Cascade Reliability for Generalized Exponential Distribution

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Abstract

Cascade reliability model is a special type of Stress- Strength model. The n- Cascade system is a hierarchical standby redundancy system, where the standby component taking the place of failed component with decreased value of stress and independently distributed strength. This paper deals with the generalized exponential distribution with cascade system.

Key Words: Stress – Strength model, Cascade system, generalized exponential distribution.

1. Introduction

The two- parameter generalized exponential (GE) distribution has been proposed by the Debasis Kundu and Rameshwar D.Gupta[1]. It has been extensively studied by Gupta and Kundu [2] and Kundu, Gupta and [3]. Note that the generalized exponential distribution is a sub- model of the exponential Weibull distribution introduced by Mudholkar and Srivastava [4] and later studied by Mudholkar, Srivastava and Freimer [5].An n - cascade system is defined as a special type of standby system with n components by Sriwastav et al .,[6]. Cascade redundancy is defined as a hierarchical standby redundancy where a standby component takes the place of a failed component with a changed stress. This changed stress is k times the preceding stress. k is the attenuation factor. Sriwastav and Pandit[6] derived the expressions for reliability of an n-cascade system when stress and strength follow exponential distribution. They computed reliability values for a 2-cascade system with gamma and normal stress and strength distributions. Raghava Char et al [7] studied the reliability of a cascade system with normal stress and strength distribution. T.S.Uma Maheswari et al [9] studied the reliability comparison of n-cascade system with addition of n-strengths system when stress and strength follow exponential distribution.

2. Statistical Model

If the r.v .X denotes the strength and the r.v. Y denotes the stress of the component, then the reliability of the component is given by

$$R = P(X > Y) = \int_{y=0}^{\infty}\left(\int_{x=y}^{\infty} f(x)\,dx\right) g(y)\,dy \qquad (1)$$

Let $X_1, X_2, X_3, \ldots, X_n$ be the strengths of the components $C_1, C_2, C_3, \ldots, C_n$ as arranged in order of activation respectively. All the $X_i{'s}$ are independently distributed random variables with probability density functions $f_i(x_i); i = 1,2,\ldots,n$. Also let Y be the stress acted on the components which is also randomly distributed with the density function $g(y)$

If $< X_1$, the first component C_1 works and hence the system survives. $Y \geq X_1$ leads to the failure of C_1; thus the second component in line viz., C_2 ,takes its place and has a strength X_2 ..Although the system has suffered the loss of one component, it survives if $Y < X_2$ and so on . In general, if the $(i-1)^{th}$ component C_{i-1} fails then the i^{th} component C_i, with the strength X_i, gets activated and will be subjected to the stress Y.

The system could survive with a loss of the first $(n-1)$ components if and only if $X_i \leq Y;\ \forall i = 1,2,3,\ldots\ldots, n-1 \text{ and } X_n > Y$. The system totally fails if all the components fail when $X_i \leq Y;\ \forall i = 1,2,\ldots\ldots n.$ The probability $R(n)$ of the system to survive with the first $(n-1)$ components failed and the n^{th} component active is

$$R(n) = P\left[\left\{\bigcap_{i=1}^{n-1}(X_i \leq Y)\right\} \cap (X_n > Y)\right] \qquad (3)$$

$R(2), R(3), \ldots\ldots\ldots, R(n)$ are the increments in reliability due to the addition of $2^{nd}, 3^{rd}, \ldots\ldots, n^{th}$ components respectively.

Then

$R(n) = P[X_1 \le Y, X_2 \le Y, \dots\dots\dots, X_{n-1} \le Y, X_n > Y]$ (4)

we can obviously associate the n^{th} component attenuation factor with Y.

Let $g(y)$ and $f_i(x_i)$ be the probability density function of Y and X_i $(i = 1,2,\dots,n)$ respectively.

The equation (4) can now be written as

$$R(n) = \int_0^\infty \left[\int_0^y f_1(x_1)dx_1 \times \int_0^y f_2(x_2)dx_2 \times \dots\dots \int_0^y f_{n-1}(x_{n-1})dx_{n-1} \times \int_y^\infty f_n(x_n)dx_n \right] g(y)dy \quad (5)$$

(or)

$$\int_0^\infty [F_1(y)F_2(y)\dots\dots\dots F_{n-1}(y)\bar{F}_n(y)]g(y)dy \quad (6)$$

$$where \quad F_i(y) = \int_0^y f_i(x_i)dx_i \quad and$$

$$\bar{F}_i(y) = 1 - F_i(y) \quad (7)$$

The two- parameter GE distribution has the following density functions

$$f(x;\alpha,\lambda) = \alpha\lambda e^{-\lambda x}\left(1 - e^{-\lambda x}\right)^{\alpha-1} ;\ for\ x > 0$$

And the distribution function

$$F(x;\alpha,\lambda) = \left(1 - e^{-\lambda x}\right)^{\alpha}\ for\ x > 0$$

$$g(y;\beta,\lambda) = \beta\lambda e^{-\lambda y}\left(1 - e^{-\lambda y}\right)^{\beta-1} ;\ for\ y > 0$$

$$The\ distribution\ function\ is \quad G(y;\beta,\lambda) = \left(1 - e^{-\lambda y}\right)^{\beta}\ for\ y > 0$$

$1 -$ Cascade system

$$R(1) = P[X > Y]$$

$$= \int_{y=0}^\infty \left(\int_{x=y}^\infty f(x)dx \right) g(y)dy$$

$$= \int_{y=0}^\infty (f_1(x_1)dx_1) g(y)dy$$

$$R(1) = \int_{y=0}^\infty \bar{F}_1(y)g(y)dy$$

$$R(1) = \int_{y=0}^\infty \left(1 - e^{-\lambda y}\right)^{\beta} \alpha\lambda e^{-\lambda y}\left(1 - e^{-\lambda y}\right)^{\alpha-1} dy$$

$$put\ 1 - e^{-\lambda y} = t$$

$$\lambda e^{-\lambda y} dy = dt\ and\ \ y = 0 \to t = 0\ and\ y = \infty \to t = 1$$

$$R(1) = \frac{\alpha}{\alpha + \beta}$$

2- Cascade system

$$R(2) = P[X_1 < Y, X_2 > Y]$$

$$R(2) = \int_{y=0}^{\infty} \left(\int_{x_1=0}^{y} (f_1(x_1) dx_1) \right) \left(\int_{x_2=y}^{\infty} (f_2(x_2) dx_2) \right) g(y) dy$$

$$= \int_{y=0}^{\infty} \left(1 - e^{-\lambda y}\right)^{\alpha} \left(1 - \left(1 - e^{-\lambda y}\right)^{\alpha}\right) \beta \lambda e^{-\lambda y} \left(1 - e^{-\lambda y}\right)^{\beta - 1} dy$$

$$R(2) = \frac{\alpha\beta}{(\alpha + \beta)(2\alpha + \beta)}$$

3- Cascade system

$$R(3) = P[X_1 < Y, X_2 < Y, X_3 > Y]$$

$$= \int_{y=0}^{\infty} \left(\int_{x_1=0}^{y} (f_1(x_1) dx_1) \right) \left(\int_{x_2=0}^{y} (f_2(x_2) dx_2) \right) \left(\int_{x_3=y}^{\infty} (f_3(x_3) dx_3) \right) g(y) dy$$

$$= \int_{y=0}^{\infty} \left(1 - e^{-\lambda y}\right)^{\alpha} \left(1 - e^{-\lambda y}\right)^{\alpha} \left(1 - \left(1 - e^{-\lambda y}\right)^{\alpha}\right) \beta \lambda e^{-\lambda y} \left(1 - e^{-\lambda y}\right)^{\beta - 1} dy$$

$$R(3) = \frac{\alpha\beta}{(2\alpha + \beta)(3\alpha + \beta)}$$

In general

$$R(n) = \frac{\alpha\beta}{((n-1)\alpha + \beta)(n\alpha + \beta)}$$

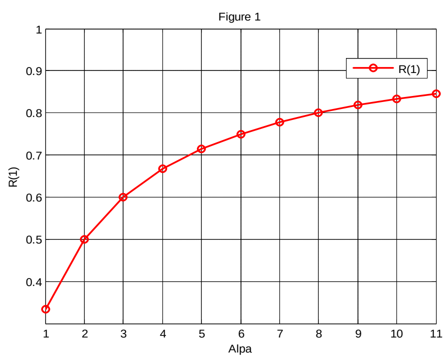
3. Reliability Computations:

Table 1

α	β	R(1)
1	2	0.333333
2	2	0.5
3	2	0.6
4	2	0.666667
5	2	0.714286
6	2	0.75
7	2	0.777778
8	2	0.8
9	2	0.818182
10	2	0.833333
11	2	0.846154

Table 2

α	β	R(1)
5	1	0.833333
5	2	0.714286
5	3	0.625
5	4	0.555556
5	5	0.5
5	6	0.454545
5	7	0.416667
5	8	0.384615
5	9	0.357143
5	10	0.333333
5	11	0.3125



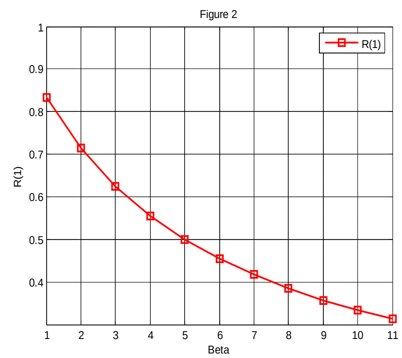
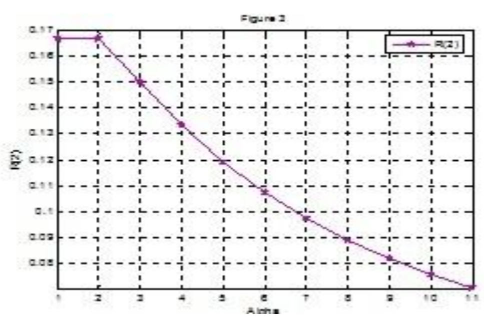


Table 3

α	β	R(2)
1	2	0.166667
2	2	0.166667
3	2	0.15
4	2	0.133333
5	2	0.119048
6	2	0.107143
7	2	0.097222
8	2	0.088889
9	2	0.081818
10	2	0.075758
11	2	0.070513

Table 4

α	β	R(2)
5	1	0.075758
5	2	0.119048
5	3	0.144231
5	4	0.15873
5	5	0.166667
5	6	0.170455
5	7	0.171569
5	8	0.17094
5	9	0.169173
5	10	0.166667
5	11	0.16369



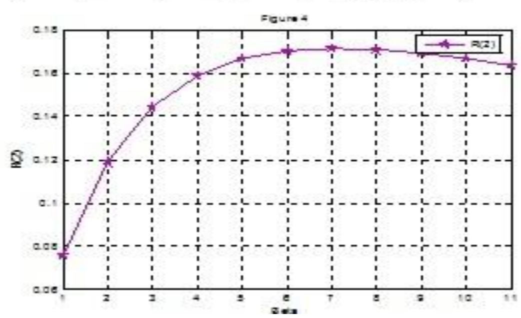
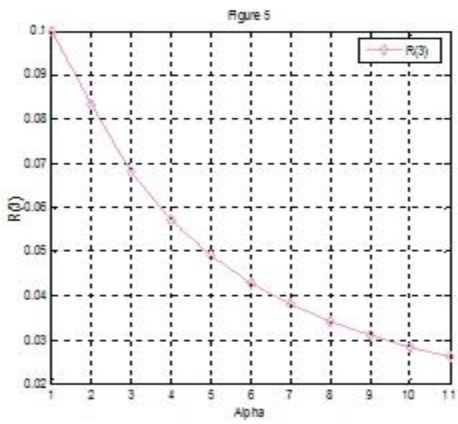


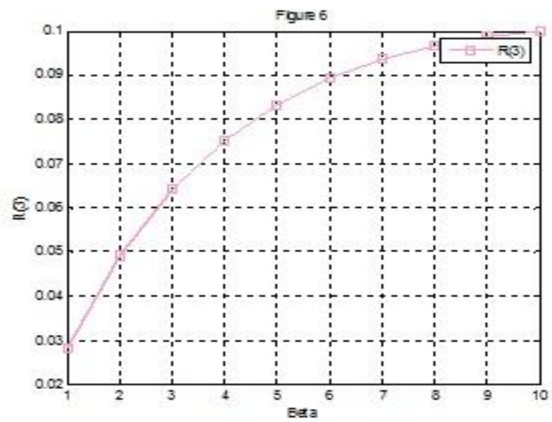
Table 5

α	β	R(3)
1	2	0.1
2	2	0.083333
3	2	0.068182
4	2	0.057143
5	2	0.04902
6	2	0.042857
7	2	0.038043
8	2	0.034188
9	2	0.031034
10	2	0.028409
11	2	0.02619

Table 6

α	β	R(3)
5	1	0.028409
5	2	0.04902
5	3	0.064103
5	4	0.075188
5	5	0.083333
5	6	0.089286
5	7	0.093583
5	8	0.096618
5	9	0.098684
5	10	0.1





4. Conclusion

The reliability of n- cascade system when stress and strength follow generalized exponential distribution. In this paper we find out the formula for n-cascade system. From computations Reliability increases in 1-cascade system whenever strength parameter α increases and reliability decreases whenever stress parameter β increases and vice-versa for 2-cascade, 3-cascade,---n-cascade system.

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