Effect of Radiation on Flow of Second Grade Fluid over a Stretching Sheet Through Porous Medium With Temperature Dependent Viscosity And Thermal Conductivity

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Abstract:

The effect of thermal radiation on boundary layer flow with temperature dependent viscosity and thermal conductivity due to a stretching sheet in porous media is investigated. The Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation. The sheet is being stretched linearly in the presence of a uniform transverse magnetic field and the flow is governed by the second –order viscoelastic fluid. The partial differential equations governing the flow and heat transfer characteristics are converted into ordinary differential equations by similarity transformations and solved numerically by fourth-order Runge-Kutta shooting method. The effects of various parameters on the velocity and temperature profiles as well as the skin-friction coefficient and Nusselt number has been shown graphically and in tabulated form and discussed in detail.

Keywords: Heat transfer, Porous medium, Radiation, Second order fluid, Stretching sheet, Thermal Conductivity, Variable viscosity

1. Introduction

The study of the flow and heat transfer created by a moving surface is relevant to several applications in the fields of metallurgy and chemical engineering, polymer processing, electro-chemistry, MHD power generators, flight magneto hydro dynamics as well as in the field of planetary magneto spheres, aeronautics and chemical engineering. Sakiadis [1] was the first to study the boundary layer flow due to a moving wall in fluid at rest. The study of flow over a stretching surface has generated much interest in recent years in view of its numerous industrial applications such as extension of polymer sheets, glass blowing, rolling and manufacturing plastic films and artificial fibers. The pioneer work on the boundary layer flows over stationary and continuously moving surfaces was initially done by Blasius [2] and Crane [3]. Ali [4] carried out a study for a stretching surface subject to suction or injection for uniform and variable surface temperatures. Rajgopal et al [5] , Dandapat and Gupta [6], Shit [7] and Reddaiah and Rao [8] extensively studied on various aspects of boundary layer flow problems over a stretching sheet.

In cooling processes, the effect of thermal radiation is also an important factor in non-isothermal systems. Hady and Mohamed [9] studied the MHD mixed convection with thermal radiation in laminar boundary layer flow over a semi-infinite flat plate embedded in porous media. Mansour [10] studied the effects of radiation and forced convection on the flow over a flat plate submersed in a porous medium of a variable viscosity. Mohammadein *et .al* [11] studied the effects of radiation with both first and second-order resistance's due to the solid matrix on some natural convection flows in fluid-saturated porous media. The effect of thermal radiation on mixed convection from horizontal surfaces in saturated porous media was investigated by Bakier and Gorla [12]. Prasad et al [13]) studied the radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate embedded in porous medium: a numerical study. Anjali Devi and Kayalvizhi [14] presented analytical solution of MHD flow with radiation over a stretching sheet embedded in a porous medium.

In most of the studies of this type of problems, the viscosity and thermal conductivity of the fluid were assumed to be constant. However, it is known that the physical properties can changed sufficiently with temperature and when the effects of variable viscosity and thermal conductivity are taken in to account, the flow characteristics are significantly changed compared to the constant property. Hassanien et al [15] revealed that the fluid viscosity and thermal conductivity might function of temperatures as well as the fluid is considering. Recently Sharma and Hazarika [16] studies the effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field.

Also, most of the practical situations demand for fluids that are non-Newtonian in nature which are mainly used in many industrial and engineering applications. It is well known that a number of fluids such as molten plastic, polymeric liquid, food stuffs etc exhibit non-Newtonian character.

In the present work, thermal radiation effects on heat transfer of second grade fluid over a stretching sheet through porous medium with temperature dependent viscosity and thermal conductivity is investigated. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, skin-friction coefficient and Nusselt number are shown in figures and tables and analyzed in detail. Numerical results are presented for velocity and temperature profiles for different parameters of the problem.

2. Mathematical Formulation

We consider the two-dimensional laminar boundary layer flow of viscous, incompressible, electrically conducting and radiating second grade fluid with temperature dependent viscosity and thermal conductivity past a semi-infinite stretching sheet coinciding with the plane $y=0$ embedded in a uniform porous medium. A uniform magnetic field of strength B_0 is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. Keeping the origin fixed, two equal and opposite forces are applied along the x- axis, so that the sheet is stretched with a velocity proportional to the distance from the fixed origin. Under the above assumptions, the basic boundary layer equations governing the flow and heat transfer of second grade fluid due to the stretching sheet are given by the following equations:

The equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum conservation:

$$\rho_\infty\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\}-\sigma B_0{}^2u-\frac{\mu}{K'}u \tag{2}$$

Thermal energy conservation:

$$\rho_\infty C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}_r\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{\partial q_r}{\partial y}-k_0\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right] \tag{3}$$

Along with the boundary conditions,

$$u=U_w=cx,\quad v=0,\quad T=T_w \quad \text{at} \quad y=0$$
$$u=0,\quad v=0,\quad T=T_\infty \quad \text{as} \quad y\to\infty \tag{4}$$

Where u and v are the flow velocity components along x- and y- directions respectively, B_0 is the applied magnetic field, μ_∞ and k_∞ are the constant viscosity and constant thermal conductivity of the free stream of the fluid respectively. T is the temperature of the fluid. μ and k are the coefficient of variable viscosity and variable thermal conductivity respectively of the fluid which are considered to vary as a function of temperature. C_p is the specific heat at constant pressure and k_0 is the coefficient of visco-elastricity. σ is the electrical conductivity. c is the constant stretching rate. T_∞ and ρ_∞ are the free stream temperature and density. K' is the permeability of the porous medium. q_r is the radiation heat flux.

Flowing Lai and Kulacki [17] We assume

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma\left(T-T_\infty\right)\right] \qquad or \qquad \frac{1}{\mu}=a\left(T-T_r\right) \tag{5}$$

where $a=\dfrac{\gamma}{\mu_\infty}$, and $T_r=T_\infty-\dfrac{1}{\gamma}$

and

$$\frac{1}{k}=\frac{1}{k_\infty}\left[1+\kappa\left(T-T_\infty\right)\right] \qquad or \qquad \frac{1}{k}=\varepsilon\left(T-T_\infty\right) \tag{6}$$

where $\varepsilon=\dfrac{\kappa}{k_\infty}$ and $T_e=T_\infty-\dfrac{1}{\kappa}$

Where a, ε, T_r, T_e are constants and their values depend on the reference state and thermal properties of the fluid i.e γ and κ . In general $a>0$ for liquids and $a<0$ for gases (the viscosity and thermal conductivity of liquid/gas usually decrease/increase with increasing temperature).

By assuming Rossenland approximation for radiation, the radiative heat flux q_r is given by

$$q_r=-\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y} \tag{7}$$

Where σ^* and K^* are the Stefan-Bolzman constant and the mean absorption coefficient respectively. We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature as shown in Chamakha [18]. Expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms we obtain

$$T^4 \cong 4T_\infty^{\ 3}T-3T_\infty^{\ 4} \tag{8}$$

Using (7) and (8) , we obtain as

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^* T_\infty^{\ 3}}{3K^*}\frac{\partial^2 T}{\partial y^2} \tag{9}$$

3. Method of Solution

The mathematical analysis of the problem is simplified by introducing the following dimensionless coordinates in terms of similarity variable η and the similarity function f as

$$u=cxf'(\eta), \qquad v=-\sqrt{c\upsilon}\, f\left(\eta\right), \qquad \eta=\sqrt{\frac{c}{\upsilon}}y, \qquad \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{10}$$

Where prime denotes the differentiation with respect to η and θ is the dimensionless temperature.

Clearly the continuity equation (1) is satisfied by u and v defined in equation (10). Substituting equation (10) in equations (2) - (3) gives the following equations

$$\left(\frac{\theta-\theta_r}{\theta_r}\right)\left[(f')^2-ff''\right]+f'''-\frac{\theta'}{\theta-\theta_r}f''+K_1\left(\frac{\theta-\theta_r}{\theta_r}\right)\left[2ff'''-(f'')^2-ff^{iv}\right]+\left[M\left(\frac{\theta-\theta_r}{\theta_r}\right)+K\right]f=0 \tag{11}$$

And

$$\left(4R+3\right)\theta''+3P_r f\theta'-3P_r E_c\left(\frac{\theta_r}{\theta-\theta_r}\right)\left(f''\right)^2-3K_1E_cP_r\left[f'\left(f''\right)^2-ff''f'''\right]=0 \tag{12}$$

The transformed boundary conditions are reduce to

$$f'(\eta)=1,\qquad f(\eta)=0,\qquad \theta(\eta)=1,\qquad at\qquad \eta=0, \tag{13}$$

$$f'(\eta)\to 0,\qquad f''(\eta)\to 0,\qquad \theta(\eta)\to 0,\qquad as\qquad \eta\to\infty, \tag{14}$$

Where prime denotes differentiation with respect to η only and

$K_1=\dfrac{k_0c}{\rho_\infty\upsilon}$ is the viscoelastic parameter, $M=\dfrac{\sigma B_0^{\,2}}{\rho_\infty c}$ is the magnetic parameter,

$\Pr=\dfrac{\mu c_p}{k}$ is the Prandtl number, $Ec=\dfrac{U_w^{\,2}}{c_p\left(T_w-T_\infty\right)}$ is the Eckert number.

$K=\dfrac{\upsilon}{K'c}$ is the porosity parameter, $R=\dfrac{4\sigma^*T_\infty^{\,3}}{kK^*}$ is the radiation parameter and

θ_r is the dimensionless parameter characterizing the influence of viscosity, where

$$\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty}=-\frac{1}{\gamma\left(T_w-T_\infty\right)} \tag{15}$$

For engineering purpose, one is usually less interested in the shape of the velocity and temperature profiles then in the value of the skin-friction, heat transfer. The expression for the local skin-friction coefficient C_f and the local Nusselt number Nu defined by:

$$C_f=\frac{\tau_w}{\mu_\infty(cx)\sqrt{\frac{c}{\upsilon}}}=-\left[\frac{\theta_r}{\theta-\theta_r}+2K_1\right]f''(0), \tag{16}$$

$$Nu=\frac{q_w}{k\sqrt{\frac{c}{\upsilon}}\left(T_w-T_\infty\right)}=-\theta'(0) \tag{17}$$

Where

$$q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}=-k\sqrt{\frac{c}{\upsilon}}\left(T_w-T_\infty\right)\theta'(0),$$

3. Numerical Results and Discussion

The system of differential equations (11) and (12) governed by boundary conditions (13) and (14) are solved numerically by applying an efficient numerical technique based on the fourth order Runge-Kutta shooting method and an iterative method. It is experienced that the convergence of the iteration process is quite rapid. The numerical computations have been carried out for various values of radiation parameter R, visco-elastic parameter K_1, Eckert number Ec, Prandtl number $\Pr$, porosity parameter K, Magnetic parameter M and the dimensionless viscosity parameter θ_r. In order to illustrate the results graphically, the numerical values of dimensionless velocity $f'(\eta)$ and dimensionless temperature $\theta(\eta)$ are plotted in Figures 1 – 14.

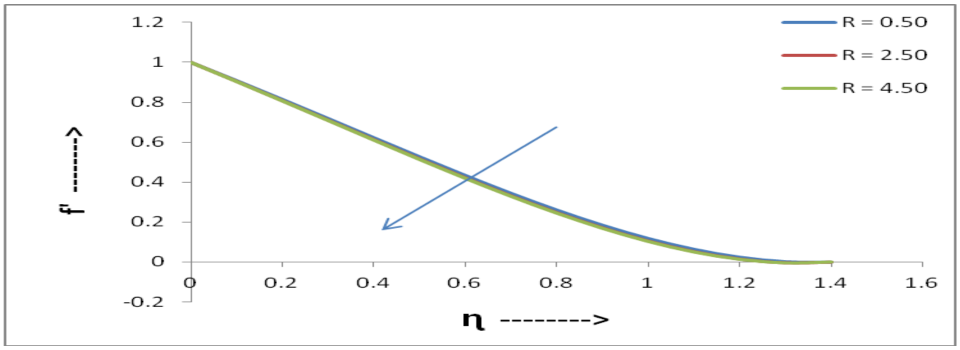

Figure 1. Variation of $f'(\eta)$ with η for different values of R

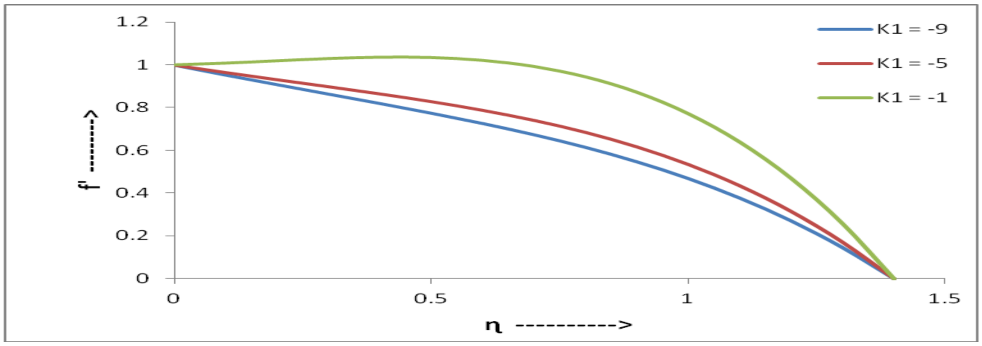

Figure 2. Variation of $f'(\eta)$ with η for different values of K_1

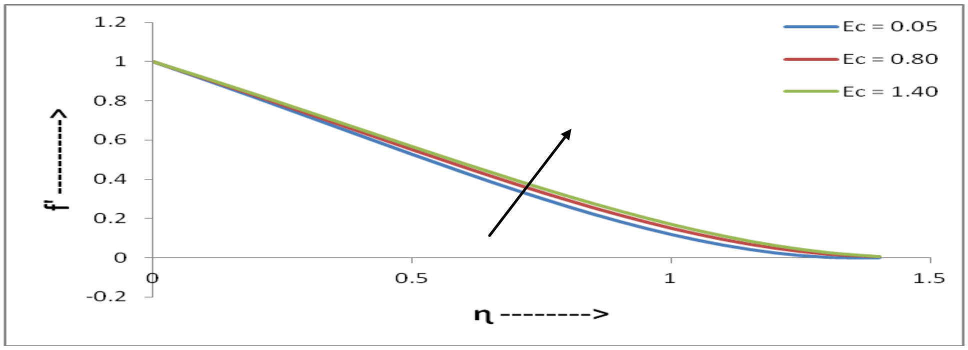

Figure 3. Variation of $f'(\eta)$ with η for different values of Ec

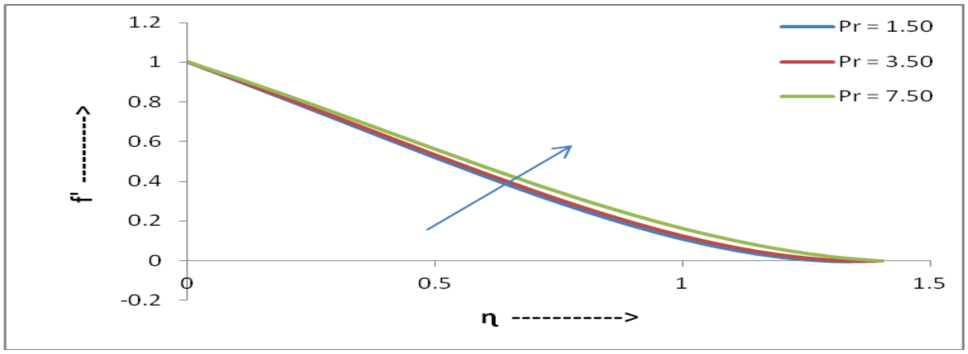

Figure 4. Variation of $f'(\eta)$ with η for different values of Pr

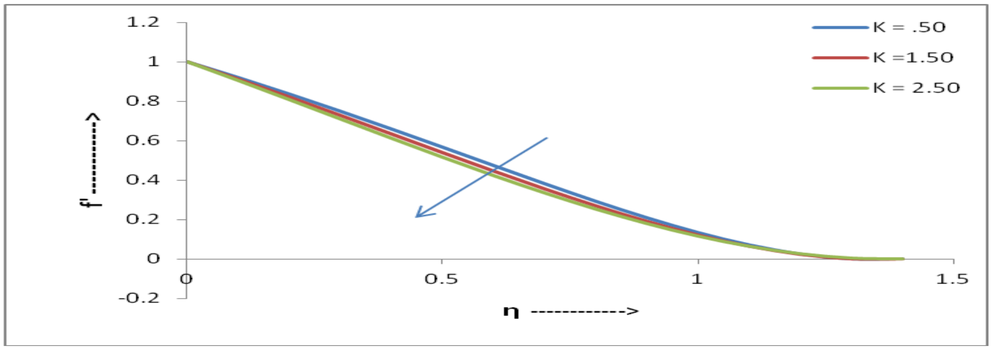

Figure 5. Variation of $f'(\eta)$ with η for different values of K

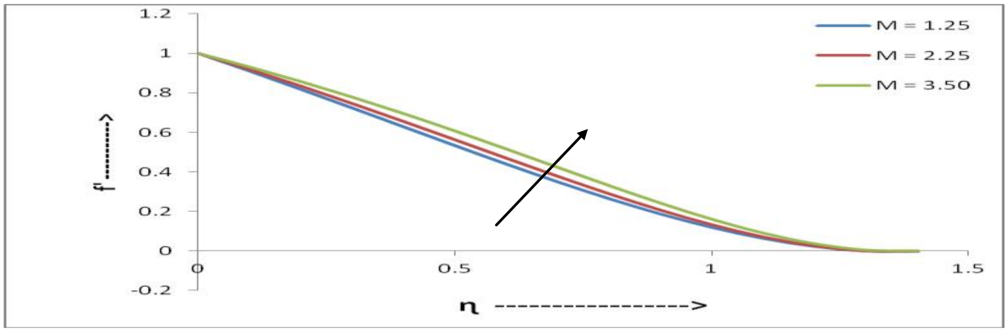

Figure 6. Variation of $f'(\eta)$ with η for different values of M

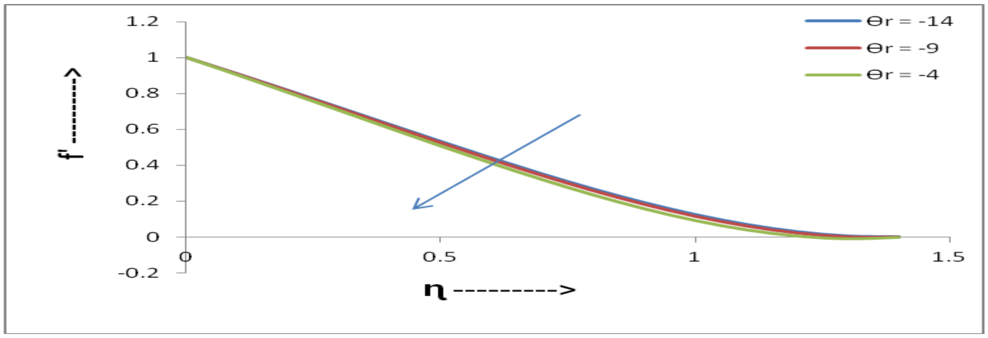

Figure 7. Variation of $f'(\eta)$ with η for different values of θ_r

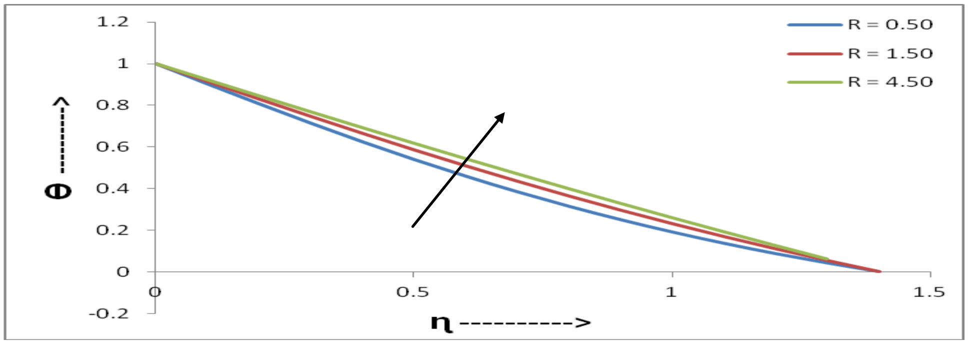

Figure 8. Variation of $\theta(\eta)$ with η for different values of R

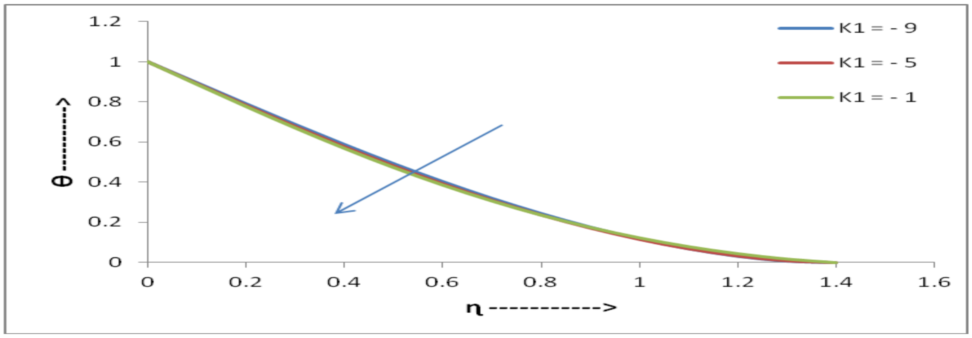

Figure 9. Variation of $\theta(\eta)$ with η for different values of K_1

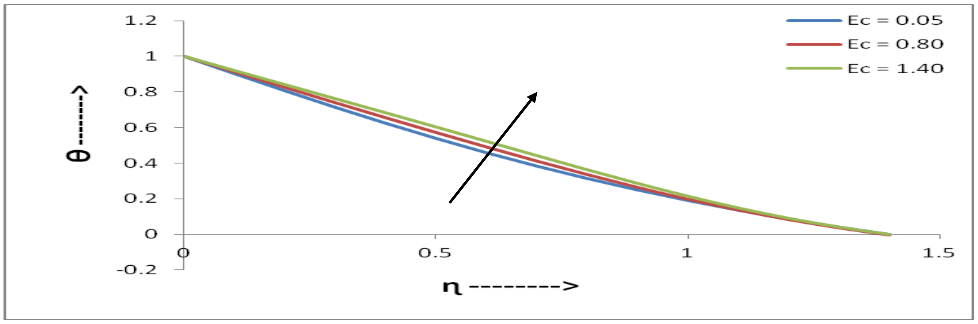

Figure 10. Variation of $\theta(\eta)$ with η for different values of Ec

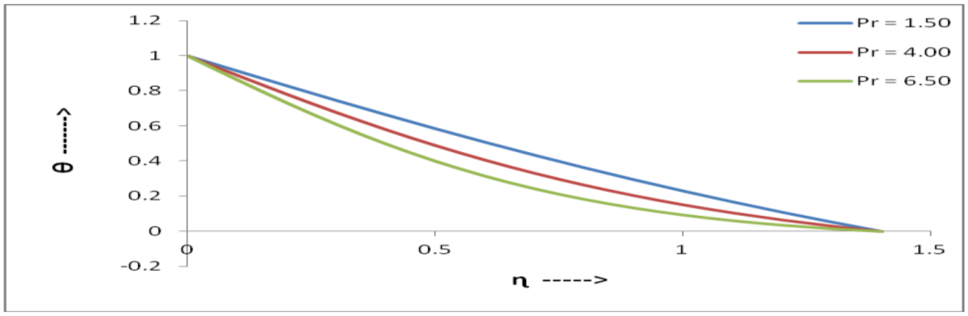

Figure 11. Variation of $\theta(\eta)$ with η for different values of $\Pr$

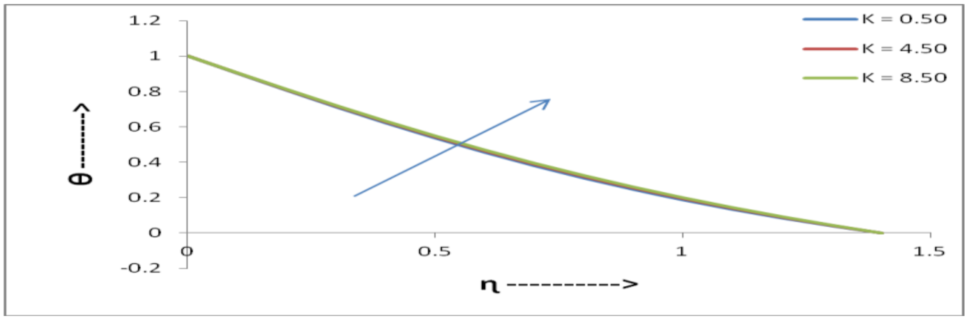

Figure 12. Variation of $\theta(\eta)$ with η for different values of K

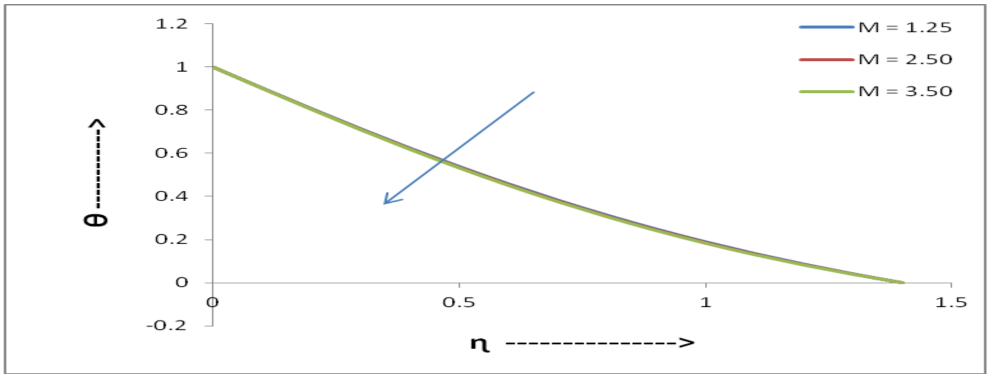

Figure 13. Variation of $\theta(\eta)$ with η for different values of M

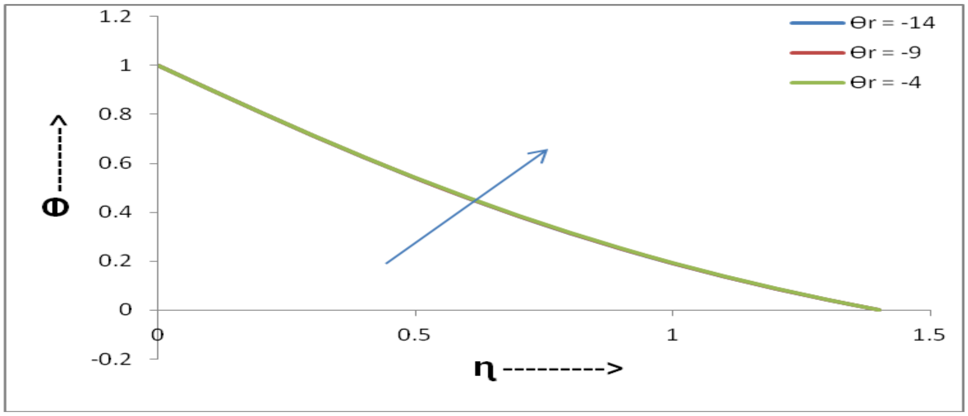

Figure 14. Variation of $\theta(\eta)$ with η for different values of θ_r

For various values of the radiation parameter *R*, the velocity profiles are plotted in Fig.1. It can be seen that as *R* increases, the velocity decreases. Fig.2. shows the effect of viscoelastic parameter K_1 on the velocity profiles. It is seen that the velocity increases as the viscoelastic parameter increases. The effect of Eckart number Ec on the velocity field is shown in Fig.3. It is noticed that the velocity profiles increases with the increase of Ekcart number. The velocity profiles for different values of Prandtl number $\Pr$ are illustrated in Fig.4. It is clear that increasing values of $\Pr$ results in increasing velocity. . Fig.5. shows the effect of permeability parameter *K* on the velocity profiles. It is seen that the velocity decreases as the permeability parameter increases. For various values of the magnetic parameter M , the velocity profiles are plotted in Fig.6. It can be seen that as M increases, the velocity increases. The effect of dimensionless viscosity parameter θ_r on the velocity profiles is shown in Fig.7. It is found that the velocity slightly decreases with an increase in θ_r . The effect of radiation parameter *R* on the temperature profiles is shown in Fig.8. It is observed that the temperature increases as *R* increases. Fig.9. shows the temperature profiles for different values of viscoelastic parameter K_1. It is obvious that an increase in K_1 results in decreasing temperature within the boundary layer. The effect of Eckart number Ec on the temperature profiles is depicted in Fig.10. It can be seen that an increase in Ec results in increase of the thermal boundary layer. Figs 11 and 12 noticed that the dimensionless temperature $\theta(\eta)$ decreases with the increase of the Prandtl number Pr and increases with the increasing values of porosity parameter K . It is interesting to note from Fig. 11 that the increase of Prandtl number Pr means decrease of thermal conductivity. The effect of the magnetic parameter M on temperature distribution shown in Fig. 13. From this figure we conclude that the temperature decreases with the increase of the magnetic parameter M . It may also observed from Fig. 14 that the effect of thermal radiation is to enhance the temperature with increase in the fluid viscosity parameter θ_r . It is interesting to note that in the presence of thermal radiation, the effect of viscosity parameter θ_r causes marginal significance.

The important characteristics in the present study are the local skin-friction coefficient C_f and the local rate of heat transfer at the sheet (Nusselt number Nu) defined in equations in (16) and (17).

Table-1. Numerical values of the local skin-friction: $C_f = \dfrac{\tau_w}{\mu_\infty (cx)\sqrt{\dfrac{c}{\upsilon}}} = -\left[\dfrac{\theta_r}{\theta - \theta_r} + 2K_1\right] f''(0),$

R	K	K_1	Pr	θ_r	Ec	$M = 0.0$	$M = 0.2$	$M = 0.4$
0.5	2	1	2.7	-10	.05	1.074144	1.057201	1.039990
2.5						1.111263	1.094620	1.077719
3.5						1.116569	1.099968	1.083112
0.5	**0.5**	1	2.7	-10	.05	0.951911	0.932854	0.913423
	1.5					1.034942	1.017369	0.999501
	2.5					1.112005	1.095630	1.079014
0.5	2	**-6**	2.7	-10	.05	-6.855943	-6.506797	-6.152434
		-4				-4.131017	-3.869648	-3.603017
		-2				-1.644921	-1.473744	-1.297443
0.5	2	1	**1.5**	-10	.05	1.101120	1.084394	1.067409
			2.5			1.078689	1.061782	1.044609
			3.5			1.055836	1.038748	1.021390
0.5	2	1	2.7	**-9**	.05	1.088707	1.071406	1.053830
				-5		1.197815	1.177733	1.157298
				-2		1.486126	1.457645	1.428535
0.5	2	1	2.7	-10	**.05**	1.074144	1.057201	1.039990
					0.15	1.064218	1.047599	1.030719
					0.25	1.054411	1.038109	1.021553

Table- 2. Numerical values of local Nusselt number : $Nu = -\theta'(0)$

R	K	K_1	Pr	θ_r	Ec	$Nu = -\theta'(0)$		
						$M = 0.0$	$M = 0.2$	$M = 0.4$
0.5	2	1	2.7	-10	.05	0.970627	0.971503	0.972407
2.5						0.809208	0.809521	0.809844
3.5						0.786403	0.786638	0.786882
0.5	**0.5**	1	2.7	-10	.05	0.977029	0.978125	0.979260
	1.5					0.972603	0.973544	0.974515
	2.5					0.968787	0.969606	0.970449
0.5	2	**-6**	2.7	-10	.05	1.003040	1.017594	1.032605
		-4				1.033991	1.046184	1.058830
		-2				1.061449	1.071274	1.081550
0.5	2	1	**1.5**	-10	.05	0.853002	0.853466	0.853944
			2.5			0.950695	0.951501	0.952333
			3.5			1.051387	1.052544	1.053736
0.5	2	1	2.7	**-9**	.05	0.970361	0.971254	0.972175
				-5		0.968634	0.969653	0.970706
				-2		0.966323	0.967656	0.969040
0.5	2	1	2.7	-10	**.05**	0.970627	0.971503	0.972407
					0.15	0.959883	0.960266	0.960672
					0.25	0.950152	0.950011	0.949887

Tables 1 and 2 exhibit the numerical values to the local skin-friction C_f and local Nusselt number Nu respectively. It has been observed empirically that for any particular values of R, K, Pr, θ_r and Ec the local skin-friction decreases with the increase in the magnetic parameter M. The skin friction is also decreases with the increase in Ec and the Prandtl number Pr. But the reversal trend is observed in the presence of fluid viscoelasticity K_1, K, θ_r and the thermal radiation R. It is worthwhile to mention here that the rate of heat transfer decreases with the increasing values of R, K, θ_r and Ec. However , the heat transfer rate increases with the increasing values of Prandtl number Pr and the viscoelastic parameter K_1.

4. Conclusions

In this paper a theoretical analysis has been done to study the effect of radiation on flow of Second grade fluid over a Stretching sheet through porous medium with temperature dependent viscosity and thermal conductivity. Some conclusions of the study are as below:

a. Velocity increases with the increase in magnetic parameter M, Eckart number Ec, Prandtl number $\Pr$ and viscoelastic parameter K_1.

b. Velocity decreases when radiation parameter R, viscosity parameter θ_r and porosity parameter K increases.

c. Skin friction decreases when magnetic field parameter M, Ec and Prandtl number $\Pr$ increases.

d. Skin friction increases when radiation parameter R, visco elastic parameter K_1 and viscosity parameter θ_r increases.

e. Temperature increases when radiation parameter R is increased. But temperature decreases when Prandtl number $\Pr$ and visco elastic parameter K_1 increases.

f. Nusselt number increases when Prandtl number $\Pr$ and visco elastic parameter K_1 increases.

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