field strength reaches a critical value, which in a rapidly varying ac field is due to the polarization body force. Maekawa *et al*. [8] considered the convective instability problem in ac and dc electric fields. Linearized perturbation equations are solved analytically.

Stiles *et al*. [9] studied the problem of convective heat transfer through polarized dielectric liquids. It is shown that for a critical voltage, as the gravitational Rayleigh number becomes increasingly negative, the critical wavenumber at the onset of convection becomes very large. As the temperature drop between the plates increases the fraction of the heat transfer associated with convection is found to pass through a maximum value when the critical horizontal wavenumber is close to four times its value when gravity is absent. Smorodin [10] analyzed the effect of an alternating arbitrary-frequency electric field on the stability of convective flow of a dielectric liquid occupying a vertical layer in the *EHD* approximation. The stability thresholds are determined in the linear approximation using Floquet theory. Maruthamanikandan [11] investigated the problem of gravitational instability in a dielectric liquid in the presence of internal heat generation, surface tension, radiation and viscoelasticity. Mikheev *et al.* [12] experimentally verified that dielectric liquids can be purified by means of turbulent electroconvection under the action of pondermotive forces arising in an inhomogenous alternating electric field. Radhakrishna and Siddheshwar [13] investigated linear and a weakly nonlinear stability analysis of thermal convection in a dielectric liquid permeated by a vertical, uniform ac electric field using the normal mode method and truncated representation of Fourier series respectively. It is found that the effect of increasing the electric number is to enhance the amplitudes and thereby the heat transport.

The propagation of thermal waves is sometimes referred to as second sound effect. The classical energy equation allows for an infinite speed for the propagation of heat, which is physically unacceptable. The energy equation to be considered in the present work is effectively a damped wave equation and is therefore hyperbolic rather than parabolic. The knowledge of second sound has provided a rich source of information for the study and understanding of the superfluid state. Second sound is not in any sense a sound wave, but a temperature or entropy wave. In ordinary or first sound, pressure and density variations propagate with very small accompanying variations in temperature; in second sound, temperature variations propagate with no appreciable variations in density or pressure. Recently, it has been realized that this is not just a low temperature phenomenon, but has important applications in such fields as skin burns, phase changes, biological materials, and in nanofluids (Straughan [14]).

Straughan and Franchi [15] investigated the effect of thermal waves (second sound) upon the onset of convective instability of a Newtonian fluid confined between a horizontal layer of finite thickness. They obtained critical Rayleigh number for the onset of convection when the Maxwell-Cattaneo heat flux law is employed, which allows for thermal waves of finite speed. Stress-free boundaries have been considered. It is found that convection is possible in both heated above and below cases and that the Bénard problem for a Maxwell-Cattaneo fluid is always less stable than the classical one and overstability only occurs in the heated below case. Lebon and Cloot [16] examined the effects resulting from the substitution of the classical Fourier law of heat conduction by the Maxwell-Cattaneo law in Bénard's and Marangoni's problems. Considering only infinitesimally small perturbations, it is shown that when buoyancy is the single factor of instability, no stationary convection can develop in a fluid layer heated from above, but oscillatory convection is possible. It is found that, in Maxwell-Cattaneo fluid, oscillatory convection does not play an important practical role.

Straughan [14] investigated the problem of thermal convection for a layer of fluid when the heat flux law of Cattaneo is adopted. The boundary conditions are taken to be rigid-rigid and isothermal. It is shown that for small Cattaneo number, the critical Rayleigh number initially increases from its classical value until a critical value of the Cattaneo number is reached. For Cattaneo numbers greater than this critical value a notable Hopf-bifurcation is observed with convection occurring at lower Rayleigh numbers and by oscillatory rather than stationary convection. It is also found that the aspect ratio of the convection cells likewise changes. Smita and Pranesh [17] studied the problem of the onset of Rayleigh-Bénard convection in a second order Colemann-Noll fluid by replacing the classical Fourier heat flux law with non-classical Maxwell-Cattaneo law. The eigenvalue problem is solved using the general boundary conditions on velocity and third type of boundary conditions on temperature. It is found that the classical Fourier heat flux law overestimates the critical Rayleigh number compared to that predicted by the non-classical law and that the results are noteworthy at short times.

While the effect of a variety of non-uniform basic temperature gradients on the onset of electroconvection has been studied intensely, this paper is devoted to studying qualitatively the effect of propagation of thermal waves upon the onset of electroconvection in a horizontal layer of dielectric fluid. The linear stability analysis is based on the normal mode technique and we allow for a thermal wave of finite speed by adopting the heat flux model of Cattaneo.

2. Mathematical Formulation

We consider a Boussinesq dielectric fluid layer of thickness 'd' with a uniform vertical ac electric field applied across the layer. The lower and upper boundaries of the fluid layer are maintained at uniform, but different temperatures T_0 and T_1 (with $T_0 > T_1$) respectively. A Cartesian coordinate system (x, y, z) is chosen with the origin at the bottom of the fluid layer and the *z*-axis normal to the fluid layer.

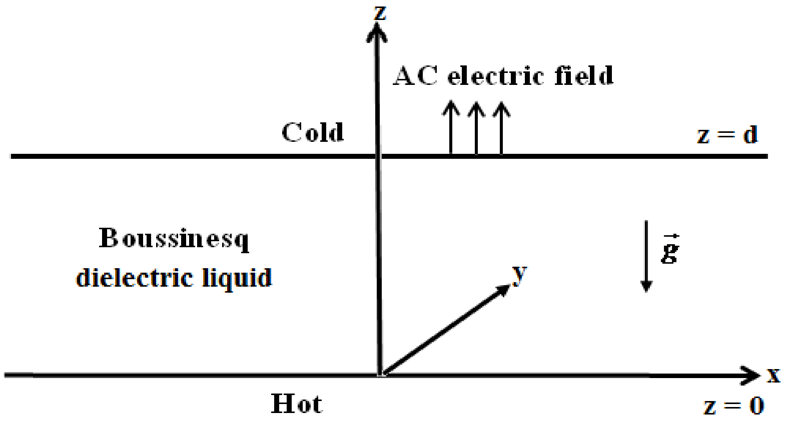

Fig. 1 Configuration of the problem.

The relevant governing equations for an incompressible dielectric fluid under the Boussinesq approximation are

$$\nabla.\vec{q}=0 \tag{1}$$

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right]=-\nabla p+\rho\vec{g}+\mu\nabla^2\vec{q}+\left(\vec{P}.\nabla\right)\vec{E} \tag{2}$$

$$\frac{\partial T}{\partial t}+(\nabla.\vec{q})T=-\nabla.\vec{Q} \tag{3}$$

$$\tau\left[\frac{\partial\vec{Q}}{\partial t}+(\nabla.\vec{q})\vec{Q}+\vec{\omega}\times\vec{Q}\right]=-\vec{Q}-\kappa\nabla T \tag{4}$$

$$\rho=\rho_0\left[1-\alpha(T-T_0)\right] \tag{5}$$

where $\vec{q}$ is the velocity vector, $\vec{P}$ the dielectric polarization, $\vec{E}$ the electric field , T the temperature, p the pressure, ρ the fluid density, κ the thermal diffusivity, μ the fluid viscosity, $\vec{g}$ the acceleration due to gravity, α the coefficient of thermal expansion, ρ_0 the density at a reference temperature $T=T_0$, $\vec{Q}$ the heat flux vector, τ the constant relaxation time and $\vec{\omega}=\frac{1}{2}(\nabla\times\vec{q})$. The electric properties involved in the Kelvin body force are the polarization $\vec{P}$ and the electric field gradient $\nabla\vec{E}$. In a dielectric liquid the polarization vector $\vec{P}$ measures the electric dipole moment per unit volume of fluid due to partial alignment of intrinsic molecular dipoles induced by the applied electric field $\vec{E}$. It should be noted that if the frequency of the electric field becomes too high there can be appreciable heating associated with dielectric loss. Fortunately, this form of dielectric heating is negligible for the frequencies discussed in this paper.

The relevant Maxwell equations are

$$\vec{P}=\varepsilon_0\left[\varepsilon_r-1\right]\vec{E} \tag{6}$$

$$\nabla.\left(\varepsilon_0\varepsilon_r\vec{E}\right)=0 \tag{7}$$

$$\nabla\times\vec{E}=0 \text{ or } \vec{E}=\nabla\phi \tag{8}$$

where ε_0 is the electric permittivity, ε_r the relative permittivity or dielectric constant and ϕ the electric potential. The dielectric constant is assumed to be a linear function of temperature according to

$$\varepsilon_r=\varepsilon_r^0-e(T-T_0) \tag{9}$$

where $e>0$ is the dielectric permittivity and $\varepsilon_r^0=1+\chi_e$ with χ_e being the electric susceptibility.

2.1 Basic state

The basic state is quiescent and is described by

$$\vec{q} = \vec{q}_b = (0,0,0),\ \ p = p_b(z),\ \ \rho = \rho_b(z),\ \ T = T_b(z),\ \ \vec{E} = \vec{E}_b = (0,0,E_b(z)),$$
$$\vec{P} = \vec{P}_b = (0,0,P_b(z)),\ \ \varepsilon_r = \varepsilon_{rb}(z),\ \ \phi = \phi_b(z),\ \ \vec{Q} = \vec{Q}_b = (0,0,\kappa\beta),\ \ \beta = \frac{T_0 - T_1}{d} \qquad (10)$$

where the subscript b denotes the basic state. The quiescent basic state has a solution given by

$$T_b = T_0 - \beta z \qquad (11)$$

$$\rho_b = \rho_o\left[1 + \alpha\beta z\right] \qquad (12)$$

$$\varepsilon_{rb} = (1+\chi_e)\left[1 + \frac{e\beta z}{1+\chi_e}\right] \qquad (13)$$

$$E_b = \frac{E_0(1+\chi_e)}{1+\chi_e + e\beta z} \qquad (14)$$

$$\vec{P}_b = \varepsilon_0 E_0\left[(1+\chi_e) - \frac{1}{\left(1+\dfrac{e\beta z}{1+\chi_e}\right)}\right]\hat{k} \qquad (15)$$

$$\phi_b = \frac{(1+\chi_e)E_0}{e\beta}\log\left[1 + \frac{e\beta z}{1+\chi_e}\right] \qquad (16)$$

where E_0 is the value of the electric field at $z = 0$.

2.2. Perturbed State

Since we are interested in the stability of the basic state, we superimpose infinitesimally small perturbations on the basic state according to

$$\vec{q} = \vec{q}_b + \vec{q}\,' = (u', v', w'),\ p = p_b + p',\ \rho = \rho_b + \rho',\ T = T_b + T',$$
$$\vec{E} = \vec{E}_b + \vec{E}\,',\ \vec{P} = \vec{P}_b + \vec{P}\,',\ \varepsilon_r = \varepsilon_{rb} + \varepsilon'_r,\ \phi = \phi_b + \phi',\ \vec{Q} = \vec{Q}_b + \vec{Q}\,' \qquad (17)$$

where primes denote perturbed quantities. Following the classical procedure of linear stability analysis, the linearized equations governing small perturbations turn out to be

$$\rho_0 \frac{\partial}{\partial t}\left(\nabla^2 w'\right) = \mu\left(\nabla^4 w'\right) + \alpha\rho_0 g\nabla_1^2 T' + \frac{\varepsilon_0 e^2 E_0^2 \beta}{1+\chi_e}\nabla_1^2 T' - \varepsilon_0 e E_0 \beta \frac{\partial}{\partial z}\left(\nabla_1^2 \phi'\right) \qquad (18)$$

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\left(\frac{\partial T'}{\partial t} - \beta w'\right) = -\kappa\nabla^2 T' - \frac{\tau\kappa\beta}{2}\nabla^2 w' \qquad (19)$$

$$(1+\chi_e)\nabla^2\phi' - eE_0\frac{\partial T'}{\partial z} = 0 \qquad (20)$$

where $\nabla_1^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$. Non-dimensionalzing (18), (19) and (20) using the length, time, velocity, temperature and electric potential scales $d,\ \dfrac{d^2}{\kappa},\ \dfrac{\kappa}{d},\ \beta d$ and $\dfrac{eE_0\beta d^2}{1+\chi_e}$, we obtain (after neglecting the primes for simplicity)

$$\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2 w\right) = \nabla^4 w + (R+L)\nabla_1^2 T - L\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right) \qquad (21)$$

$$\left(1 + 2C\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t} - w\right) = \nabla^2 T - C\nabla^2 w \qquad (22)$$

$$\nabla^2\phi - \frac{\partial T}{\partial z} = 0 \qquad (23)$$

where $Pr = \dfrac{\mu}{\rho_o \kappa}$ is the Prandtl number, $R = \dfrac{\alpha \rho_0 g \beta d^4}{\mu \kappa}$ is the thermal Rayleigh number, $L = \dfrac{\varepsilon_o \left(eE_o \beta d^2\right)^2}{\mu \kappa (1+\chi_e)}$ is the Roberts number and $C = \dfrac{\tau \kappa}{2d^2}$ is the Cattaneo number. The boundary conditions are taken to be free-free, isothermal and their importance will be made clear in Section 5. So the boundary conditions are

$$w = D^2 w = T = D\phi = 0 \quad \text{at} \quad z = 0, 1. \tag{24}$$

It should be mentioned that Takashima and Ghosh [18] used the boundary condition $D\phi = 0$ in order to obtain exact solution to the problem of electrohydrodynamic instability in a viscoelastic fluid with free-free, isothermal boundaries. The derivation of the general boundary conditions for the electric potential ϕ is given in the work of Maruthamanikandan [11].

3. Linear Stability Analysis

We use the normal mode solution for the dependent variables according to

$$\left[w, T, \phi\right] = \left[W(z), \Theta(z), \Phi(z)\right] \exp\left\{i(lx + my) + \sigma t\right\} \tag{25}$$

where l and m are the dimensionless wavenumbers in the x and y directions respectively and σ is the growth rate. Because the linear eigenvalue system (21) – (23) has constant coefficients, it has a general solution with an exponential dependence on z. Substituting (25) into (21), (22) and (23), noting that the principle of exchange of stabilities is valid [6, 16], we arrive at the following stability equations

$$\left(D^2 - a^2\right)^2 W - (R+L) a^2 \Theta + L a^2 D\Phi = 0 \tag{26}$$

$$\left(D^2 - a^2\right)\Theta = \left[C\left(D^2 - a^2\right) - 1\right] W \tag{27}$$

$$\left(D^2 - a^2\right)\Phi - D\Theta = 0 \tag{28}$$

where $D = \dfrac{d}{dz}$ and $a = \sqrt{l^2 + m^2}$ is the non-dimensional wavenumber of the convective disturbance. In view of (25), the boundary conditions (24) take the form

$$W = D^2 W = \Theta = D\Phi = 0 \quad \text{at} \quad z = 0, 1. \tag{29}$$

4. Exact Solution

Equations (26), (27) and (28) together with boundary conditions (29) constitute an eigenvalue problem with R being the eigenvalue. Let us assume the solution in the following form so that they satisfy the boundary conditions (29). The solution is therefore given by

$$W = A_1 \sin\pi z, \quad \Theta = A_2 \sin\pi z, \quad D\Phi = A_3 \sin\pi z, \tag{30}$$

where A_1, A_2 and A_3 are constants. The condition for the existence of a non-trivial eigenvalue leads to the following expression for R

$$R = \frac{(\pi^2 + a^2)^3}{a^2\left[1 + C(\pi^2 + a^2)\right]} - \frac{La^2}{\pi^2 + a^2}. \tag{31}$$

Before developing the consequences of (31), we mention a couple of limiting cases that can be derived from (31). In the limiting case of $L = 0$, one recovers the result of Lebon and Cloot [16] and the associated Rayleigh number R is given by

$$R = \frac{(\pi^2 + a^2)^3}{a^2\left[1 + C(\pi^2 + a^2)\right]}. \tag{32}$$

In this case R assumes its minimum value at the critical wavenumber $a_c^2 = \frac{\sqrt{1+C\pi^2\left(1+C\pi^2\right)}-1}{C}$. In the limiting case of $C=0$, one recovers the classical result established for Fourier dielectric liquid (Turnbull [5]) and the corresponding Rayleigh number R is given by

$$R = \frac{(\pi^2+a^2)^3}{a^2} - \frac{La^2}{\pi^2+a^2}. \tag{33}$$

Moreover, in the limiting case when $L=C=0$, we obtain

$$R = \frac{(\pi^2+a^2)^3}{a^2} \tag{34}$$

which is the well-known expression concerning the problem of Rayleigh-Bénard convection in a Newtonian Fourier fluid [19]. We find that R assumes its minimum value $R_c = 27\pi^4/4$ at $a_c = \pi/\sqrt{2}$.

5. Results and Discussion

The problem of convective instability induced by a coupling of thermal and dielectrophoretic effects in an initially quiescent polarised dielectric liquid is investigated analytically by the method of small perturbation. The non-classical Maxwell-Cattaneo heat flux law involves a wave type heat transport and does not suffer from the physically unacceptable drawback of infinite heat propagation speed. The eigenvalues are obtained for free-free, isothermal boundary conditions. It was corroborated, in Maxwell-Catteneo fluid convection and dielectric liquid convection, that oscillatory convection does not play an important practical role [8, 16]. Indeed, it is known from thermodynamics that the relaxation time and consequently the Cattaneo number C are positive quantities. It has been established that oscillatory convection occurs only for C above a threshold value and since the C values encountered with laboratory fluids appear well below this threshold, it is advantageous to concentrate on stationary convection.

It should be remarked that the use of realistic flow boundary conditions does not qualitatively, but quantitatively change the critical values (Chandrasekhar [19]). Similarly the use of realistic boundary conditions on the electric potential is of only very limited impact on the stability of the system [5]. It is well known that rigid-rigid boundaries offer most stabilizing effect against the fluid motion and the least suppression is offered by free-free boundaries [19]. For non-dissipative flows there is an alternative variational approach to stability which relies on determining whether or not the energy of the flow is a minimum at equilibrium (Straughan [20]). Neutral stability curves in the (R, a) plane are plotted for different values of the governing parameters. The coordinates of the lowest point on these curves designate the critical values of R and a.

Before we embark upon a discussion of the results obtained, it should be noted that the heat is transferred purely by conduction in the quiescent state and by both conduction and convection in the steady convective state. The thermal Rayleigh number R, characterising the stability of the system, is calculated as a function of the Roberts number L and the Cattaneo number C. The variation of thermal Rayleigh number R with the wavenumber a for different values of the Roberts number L and for the Cattaneo number $C=0$ is shown in Fig. 2. The Roberts number L is the measure of the ratio of electric to dissipative forces. Dissipative forces can be neglected when L is extremely large. It is observed that as L increases, R_c decreases monotonically. From Figs. 3 and 4, it is seen that this trend continues to be the same even for $C=0.01$ and $C=0.1$. This means that the dielectrophoretic force has a destabilizing influence on the system. So higher the electric field strength, the less stable the system due to an increase in the destabilizing electrostatic energy to the system. In other words, the presence of electric field facilitates heat transfer more effectively and hence hastens the onset of electroconvection at a lower value of the thermal Rayleigh number.

Moreover, as can be seen from Figs. 2 through 4, R_c decreases monotonically with an increase in the Cattaneo number C. The reason for the destabilizing effect of second sound is that the energy equation considered is effectively a damped wave equation and is, therefore, hyperbolic rather than parabolic. Noticeably, the band width of the neutral stability curves increases with an increase in C indicating that large values of C augment the destabilizing influence of the electric forces and vice versa. It is also obvious that maximum destabilization is achieved when both L and C are large. Consequently, the onset of electroconvective instability in a dielectric fluid layer is hastened by increasing the magnitudes of second sound and electric forces.

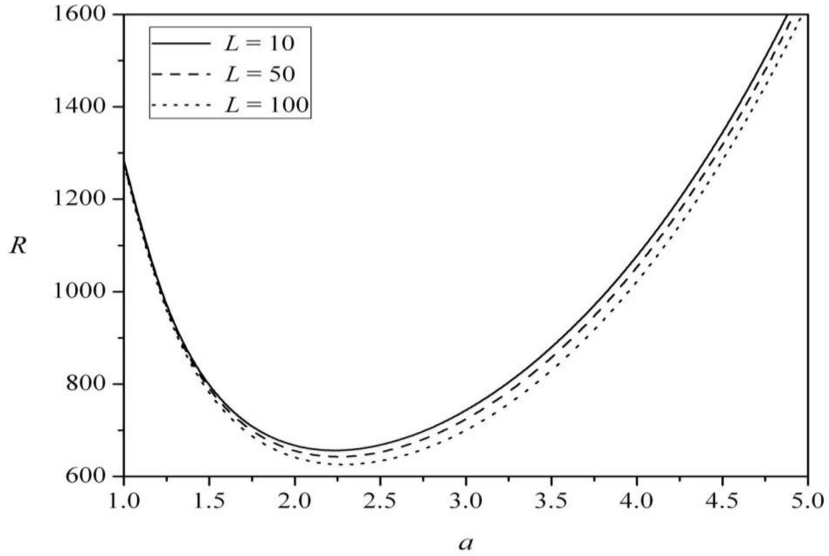

Fig. 2 Variation of thermal Rayleigh number R with a for different values of Roberts number L when $C = 0$.

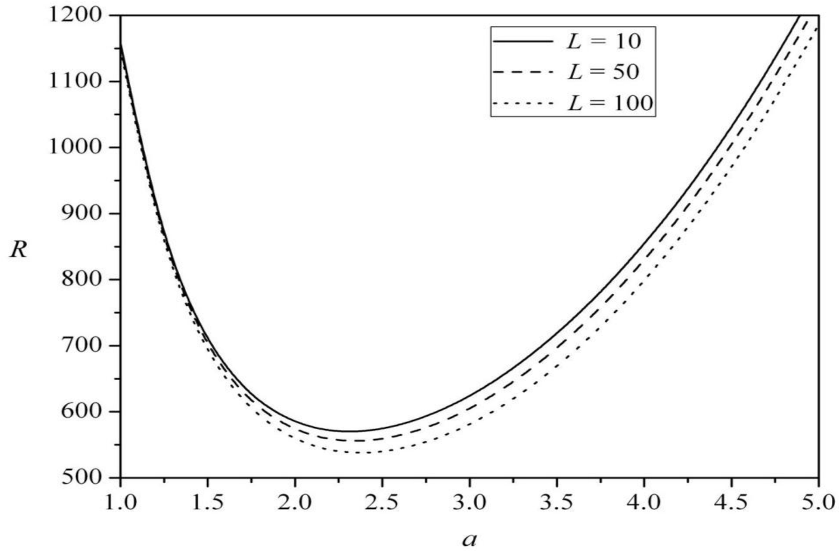

Fig. 3 Variation of thermal Rayleigh number R with a for different values of Roberts number L when $C = 0.01$.

On the other hand, the dimensionless number a is the characteristic of the convection cell shape and size. It is evident from Figs. 2 through 4 that the effect of increasing the values of both L and C is to increase a_c monotonically. This means that the convection cell size is contracted when both L and C are increased. It follows that the transition from equilibrium state to destabilization is accompanied by the shorter wavelength electroconvection on account of the presence of second sound and the dielectrophoretic force. We believe that the present study offers some insights into the heat transfer mechanism that can take place in devices wherein dielectric fluids play a vital role. Undoubtedly, the problem would be more challenging if the magnetic and couple-stress effects are taken into account (Saravanan [21, 22]).

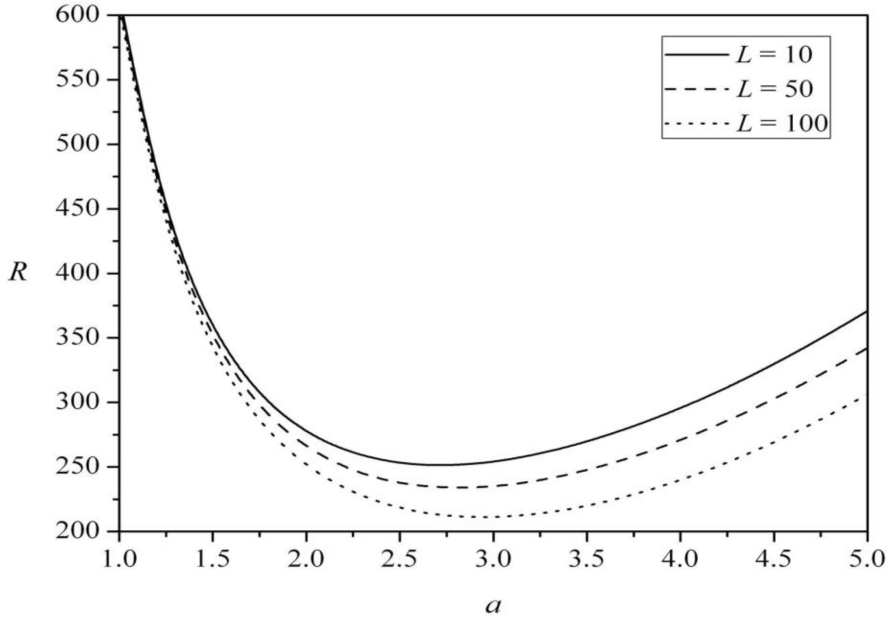

Fig. 4 Variation of thermal Rayleigh number R with a for different values of Roberts number L when $C = 0.1$.

6. Conclusions

The effect of non-classical heat conduction on the onset of Rayleigh-Bénard instability in a horizontal layer of a Cattaneo-dielectric fluid subject to the simultaneous action of a vertical ac electric field and a vertical temperature gradient is investigated analytically by the method of small perturbation. The instability criteria are determined in terms of the thermal Rayleigh number, wavenumber, the Cattaneo number and Roberts number. The following conclusions are drawn:

(i) The Rayleigh-Bénard problem for a Cattaneo-dielectric fluid layer is always less stable than that with the Fourier-dielectric fluid.
(ii) The system is considerably influenced by the effect of second sound in the presence of dielctrophoretic forces.
(iii) The effect of second sound reinforces the destabilising influence of dielectrophoretic forces and vice versa, and maximum destabilization is achieved for large values of the Roberts number and the Cattaneo number.
(iv) The threshold for the stationary instability decreases with increase in the electric forces and the Cattaneo number.
(v) Increase in the magnitude of electric forces and second sound causes the convective motion to occur at shorter wavelengths.

7. Acknowledgements

The authors would like to thank the management of Christ University, Bangalore, India for their support and encouragement. The authors are appreciative of the comments of the anonymous reviewers.

References

[1] J.R. Melcher. Continuum electromechanics. MIT Press, 1981.
[2] T.B. Jones. Electrodynamically enhanced heat transfer in liquids: A review. Adv. Heat Transfer, 14: 107-148, 1978.
[3] X. Chen, J. Cheng and X. Yin. Advances and applications of electrohydrodynamics. Chinese Science Bulletin, 48: 1055-1063, 2003.
[4] M.J. Gross and J.E. Porter. Electrically induced convection in dielectric liquids. Nature, 212: 1343-1345, 1966.
[5] R.J. Turnbull. Effect of dielectrophoretic forces on the Bénard instability. Phys. Fluids, 12: 1809-1815, 1969.
[6] M. Takashima. The effect of rotation on the electrohydrodynamic instability. Can. J. Phys., 54: 342-347, 1976.
[7] P.J. Stiles. Electrothermal convection in dielectric liquids. Chem. Phys. Lett., 179: 311-315, 1991.
[8] T. Maekawa, K. Abe and I. Tanasawa. Onset of natural convection under an electric field. Int. J. Heat Mass Trans., 35: 613-621, 1992.

[9] P.J. Stiles, F. Lin and P.J. Blennerhassett. Convective heat transfer through polarized dielectric liquids. Phys. Fluids, 5: 3273-3279, 1993.
[10] B.L. Smorodin. Stability of plane flow of a liquid dielectric in a transverse alternating electric field. Fluid Dynamics, 36: 548–555, 2001.
[11] S. Maruthamanikandan. Convective instabilities in ferromagnetic, dielectric and other complex liquids. Ph.D. Thesis, Bangalore University, India, 2005.
[12] G.M. Mikheev, Gr.M. Mikheev, V.A. Tarasov. and T.G. Mikheeva. Electroconvective purification of dielectric liquids. Tech. Phys. Lett., 34: 391-393, 2008.
[13] D. Radhakrishna. and P.G. Siddeshwar. Linear and nonlinear electroconvection under ac electric field. Commun. Nonliner. Sci. Numer. Simul., 17: 2883-2895, 2012.
[14] B. Straughan. Oscillatory convection and the Cattaneo law of heat conduction. Ricerche. Mat., 58: 157-162, 2009.
[15] B. Straughan and F. Franchi. Bénard convection and the Cattaneo law of heat conduction. Proc. Roy. Soc. Edinburgh, 96: 175-178, 1984.
[16] G. Lebon and A. Cloot. Bénard-Marangoni instability in Maxwell-Cattaneo fluid. Phys. Lett., 105A: 361-364, 1984.
[17] S.N. Smita and S. Pranesh. Rayleigh-Bénard convection in a second-order fluid with Maxwell-Cattaneo Law. Bull. Soc. Math. Services and Standards, 1: 33-48, 2012.
[18] M. Takashima and A.K. Ghosh. Electrohydrodynamic instability in a viscoelastic liquid layer. J. Phys. Soc. Japan, 47: 1717-1722, 1979.
[19] S. Chandrasekhar. Hydrodynamic and hydromagnetic stability. Oxford University Press, 1961.
[20] B. Straughan. The energy method, stability and nonlinear convection. Springer, 2004.
[21] S. Saravanan. Centrifugal acceleration induced convection in a magnetic fluid saturated anisotropic rotating porous medium. Trans. Porous Media, 77: 79-86, 2009.
[22] S. Saravanan and D. Premalatha. Effect of couple stress on the onset of thermovibrational convection in a porous medium. Int. J. Thermal Sci., 57: 71-77, 2012.