Five Dimensional String Cosmological Model In General Relativity

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Abstract:
In this paper we have investigated Five Dimensional String Cosmological Models with bulk viscosity and determined it's solution for three different cases. The various physical and kinematical properties are also studied.

Keywords: Bulk viscosity, Cosmic string, Five dimensional cosmological models.

1. Introduction

The study of cosmic strings attached considerable interest of cosmologists in the frame work of general relativity as it play an important role in the description of the universe in the early stages of its evolution (Kible 1976) and give rise to density perturbations leading to the formation of galaxies (Zel'dovich 1980). The existence of a large scale network of strings in the early universe is not a contradiction with the present day observations of the universe. Various authors (Hogan and Ress 1984; Myung et al.1990; Banerjee et al. 1990; Gundalach and Ortiz 1990; Barros and Romero 1995; Yavuz and Yilmaz 1996; Sen et al.1997;Sen 2000;Bhattachajee and Baruah 2001; Barros et al.2001;Rahaman et al.2003; Reddy 2005) constructed string cosmological models in various theories of gravitation. Seaz and Ballester (1986) formulated a scalar tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. Singh and Agrawal (1991), Shri Ram and Singh (1995) and Mohanty and Sahu (2003, 2004, 2005) studied various aspects of Seaz and Ballester scalar tensor theory of gravitation. In particular Reddy (2003, 2006) studied some string cosmological models in Seaz and Ballester scalar tensor theory of gravitation with the help of an excess ad hoc condition in five dimensional Kaluza-Klein space time, even though these solutions may not reveal the early universe.

2. The Field Equations

We consider the five dimensional line element in the form

(1) $ds^2 = -A^2(dX^2 + dY^2 + dZ^2) - B^2 dM^2 + dt^2,$

where A and B are functions of cosmic time t only.

The energy momentum tensor for a cloud of string dust with a bulk viscous fluid is given by Letelier (1979), Landau and Lifshitz (1963) as

(2) $T^j_{\ i} = \rho u_i u^j - \lambda\, x_i x^j - \xi u^k_{;k}(g^j_i + u_i u^j),$

where 'ρ' is the proper energy density for a cloud of strings with particles attached to them, 'λ' is the string tension density, 'ξ' is the bulk coefficient of viscosity, u^i is the five velocity of the cloud of particles and x^i is the direction of string i.e. direction of anisotropy satisfying the relation

(3) $u_i u^i = -1,\ x^i x_i = 1$ and $u^i x_i = 0$

The field equation in normal gauge for Lyra's manifold as obtained by Sen and Dunn are

(4) $R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij}\phi_k \phi^k = -\chi T_{ij}$

where ϕ_i is the displacement vector given by $\phi_i = (\beta(t),0,0,0,0)$.

The field equation (4) for the line element (1) leads to

(5) $\frac{-3A_4^2}{A^2} - \frac{3A_4B_4}{AB} + \frac{3\beta^2}{4} = \chi\rho,$

(6) $\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} + \frac{2A_4B_4}{AB} + \frac{B_{44}}{B} + \frac{3\beta^2}{4} = -\chi(\lambda + \xi\theta),$

(7) $\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} + \frac{2A_4B_4}{AB} + \frac{B_{44}}{B} + \frac{3\beta^2}{4} = -\chi\xi\theta$ and

(8) $\frac{3A_{44}}{A} + \frac{3A_4^2}{A^2} + \frac{3\beta^2}{4} = -\chi\xi\theta\,,$

where the suffix '4' denotes ordinary differentiation with respect to 't'.

3. Solutions of Field Equations and Physical Properties

Here we have four independent field equations in six unknowns. Hence in order to get determinate solution, we consider the relation.

(9) $B=g(A(t))$.

From equations (7) & (8), we have

(10) $\frac{A_{44}}{A}-\frac{B_{44}}{B}+\frac{2A_4^2}{A^2}-\frac{2A_4B_4}{AB}=0.$

Using (9), we have

(11) $A_{44}\left\{\frac{1}{A}-\frac{g'}{g}\right\}+A_4^2\left\{-\frac{g''}{g}-\frac{2g'}{Ag}+\frac{2}{A^2}\right\}=0$

Here prime indicates the differentiation with respect to the argument.

Equation (11) is satisfied for the following cases:

Case I: $A_4=0$

Case II: $A_{44}=0$ and $\frac{g''}{g}+\frac{2g'}{Ag}-\frac{2}{A^2}=0$

Case III: $\frac{g'}{g}-\frac{1}{A}=0$ and $\frac{g''}{g}+\frac{2g'}{Ag}-\frac{2}{A^2}=0$

Case I: $A_4=0$, leads to A = K where K ≠ 0 is a constant of integration.

With the help of (9), we have $B=g=K$

Hence five dimensional string models in this case, after using transformations, reduces to the form

(12) $dS^2=dT^2-(dX^2+dY^2+dZ^2)-dM^2.$

The energy density ρ, tensor density λ and the bulk viscosity ξ for the model (12) is given by

$\rho=\frac{3\beta^2}{4\chi},$

λ = 0,

$\xi=-\frac{3\beta^2}{4\chi\theta}.$

Since θ = 0, therefore we conclude that the Lyra geometry and cosmic strings do not survive in this case and space-time becomes Minkowaskian.

Case II: $A_{44}=0$ and $\frac{g''}{g}+\frac{2g'}{Ag}-\frac{2}{A^2}=0.$

Here $A_{44}=0$ leads to

(13) $A=L_1t+L_2,$

where $L_1\neq 0$and L_2 is a constants of integration.

Where as solving

$\frac{g''}{g}+\frac{2g'}{Ag}-\frac{2}{A^2}=0,$

After multiplying by A^2g and solving , we get

$g=C_1A+C_2A^{-2}.$

(14) $B=C_1A+C_2A^{-2}.$

After using some transformations the five dimensional string cosmological model in this case reduces to

(15) $dS^2=-T^2(dX^2+dY^2+dZ^2)-\left[C_1T+\frac{C_2}{L_1{}^3T^2}\right]^2 dM^2+dT^2.$

The energy density ρ, tensor density λ and bulk viscosity ξ for this model are given by

$\rho=\frac{3\beta^2}{4\chi}-\frac{3}{\chi T^2}\left[\frac{2C_1L_1^3T^3-C_2}{C_1L_1^3T^3+C_2}\right],\quad \lambda=0$

Bulk viscosity is given by the relation

$\xi\theta+\rho=0$

The scalar of expansion, spatial volume, deceleration parameter and shear for the model are given by

$\theta=-\left[\frac{3}{T}+\frac{C_1L_1^3T^3-2C_2}{T(C_1L_1^3T^3+C_2)}\right],$

$V=C_1L_1^4T^4+C_2L_1T,$

$q=\frac{3}{(4C_1L_1^3T^3+C_2)^2}[3C_1^2L_1^6T^6-2C_1C_2L_1^3T^3+C_2^2+C_1^3L_1^9T^6-2C_1^2C_2L_1^6T^3]-1\,,$

$\sigma=\frac{2}{3}\left[\frac{3}{T}+\frac{C_1L_1^3T^2}{C_1L_1^3T^3+C_2}-\frac{2C_2}{C_1L_1^3T^4+C_2T}\right]$

In this model string tensor density vanishes but energy density is present and it is only due to bulk viscosity. Lyra geometry survives only due to the presence of bulk viscosity. In the absence of bulk the model reduces to five dimensional vacuum model in Einstein's theory. The model starts to expand with a big-bang and stops at $T = \infty$. Volume increases with time. As $T \to \infty$, the string energy density $\rho \to \frac{3\beta^2}{4\chi}$. Moreover string tension density $\lambda = 0$, therefore we can conclude that energy density is only due to the Lyra geometry. Shear decreases as time increases. Since, $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, therefore the model is anisotropic for large values of T.

$$\Rightarrow \frac{g^{'}}{g} = -\frac{1}{A}$$

$$\therefore \frac{g^{''}}{g} + \frac{2}{A^2} - \frac{2}{A^2} = 0$$

$g = zA + z_1$,

(16) $\therefore B = zA + z_1$,

where $z \neq 0$, z_1 is a constant of integration.

In particular if we take $z_1 = 0$,then for this case, we arrive at the following mathematical models.

If $A = \sin(at + b)$, where $a \neq 0, b$ are constants, then after suitable transformations we have the metric in the form

(17) $dS^2 = -\sin^2(aT)\{dX^2 + dY^2 + dZ^2 - dM^2\} + dT^2$,

For $T = \left(\frac{2n+1}{a}\right)\frac{\pi}{2}$, for $n = 0, 1, 2, ...$ reduces to flat 5-dimension space-time.

If $A = e^{(at+b)}$, where $a \neq 0, b$ are constants, then after some Transformations metric reduces to the form

(18) $dS^2 = -e^{2aT}\{dX^2 + dY^2 + dZ^2 - dM^2\} + dT^2$,

Here for $a < 0$, as T increases, model contracts in four directions. For $a > 0$, as T increases, model expands in four directions. The model becomes flat at initial epoch. The model follows exponential expansion.

If we take $A = (at + b)^m$, where $m \neq 0, a \neq 0, z_1 = 0$ are constants. Then model reduces to the form

(19) $dS^2 = -T^{2m}\{dX^2 + dY^2 + dZ^2 - dM^2\} + dT^2$.

Here model follows power law inflation.

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