Computation of Least Cost Pipe Network –An Alternate Method

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Abstract

The paper is based on the optimal design of pipe networks for the water distribution. The treated water has to be supplied to the consumers in their individual homes. This function of carrying water is accomplished through well planned distribution system with optimal design of pipes as it comprises the major investment in the system. So for economy and cutting down huge expenditures, design of water distribution networks has to be such that the cost incurred is minimal and simultaneously it meets the demands and discharges at various outlets of the network. The problem in this paper has thus been solved with a view to reduce the cost of pipe networking with the required amount discharge in the outlet, Hardy Cross Method has been used for estimating the required discharge in each outlet of the pipe network, and optimization of the system has been done to reduce the cost with the help of Microsoft-excel. The proposed optimization setup has been very close to the original value, thereby validating its use for optimization.

1. Introduction

A water distribution network is a system by which the water treated in a treatment plant is distributed to the consumers in their individual homes. Therefore the water to be supplied in the houses is carried through a system, this system for distributing water contains pipes, reservoirs, pumps, valves of different types, which are connected to each other to provide water to consumers or when expanding the existing system to larger population. It is a vital component of the urban infrastructure and requires significant investment. The process of distributing water generally consists of different phases like designing of layout for distributing system, designing of pipe network and process of operation. The problem of optimal design of water distribution networks has various aspects to be considered such as hydraulics, reliability, material availability, water quality, and infrastructure and demand patterns.The objective here is to determine the optimal diameters of pipes in a network with a predetermined layout. This includes providing the pressure and quantity of the water required at every demand node. The problem of optimizing network requires the determination of pipe sizes from a set of commercially available diameters ensuring a feasible least cost solution. Here we have considered a two – loop network supplied by gravity, with the objective of determining the minimum cost for a given layout. Here we will be dealing with the determination of the optimal diameters of pipes in a network with a predetermined layout. The cost of realizing the network is a function of the diameters. The smaller the diameter, the lower is the price. However the energy head at the consumers also decrease, therefore the problem is to minimize the cost under the constraint that the energy heads at the interior nodes are above some given lower limits.The loss of head and the discharge in every loop forming the pipe network system is determined through Hardy Cross Method where trial distribution of discharge at each node is done in such a way that continuity equation is satisfied. The algebraic sum of the pressure drops around a closed loop must be zero, i.e. there can be no discontinuity in pressure. This secures the overall mass balance in the network. For n nodes in the network, this can be written as

$$\sum_{i=0}^{n} Q_i = 0 \qquad 1$$

Where Q_i represents the discharges into or out of the node i. The desired discharge value for the predetermined loop of the network system is then being optimized, considering the diameters of the pipes in the network as decision variables, the problems can be considered as a parameter optimization problem with dimension equal to the number of pipes in the network. Market constraints, however, dictate the use of commercially available pipe diameters. With this constraint the problem can be formulated in Microsoft Excel, where the optimization is done by Newton – Raphson Method.

2. Literature Review

As the pipe networking works involve a huge amount of money, so there have been many endeavors to optimize the pipe networks so that the cost gets lowered. Various methods of optimization have been developed, implemented and validated on many different pipe networks by many researchers so far. Most of the works of optimization have been applied on some standard water distribution networks like the Two-loop water distribution network (first presented by Alperovits and Shamir in 1977 consisting of 7 nodes, 8 pipes and two loops, fed by gravity from a reservoir with a 210m fixed head. As the scope of this work deals with the optimization of two loop pipe network presented by Alperovits and Shamir, we will be discussing the various works carried out earlier on optimization of two-loop network.

The two-loop network, shown in figure 1, was originally presented by Alperovits and Shamir (1977), followed by Goulter *et al*. (1986), Kessler and Shamir (1989), Savic and Walters (1997), and Cunha and Sousa (1999). The network has seven nodes and eight pipes with two loops, and is fed by gravity from a reservoir with a 210-m (=689 ft.) fixed head. The pipes are all 1000m (=3281 ft.) long with a Hazen-Williams coefficient *C* of 130. The minimum head limitation is 30m (=98.4 ft.) above ground level.

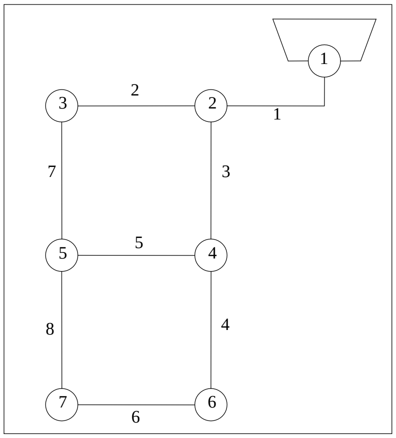

Figure 1. A Two loop network

Alperovits & Shamir (1977) methods for optimal design of looped systems which was divided into two methods. In first method they used An optimization solver at each iteration of the optimization. The solver first solves for the head loss and calculate discharges in the pipe network, then uses the solutions in some procedure to modify the design. The second method does not use a conventional network solver. The minimum cost of the proposed two loop pipe network was obtained as $ 497,525 by Alpoerovits and Shamir. Goulter et al. (1986) further modified the work following LP method and the result obtained was $435,015. Kessler and Shamir also modified the work and obtained the results as $ 417,500. Both Goulter et al. and Kessler and Shamir used the LP approach and modified the work by optimizing the network by changing the pipe diameters. Cunha and Sousa (1999) used the Simulated annealing (SA) method and found the cost $ 419,000. Savic and Walters (1997) took Genetic algorithm (GA) for optimization and obtained results of $ 419,000 each. A genetic algorithm is a member of class of search algorithms based on artificial evolution.

3. Objective Of The Present Study

Objective of the present study is to provide a solution for optimization using Microsoft Excel solver tool. The objective here is to determine the optimal diameters of pipes in a network with a predetermined layout. This includes providing the water required at every demand node satisfying the minimum required conditions of pressure and quantity (discharge). The objective here, requires the determination of pipe sizes from a set of commercially available diameters ensuring a feasible least cost solution and that too without the involvement of such technical complexities which are present in most of the complex programs, algorithms and search mechanisms employed for optimization so far.

3.1 Methodology

In the present study, the two-loop network where flow occurs due to gravity is taken into account. It was first formulated using Linear Programming Gradient method by E. Alperovits and U. Shamir (1977).The aim of the water distribution network analysis is to find least cost pipe network by optimizing pipe diameters in such a way that the analysis fulfills water demand and required pressure head in every node. To find out the optimal values, two modules, namely hydraulic module and an optimization module are brought into consideration. Both the process has been compiled using a solver application of excel spreadsheet.

3.2 Model Formulation

The model which has been formulated to accomplish the required task is done by formulating a hydraulic module which deals with the hydraulic aspects and the optimization module which deals with the optimization aspect and then compiling both the processes using a solver application in Microsoft excel spreadsheet. Flow chart of the model is shown in figure 2.

3.3 Hydraulic module:

Analysis of pipe network

For the analysis of pipe network, the following two necessary conditions must be satisfied.

1. The algebraic sum of the pressure drops around a closed loop must be zero,i.e. there can be no discontinuity in pressure.
2. The flow entering a junction must be equal to the flow leaving the same junction; i.e. the law of continuity must be satisfied. Based upon these two basic principles, the pipe networks are generally solved by the method of successive approximation because any direct analytical solution is not possible. The analysis of a pipe network requires many equations, most of which being nonlinear, to be solved simultaneously. The important methods used for solving such problems are briefly discussed in the next section 4.1.3.

4. Hardy-Cross Method

The procedure suggested by Hardy and Cross (Garg S.K, 1977) requires that the flow in each pipe be assumed by the designer (in magnitude as well as direction) in such a way that the principle of continuity is satisfied at each junction (i.e the inflow at any junction becomes equal to the outflow at that junction).Correction to these assumed flows is then computed successively for each pipe loop in the network, until the correction is reduced to an acceptable magnitude.

If Q_a is the assumed flow and Q is the actual flow in the pipe, then the correction ΔQ is given by

$$\Delta Q = Q - Q_a \qquad 2$$

$$\Rightarrow Q = Q_a + \Delta Q \qquad 3$$

Now expressing the head loss (H_L) as

$$H_L = K \cdot Q^x \qquad 4$$

where, $K = \frac{L}{470 \cdot d^{4.87}}$ (for Hazem-William formula)

L = length of pipe between two node.

x = a constant (1.852, for Hazen Williams formula ; 2, for Mannings or Darcy Weisbach formula)

the head loss in a pipe can be calculated as

$$H_L = K \cdot (Q_a + \Delta Q)^x$$

$$H_L = K \cdot [Q_a^x + x \cdot Q_a^{x-1} \cdot \Delta Q + \cdots\cdots + \text{negligible terms of higher power}$$

$$H_L = K \cdot \left[Q_a^x + x \cdot Q_a^{x-1} \cdot \Delta Q\right] \qquad 5$$

Now around a closed loop, the summation of head loss must be zero.

i.e $\sum K \cdot \left[Q_a^x + x \cdot Q_a^{x-1} \cdot \Delta Q\right] = 0$

Since ΔQ is same for the all the pipes of the considered loop, it can be taken out of the summation.

Therefore,

$$\Delta Q = -\frac{\sum K \cdot Q_a^x}{\sum x \cdot K \cdot Q_a^{x-1}} \qquad 6$$

Since ΔQ is given the same sign (or direction) in all pipes of the loop, the denominator of the above equation is taken as the absolute sum of the individual items in the summation. Hence

$$\Delta Q = -\frac{\sum K \cdot Q_a^x}{\sum \left|x \cdot K \cdot Q_a^{x-1}\right|} \qquad 7$$

$$\Delta Q = -\frac{-\sum H_L}{x \sum \left|\frac{H_L}{Q_a}\right|} \qquad 8$$

Where H_L = head loss for the assumed flow Q_a

The numerator of the above equation is the algebraic sum of the head losses in the various pipes of the closed loop computed with the assumed flow. Since the direction and magnitude of flow in these pipes is already assumed, their respective head losses with due regard to sign (The head loss in clockwise direction may be taken as +ve and that in the anti-clockwise direction as -ve) can be easily calculated after assuming their diameters. The absolute sum of respective $K \cdot Q_a^{x-1}$

or $\frac{H_L}{Q_a}$ is then calculated. Finally the value of ΔQ is found out for each loop, and the assumed flows in each pipe are corrected by using equation (8). Pipes common to two loops will receive both corrections with due attention to sign.
After correcting the flows in the entire pipe network in the first iteration, the second correction can be applied to the already corrected flows in the previous step, and the re-corrected flows are again worked out in the entire network (consisting of one or more loops). The flows in pipes, common to two loops, should be corrected for the computed corrections of both the loops, as stated earlier. The procedure can be repeated to obtain more accurate results.The hydraulic module selects the optimal pipe sizes in the final network satisfying all constraints such as conservations of mass and energy and on the other hand pressure head and design constraints. The hydraulic constraints, for example, deal with hydraulic head at certain nodes to meet a specified minimum value. However, diameter constraints enforce the algorithms to select the trial solution within a predefined limit. A hydraulic network solver handles the implicit constraints and simultaneously evaluates the hydraulic performance of each trial solution that is a member of population of points. The hydraulic model first checks the head across each node of whether it satisfies the minimum pressure head conditions and then keeps on iterating until the minimum pressure head condition is satisfied by changing the diameter of each pipe within a given diameter range. Optimization module selects best fitted diameters from a set of diameters and minimizes the total cost of the pipe network.

4.1 Optimization Module

The optimization model involves the use of an excel solver which estimates the cost of the network and settles with the least cost satisfying all the constraints. The network cost is calculated as the sum of the pipe costs where pipe costs are expressed in terms of cost per unit length. Total network cost is computed as follows:

$$C = \sum c_k(D_k) \cdot L_k \qquad 9$$

where, $c_k(D_k)$ = cost per unit length of the k^{th} pipe with diameter D_k,

L_k = length of the k^{th} pipe.

The optimization module keeps on checking the combination of pipe diameters satisfying the head conditions and resulting in the least cost of the network.

While using solver for the optimization the following parameters was kept as constraints:

(i) Pressure head across each node must be at least 30m.
(ii) The diameter of the any of the pipe must be within the range of 0.025m-0.508m.

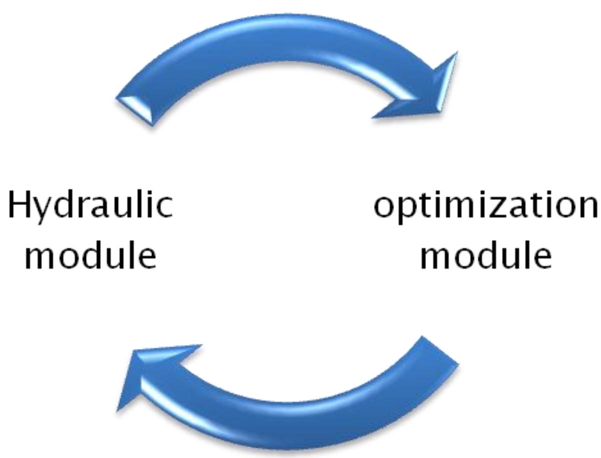

Figure 2 Flow chart of the model

5 Model Application

The application part involves the application of the two modules viz. the hydraulic module and the optimization module compiled using a solver application of excel spreadsheet onto the Two-loop pipe network proposed by Alperovits and Shamir. For finding the cost incurred, data provided in table 1 have been taken for pipe diameters available in the market and their unit costs per meter length.

Table 1 Pipe diameters and their corresponding costs

Diameter (inches)	Diameter (m)	Unit cost (per meter length)
1	0.0254	2
2	0.0508	5
3	0.0762	8
4	0.1016	11
6	0.1524	16
8	0.2032	23
10	0.254	32
12	0.3048	50
14	0.3556	60
16	0.4064	90
18	0.4572	130
20	0.508	170

Then the optimization for least cost was carried out based on the above data in Microsoft Excel and the sizes of the respective pipes and the total cost incurred was determined which was compared with some earlier works and the results are shown in table 2.

Table 2 Comparison of pipe diameters and total cost for two loop network

Pipe number	Alperovits and Shamir	Goulter Et al.	Kessler and Shamir	Present study
1	20 - 18	20 – 18	18	20
2	8 - 6	10	12 - 10	12
3	18	16	16	14
4	8 - 6	6 – 4	3 - 2	10
5	16	16 – 14	16 - 14	12
6	12 - 10	12 – 10	12 - 10	10
7	6	10 – 8	10 - 8	12
8	6 - 4	2 – 1	3 - 2	12
Cost($)	497,525	435,015	417,500	440,000

6. Conclusion & Discussion

Any water distribution system consists of basic three components pumps, storage tanks and distributing pipe networking. So, the process of optimization helps in reducing the cost of pipe networks by selecting and recognizing to adopt the best possible diameter to guarantee the best flow rate. The design for optimal distribution of the network is a complex task,various search methods, complex programs and algorithms have been proposed and attempted for the main concern of designing the most least cost network simultaneously satisfying the required minimum pressure head and discharge at the demand nodes. However, Microsoft Excel was used here for optimization to achieve the minimum cost but at the same time it also holds some drawbacks as it does not involve complex mechanisms for optimization as in the case of many algorithms which employ complex mechanisms for search of global optimal solution. As these methods involve complex algorithms, programs and function which require a lot of technical know-how's it becomes difficult to implement such mechanisms for optimization by everyone in many cases. So our approach was to provide with an easy method for optimization which doesn't involve such complexities. As is clear from the results embodied in this report, the cost incurred was lesser than that of Alperovits and Shamir; this is a good option for optimization if one doesn't want to go into such complexities. The total cost however could have been a bit lower as well and well near about the range of some other works on the same network. But the reasons for such fluctuations might be because of the following reasons:

1. As we have used Darcy Weisbach's formula for determination of head loss and the value of n is assumed to be 2 for turbulent flows, whereas in many cases Hazen-William Equation has also been used and the value of n is taken near about 1.85, so this might be a cause for fluctuations in the total cost.
2. Further another reason might be that we have assumed $\Delta Q \rightarrow 0$ instead of $\Delta Q = 0$.

6.1 Future Work

There have been a lot of works on the optimization of pipe networks and the results have improved over the time. There are still a lot of works which can be done on optimization of pipe networks, various algorithms can further be improved to further improve the effectiveness of optimization, make them simpler, less tedious and user friendly. Further diversification of this work can be done by devising on developing an application which optimizes the cost of pipe networks and provide a solution for least cost pipe network for an area by considering different locations of reservoir and then calculating the cost on the basis of length and diameters of the pipes required for satisfying the requirements of pressure and discharge at every node for every possible alternative and then selecting the alternative which requires the least cost.

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