Robust LMI-Based Controller Design using H_∞ and Mixed H_2/H_∞ for Semi Active Suspension System

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Abstract:
Control of vehicle suspension systems has been the focus of extensive work in the past two decades. Many control strategies have been developed to improve the overall vehicle performance, including both ride quality and stability conditions. In this paper, we use the H_∞ and mixed H_2/H_∞ techniques with a semi active suspension system to increase the passenger's ride comfort performance A two degree of freedom dynamic model of a vehicle semi-active suspension system was presented. The role of $H\infty$ is used to minimize the disturbance effect on the system output whereas H_2 is used to improve the transients against random disturbances. The capability of the system of improving comfort of operators has been evaluated through simulations carried out with a validated model of the entire vehicle. The results of this study revealed that the use of mixed H_2/H_∞ with pole placement for a semi-active suspension system showed a great improvement compared with H_∞ systems. In addition, the results of the simulation showed that ride comfortable of the vehicle can be improved effectively by using the semi-active suspension with mixed H_2/H_∞ control method, and also the mixed H_2/H_∞ control method more effective than the H_∞ control method. Finally, this paper showed a robust use of both H_∞ and mixed H_2/H_∞ problem which can be solved using linear matrix inequality (LMI) techniques.

Keywords*:* H_∞ control; mixed H_2/H_∞ control; robust control; semi-active suspension; ride comfort; simulation; pole placement.

I Introduction

All motor manufactures are currently engaged in research and development to ensure that they remain at the competitive edge, in terms of both vehicle performance and perceived human factors such as comfort and drivability. Conventional vehicle suspension systems consists of a passive spring in parallel with a damper, their main functions being to support the body mass and to provide both passenger comfort and road holding. These have a number of limitations due to the fixed nature of the components used and requirements for the vehicle to function over a wide variety of operating conditions. A suspension system is also required to react to changes in vehicle load, a constraint which requires a stiff suspension. The introduction of active elements into the suspension allows the compromise to redefined, providing an all round improvement in performance. The topic of this paper is the using of H_∞ and mixed H_2/H_∞ control with a semi active suspension system. The design of control algorithms for semi-active vehicle suspensions has been an active research field for over forty years [1,2]. Numerous control algorithms have been developed for semi-active suspensions [1,3]. The principle of semi-active damping is the control of variable dampers for the purpose of vibration isolation. Semi-active damping has been shown to significantly improve vibration isolation in comparison to passive damping for a range of mechanical and civil engineering applications see for example [4–8]. The semi-active suspension of vehicles uses the damping components that can be controlled and the closed loop system, which can regulate the damping force according to the feedback signals generated by suspension working space ,and acceleration of the car body, so that the damping suspension stay in the best condition and improve the ride comfort ability. Many active suspension control approaches have been proposed such as Linear Quadratic Gaussian (LQG) control, adaptive control, and non-linear control to overcome these suspension systems problems [9-11]. Stability represents the minimum requirement for control systems. However, in most cases, a good controller should act sufficiently fast with well-damped response beside the disturbance attenuation on selected system outputs. If the controller design is not robust against disturbance and parameters change, the system may become unstable [12,14]. Mixed H_2/H_∞ robust control alleviates such handicap [15-19]. Linear matrix inequality (LMI) [20] is one of the most effective and efficient tools in controller design. Many LMI-based design methods of static out feedback (SOF) design have been proposed over the last decade. The main advantage of the H_∞ control is that it provides maximum robustness to the most destabilizing uncertainty, which is modeled as disturbance input. The H_2 performance criterion introduced above is extended with an H_∞ criterion for the body mass acceleration. This idea leads to an attempt of the mixed H_2/H_∞ control design scheme.

II. Mathematical Model Formulation

A two-degree-of-freedom "quarter-car" vehicle suspension system is shown in Figure 1. An advantage of this model is that many published results are available, which makes it easy to verify and compare the results which those other researches. It represents the vehicle system at each wheel i.e. the motion of the axle and of the vehicle body at any one of the four wheels of the vehicle. The suspension itself is shown to consist of a spring k_s, a damper c_s and an active force actuator u. The active force u can be set to zero in a passive suspension. The sprung mass m_s represents the quarter car equivalent of the vehicle body mass. The unsprung mass mu represents the equivalent mass due to the axle and tire. The vertical stiffness of the tire is represented by the spring k_t. The variables z_b , z_w and z_o represent the vertical displacements from static equilibrium of the sprung mass, unsprung mass and the road respectively[21]. In this paper it was assumed that only the suspension deflection could be measured and used by the controllers.

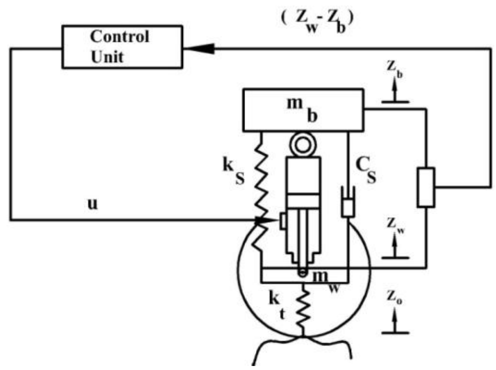


Figure 1. Semi-active suspension system.

$$m_b\ddot{z}_b + C_s(\dot{z}_b - \dot{z}_w) + k_s(z_b - z_w) = u \qquad (1)$$

$$m_w\ddot{z}_w + k_t(z_o - z_w) - c_s(\dot{z}_b - \dot{z}_w) - k_s(z_b - z_w) = -u \qquad (2)$$

Assume the following

$x_1 = Z_o - Z_w$,

$x_2 = \dot{Z}_b$,

$x_3 = Z_u - Z_r$, $x_4 = \dot{Z}_u$

Where :

$x_1 = z_s - z_w$ is the suspension deflection (rattle space)

$x_2 = \dot{z}_b$ is the absolute velocity of sprung mass

$x_3 = z_o - z_w$ is the tire deflection

$x_4 = \dot{z}_w$ is the absolute velocity of unsprung mass

The state equations of the sample power system can be written in the vector-matrix differential equation form as:

$$\dot{x} = Ax + B_1 z_o + B_2 u$$

The system matrix A, the control matrix B_1, and the road input matrix B_2 are, respectively, denoted as

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{(k_t+k_s)}{m_b} & 0 & 0 & 1 \\ \frac{k_s}{m_w} & \frac{c_s}{m_w} & -\frac{k_t}{m_w} & -\frac{(c_s)}{m_w} \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{m_b} \\ 0 \\ -\frac{1}{m_w} \end{bmatrix}, \text{ and } \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -\frac{1}{m_w} \end{bmatrix}$$

The suspension parameters are shown the Table 1.

Table 1. Quarter car parameters

Parameters	**Symbols**	**Quantities**
Body mass	m_b	250 kg
Wheel mass	m_w	50 kg
Stiffness of the body	k_s	16 kN/m
Stiffness of the wheel	k_t	160 kN/m
Stiffness of the damper	c_s	1.5 kN.s/m

III. Input Profile Excitation

The representation of the road profile is vital for vehicle dynamic simulations because it is the main source of excitation. An accurate road model is as important as a good vehicle model. The Excitation input from the road is transmitted to the vehicle floor. For the simplification of the dynamic modeling, it is assumed that there exists only the vertical motion of the vehicle. Both pitching and rolling motions are ignored in this study. The reduction of forces transmitted to the road by moving vehicles (particularly for heavy vehicles) is also an important issue responsible for road damage. Heavy vehicle suspensions should be designed accounting also for this constraint. In this work, A periodic road excitation input has been used for simulation of suspension systems. The periodic input is used for smooth road in order to evaluate ride comfort as shown in Figure 2. It is widely recognized that the road surfaces approximate to Gaussian processes, having a power spectral density (PSD) of the form [22]:

$$PSD(f) = \frac{R_C\, V^{n-1}}{f^n} \qquad (3)$$

Where:
R_C : Road Roughness Coefficient.
f : Road Excitation Frequency, Hz.

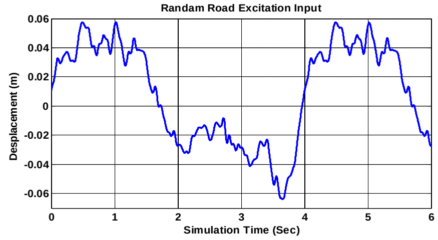


Figure 2. Road excitation.

IV. Robust H∞ Controller (RH∞)

In a typical H_∞ design problem, the nominal plant model represented by its transfer function P(s) is usually known and the design problem for an output feedback control is formulated as a standard H_∞ problem, as described by the block diagram of Figure 3. P(s) represents the plant and K(s) the controller transfer function in Laplace domain. The controller is aimed to be designed using the H_∞ design technique. In the block diagram, w represents the external disturbances, z the regulated outputs and y the measured outputs. The vector u consists of the controlled inputs[23].

Let:

$$P(s):\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z_1 = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} u \end{cases} \qquad (4)$$

Controller:

$$K(s):\begin{cases} x_K = A_K x_K + B_K y \\ u = C_K x + D_K y \end{cases} \qquad (5)$$

be state-space realizations of the plant P(s) and controller K(s), respectively, and let

$$\begin{cases} \dot{x}_{CL} = A_{CL} x_{CL} + B_{CL} w \\ z = C_{CL} x_{CL} + D_{CL} w \end{cases} \qquad (6)$$

be the corresponding closed-loop state-space equations with

$$X_{CL} = [X \quad X_K]^T$$

The design objective for finding K(s) is to optimize the H_∞-norm of the closed-loop transfer G(s) from (w) to (z), i.e.,

$$G(s) = C_{CL}\left(s - A_{CL}\right)^{-1} B_{CL} + D_{CL} \qquad (7)$$

and $\left| G(s)_{ZW} \right| < \gamma$

using the LMI technique. □ is a specific number. This can be fulfilled if and only if there exists a symmetric matrix X such that the following LMIs are satisfied.

$$\begin{bmatrix} A_{CL}X + XA_{CL}^T & B_{CL} & XC_{CL}^T \\ B_{CL}^T & -1 & C_{CL}^T \\ C_{CL}X & D_{CL} & -\gamma^2 I \end{bmatrix} < 0 \qquad (8)$$

$$X > 0$$

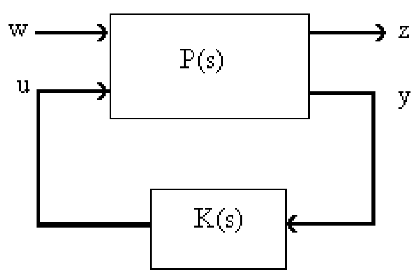


Figure 3. Block diagram of output feedback.

It represents the system disturbance re... t-case disturbance on the output. LMI toolbox can be used for such controller design [24]. Where,

$$A_{CL} = \begin{bmatrix} A + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix}$$

$$B_{CL} = \begin{bmatrix} B_1 + B_2 D_K D_{21} \\ B_K D_{21} \end{bmatrix}$$

$$C_{CL} = \left[(C_1 + D_{12} D_K C_2) \quad D_{12} C_K \right]$$

$$D_{CL} = \left[D_{11} + D_{12} D_K D_{21} \right]$$

LMI constraints defined by (8) can be derived from: Stability condition based on Lyapunov energy function;

$$V(X) = x^T X x > 0 \qquad (9)$$

$$\frac{dV}{dt} = x^T\left(A^T X + XA\right)x + x^T (XB) + u + u^T (B^T X)x < 0 \quad (10)$$

From Eq. (10) the stability LMI constraints are;

$$\begin{pmatrix} A_{CL}^T X + XA_{CL} & XB_{CL} \\ B_{CL}^T & -\gamma^2 I \end{pmatrix} < 0 \quad (11)$$

$$X > 0$$

Minimization of the disturbance effect condition on the selected outputs based on infinity norm (H_∞) that equals;

$$yTy - \gamma^2 uTu < 0 \quad (12)$$

From Eq. (12) the disturbance effect under LMI constraints is;

$$\begin{pmatrix} C_{CL}^T C_{CL} & C_{CL}^T D_{CL} \\ D_{CL}^T C_{CL} & D_{CL}^T D_{CL} \end{pmatrix} < 0 \quad (13)$$

From Eqs.(11) and (13) LMI constraints become;

$$\begin{pmatrix} A_{CL}^T X + XA_{CL} + C_{CL}^T C_{CL} & XB_{CL} + C_{CL}^T D_{CL} \\ B_{CL}^T P + D_{CL}^T C_{CL} & D_{CL}^T D_{CL} - \gamma^2 I \end{pmatrix} < 0 \quad (14)$$

$X = X^T > 0$ (Positive definite matrix)

According to the Schur complement LMI constraints defined by (14) become as given in (8).

5. Mixed H_2/H_∞ Controller Design

The H_2 and H_∞ control strategies based on the LMI were derived, respectively. Now we will combine these two constraints into one design expression. The mixed H_2/H_∞ control problem is to minimize the H_2 norm of T_{z2w} over all state feedback gains k such that what also satisfies the H_∞ norm constraint. Mixed H_2/H_∞-synthesis with regional pole placement is one example of multi-objective design addressed by the LMI. The control problem is sketched in Fig. 4. The output channel z_∞ is associated with the H_∞ performance while the channel z_2 is associated with the H_2 performance (LQG aspects)[25].

A. System Representation

Figure 4. shows the standard representation of the robust output-feedback control block diagram where P(s) is the plant and K(s) represents the controller that is usually of the same order as the plant let:

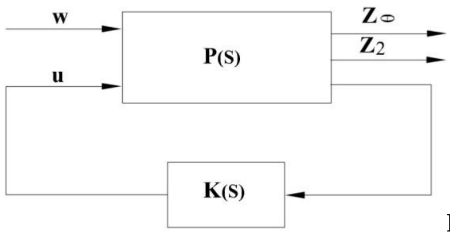
$$P(s): \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z_\infty = C_\infty x + D_{\infty 1} w + D_{\infty 2} u \\ z_2 = C_2 x + D_{21} w + D_{22} u \\ y = C_y x + D_{y1} w + D_{y2} u \end{cases} \quad (15)$$

$$K(s): \begin{cases} \dot{\zeta} = A_K \zeta + B_K y \\ u = C_K \zeta + D_K y \end{cases} \quad (16)$$

and let

$$CL:\begin{cases}\dot{x}_{cl} = A_{cl}x_{cl} + B_{cl}w \\ z_{\infty} = C_{cl\infty}x_{cl} + D_{cl\infty}w \\ z_2 = C_{cl2}x_{cl} + D_{cl2}w\end{cases} \tag{17}$$

be the corresponding closed-loop state-space equations with $x_{cl} = [x \quad \zeta]^t$



Figur e 4. Output feedback block diagram.

B. Pole-Placement Technique[25]

The concept of LMI region [26]. is useful to formulate pole-placement objectives in LMI terms. They are convex subsets D of the complex plane C characterized by

$D = \{z \in C \text{ such that } f_D(z) = L + Mz + M^t z < 0\}$ where M and L=L^t are fixed real matrices,

$L = L^t = [\lambda_{ij}]$ and $M = [\mu_{ij}]$ where $1 \le i,j \le m$

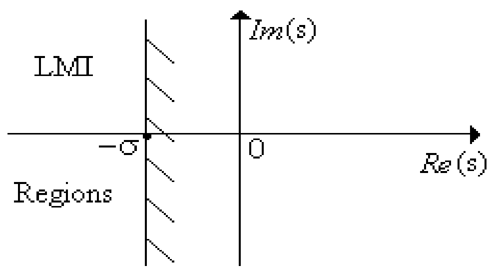


Figure 5. Pole-placement region.

z= x+iy a complex number. More practically, LMI regions include relevant regions such as sectors, disks, conics, strips, etc., as well as any intersection of the above. Only a shift in the left-hand side plane, shown in Figure 5. is considered. Its characteristic function with $Re(z) = x<-\sigma$, is $f_D(z) = z + \bar{z} + 2\sigma < 0$, thus L=2σ, M=1.

From a Theorem in [25], the pole-placement constraint is satisfied if and only if there exists $X_p > 0$ such that $[\lambda_{ij}X_p + \mu_{ij}A_{cl}X_p + \mu_{ji}X_pA_{cl}^t] < 0$ with $1 \le i,j \le m$

V. Results and Discussions

The digital simulation results are obtained using MATLAB Platform. The aim of a suspension system for automotive applications is to isolate the passengers or load from vibrations generated by uneven roads. The suspension working space must not be too large because the working space for the suspension mechanism is limited. In this paper some paramters were investigateted its effect on the suspension systems performance. With H_∞ controller technique it is observed that parameter gama **(**γ *)* has most significant effect on the dynamic performance firstly the effect of the tuning variables of the LMI algorithms on the suspension performance are shown in table 2. which illustrates the root mean square value (RMS) of suspension working space, body acceleration, and dynamic tire load. It is clear that the parameter gama γ has a large effect on the system dynamic responses. From the table it can be noted that the optimal value of gama is 105. Figure 6 illustres the effect of gama on the suspension working space.

Table 2. Effect of the parameter (□ □ □on the Suspension Performance.

Case No.	□	SWS (m)	BAC (m/s^2)	WAC (m/s^2)	DTL (N)
1	50	0.0181	2.88	5.326	878.6
3	105	0.0087	1.48	6.340	691.9
4	120	0.0197	2.6258	5.6331	809.8
5	150	0.0207	2.6312	5.652	812

- SWS : Suspension working space (m).
- BAC : Body acceleration (m/s^2).
- WAC : Wheel acceleration (m/s^2).
- DTL : Dynamic tire load (N).

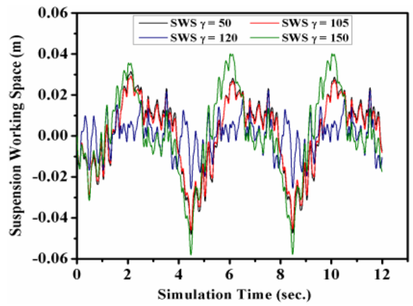


Figure 6. Suspension working space with different gama γ values

With H_2/H_∞ technique there are some parameters effected on the performance of suspension system and investigated as shown in Table 3. Shows the different values of the tuning variable of H_2/H_∞.

Table 3. RMS of suspension system performance.

Parameters	Values	SWS (m)	BAC(m/s2)	DTL (N)
□	100	0.0117	2.14	616.4
	200	0.0085	1.82	594.4
□	3	0.0117	5.45	1635
	50	0.0085	1.82	594.4
□	2	0.0117	2.72	799.5
	20	0.0085	1.82	594.4
□	0.1000	0.0117	1.9	1635
	0.0001	0.0085	1.82	594.4

Table 4. RMS Analysis random excitation.

System		SWS (m)	BAC (m/s^2))	DTL (N)
Passive System		0.0176	3.09	938
Semi Acti	H_∞	0.0150	2.72	855
	H_2/H_∞	0.0069	1.48	711.8
Improvement%		**54**	**46**	**17**

The vehicle body acceleration is an important index while evaluating vehicle ride comfort. The proposed of active suspension system with mixed H_2/H_∞ controller is effective in reducing vehicle body acceleration. Table 3. shows the RMS values of suspension working space, body acceleration, and dynamic tire load. The simulation results show that the vehicle body acceleration reduced from 2.72 m/s2 to 1.48 m/s2, and the suspension working space reduced from 0.015 mm to 0.0069 mm, and the tire dynamic load reduced from 855 N to 711.8 N, so the improvement are (54% , 46% ,and 17% respectively).

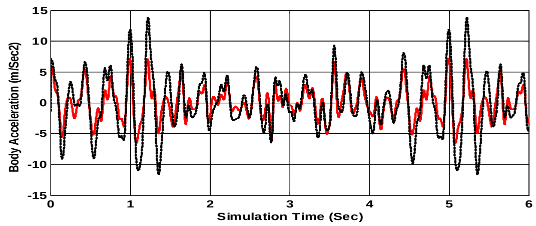


Figure 7. Body acceleration.

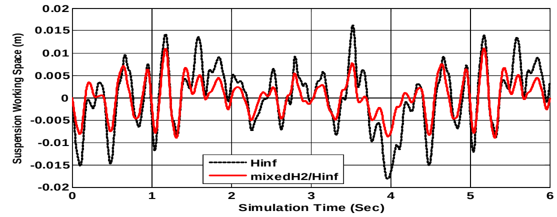


Figure 6. Suspension working space.

Simulation results indicate that the proposed of semi- active suspension system proves to be effective in improving riding comfort and holding ability as shown from Figures (7-9) which illusterate the comparsion between the two methods of controls.

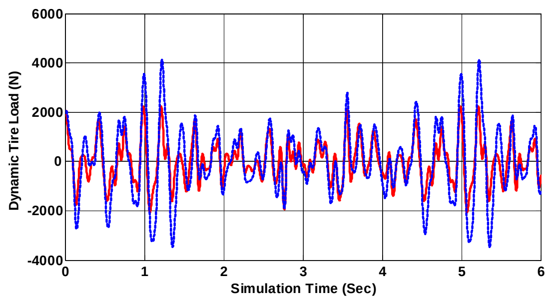


Figure 8. Dynamic tire load.

VI. Conclusions

A qurter car model (2DOF) is devolped in order to investigate the influnce of different control techneques on the suspension system performance, H_∞ and H_2/H_∞ . From the simulations results we can clearly see that the sime-active controlled suspension with both H_∞ and H_2/H_∞ control techniques offers a much better suspension performances than the passive system as compared in time domain, and a comparison between the two techniques H_∞ and H_2/H_∞ was done. It can noted that the mixed technique control method is offer a much better perfomance than H_∞ technique.

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