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Control Of Unstable Periodic Orbits Coexisted With The Strange Attractor

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Abstract

It is known that the frame of a chaotic attractor is given by infinitely many unstable periodic orbits, which coexist with the strange attractor and play an important role in the system dynamics. There are many methods available for controlling chaos. The periodic proportional pulses technique is interesting one. In this paper it is aimed to apply the periodic proportional pulses technique to stabilize unstable periodic orbits embedded in the chaotic attractor of the nonlinear dynamics: $f(x) = ax^2 - bx$, where $x \in [0,4]$, a and b and are tunable parameters, and obtain some illuminating results.

Key Words: Controlling chaos, periodic proportional pulse, unstable periodic orbits, chaotic attractor, discrete model.
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1. Introduction

In chaos theory, **control of chaos** is based on the fact that any chaotic attractor contains an infinite number of unstable periodic orbits. Chaotic dynamics then consists of a motion where the system state moves in the neighborhood of one of these orbits for a while, then falls close to a different unstable periodic orbit where it remains for a limited time, and so forth. This results in a complicated and unpredictable wandering over longer periods of time Control of chaos is the stabilization, by means of small system perturbations, of one of the unstable periodic orbits. The result is to render an otherwise chaotic motion more stable and predictable, which is often an advantage. The perturbation must be tiny, to avoid significant modification of the system's natural dynamics.It is known that the frame of a chaotic attractor is given by infinitely many unstable periodic orbits, which coexist with the strange attractor and play an important role in the system dynamics. The task is to use the unstable periodic orbits to control chaos. The idea of controlling chaos consists of stabilizing some of these unstable orbits, thus leading to regular and predictable behavior. However, in many practical situations one does not have access to system equations and must deal directly with experimental data in the form of a **time series** [3, 10]. Publication of the seminal paper [1] entitled "Controlling chaos" by Ott, Grebogi and Yorke in 1990, has created powerful insight in the development of techniques for the control of chaotic phenomena in dynamical systems. There are many methods available [9] to control chaos on different models, but we take the advantage of the periodic proportional pulses technique [7], to control unstable periodic orbits in strange attractor by considering the one-dimensional nonlinear chaotic dynamics:

$$x_{n+1} = f(x_n) = ax_n^2 - bx_n, n = o,1,2,... \qquad (1.1)$$

We now highlight some useful concepts which are absolutely useful for our purpose.

1.1 Discrete dynamical systems

Any $C^k (k \geq 1)$ map $E : U \to \Re^n$ on the open set $U \subset R^n$ defines an n-dimensional **discrete-time** (autonomous) smooth dynamical system by the state equation

$$\overline{x}_{t+1} = E(\overline{x}_t), t = 1,2,3,.....$$

where $\overline{x}_t \in \Re^n$ is the state of the system at time t and E maps $\overline{x}_t$ to the next state $\overline{x}_{t+1}$. Starting with an initial data $\overline{x}_0$, repeated applications (iterates) of E generate a discrete set of points (the orbits) $\{E^t(\overline{x}_0) : t = 0,1,2,3,.....\}$, where $E^t(\overline{x}) = \underbrace{E \circ E \circ ... \circ E(\overline{x})}_{t \text{ times}}$ [6].

1.2 Definition: A point $\overline{x}^* \in \Re^n$ is called a **fixed point** of E if $E^m(\overline{x}^*) = \overline{x}^*$, for all $m \in$ ☪.

1.3 Definition: A point $\overline{x}^* \in \Re^n$ is called a **periodic point** of E if $E^q(\overline{x}^*) = \overline{x}^*$, for some integer $q \geq 1$.

1.4 Definition: The closed set $\boldsymbol{A} \in \Re^n$ is called the **attractor** of the system $\overline{x}_{t+1} = E(\overline{x}_t)$, if (i) there exists an open set $\boldsymbol{A}_0 \supset \boldsymbol{A}$ such that all trajectories $\overline{x}_t$ of system beginning in $\boldsymbol{A}_0$ are definite for all $t \geq 0$ and tend to $\boldsymbol{A}$ for $t \to \infty$, that is, $\operatorname{dist}(\overline{x}_t, \boldsymbol{A}) \to 0$ for $t \to \infty$, if $\overline{x}_0 \in \boldsymbol{A}_0$, where $\operatorname{dist}(\overline{x}, \boldsymbol{A}) = \inf_{\overline{y} \in \boldsymbol{A}} \|\overline{x} - \overline{y}\|$ is the distance from the point $\overline{x}$ to the set $\boldsymbol{A}$, and (ii) no eigensubset of $\boldsymbol{A}$ has this property.

1.5 Definition: A system is called **chaotic** if it has at least one chaotic attractor.

Armed with all these ideas and concepts, we now proceed to concentrate to our main aim and objectives

2. Control of Chaos by periodic proportional pulses

In N. P. Chua's paper [7] it is shown that periodic proportional pulses,

$x_i \to \lambda x_i$ (i is a multiple of q, where λ is a constant), (1.2)

applied once every q iterations to chaotic dynamics,

$$x_{n+1} = f(x_n), \tag{1.3}$$

may stabilize the dynamics at a periodic orbit. We note that a fixed point of (1.3) is any solution x^* of the equation

$$x^* = f(x^*) \tag{1.4}$$

and the fixed point is locally stable if

$$\left|\frac{df(x)}{dx}\right|_{x=x^*} < 1. \tag{1.5}$$

The composite function $g(x)$ is given by

$$g(x) = \lambda f^q(x). \tag{1.6}$$

where the dynamics is kicked by multiplying its value by the factor λ, once every q iterations. As above a fixed point of $g(x)$ is any solution x^* of

$$\lambda f^q(x^*) = x^* \tag{1.7}$$

and this fixed point is locally stable if

$$\left|\lambda \frac{df^q(x^*)}{dx}\right| < 1 \tag{1.8}$$

We note that a stable fixed point of g can be viewed as a stable periodic point of period q of the original dynamics f, kicked by the control procedure. Now the dynamics f is chaotic and wanted to control it so as to obtain stable periodic orbits of period q, by kicking once every q iterations, following equation (1.1).

To find a suitable point x^* and a factor λ satisfying (1.7) and (1.8), the function $Hq(x)$ is defined as

$$Hq(x) = \frac{x}{f^q(x)} \frac{df^q(x)}{dx} \tag{1.9}$$

Substituting from (1.7), equation (1.8) becomes

$$\left|Hq(x^*)\right| < 1. \tag{1.10}$$

Interestingly, if a point x^* satisfies the inequalities (1.10), then with the kicking factor λ defined by equation (1.6), the control procedure will stabilize the dynamics at a periodic orbit of period q, passing through the given point. It is important to note that, if the the impulse λ is too strong, it may kick the dynamics out of the basin of attraction, and in that case, the orbit may escape to infinity. In performing pulse control, one must have this precaution in mind.

3. Periodic proportional pulses on the concerned model

Periodic proportional pulses for stabilizing unstable periodic orbits embedded in a chaotic attractor can be well demonstrated by the nonlinear chaotic model (1.1), that is,

$$x_{n+1} = ax_n^2 - bx_n, n = o,1,2,...$$

with the control parameter value $b = -3.9$. Here the parameter a is fixed as $\boldsymbol{a} = \mathbf{1}$ and for this fixed value, it is observed that model (1.1) develops chaos via the period-doubling bifurcation route.

Period-doubling cascade for the model (1.1):

Table 1.1

Period	One of the Periodic points	Bifurcation Points.
1	x_1 =2.000000000000...	b_1 = -3.000000000000...
2	x_2 =1.517638090205...	b_2 =-3.449489742783...
4	x_3 =2.905392825125...	b_3 =-3.544090359552...
8	x_4 =3.138826940664...	b_4 =-3.564407266095...
16	x_5 =1.241736888630...	b_5 =-3.568759419544...
32	x_6 =3.178136193507...	b_6 =-3.569691609801...
64	x_7 =3.178152098553...	b_7 =-3.569891259378...
128	x_8 =3.178158223315...	b_8 =-3.569934018374...
256	x_9 =3.178160120824...	b_9 =-3.569943176048...
512	x_{10} =1.696110052289...	b_{10} =-3.569945137342...
1024	x_{11}=1.696240778303...	b_{11} =-3.569945557391...
...	... ...	... ...

[Periodic points and period-doubling points are calculated using numerical mechanisms discussed in [8, 11] taking the fixed parameter value a = 1]

The period-doubling cascade accumulates at the accumulation point *b* = *-3.569945672…..*, after which chaos arise. For the parameter $b = -3.9$ the system (1.1) is chaotic. The time series graph in the following figure (1.1) shows the chaotic behavior of the system:

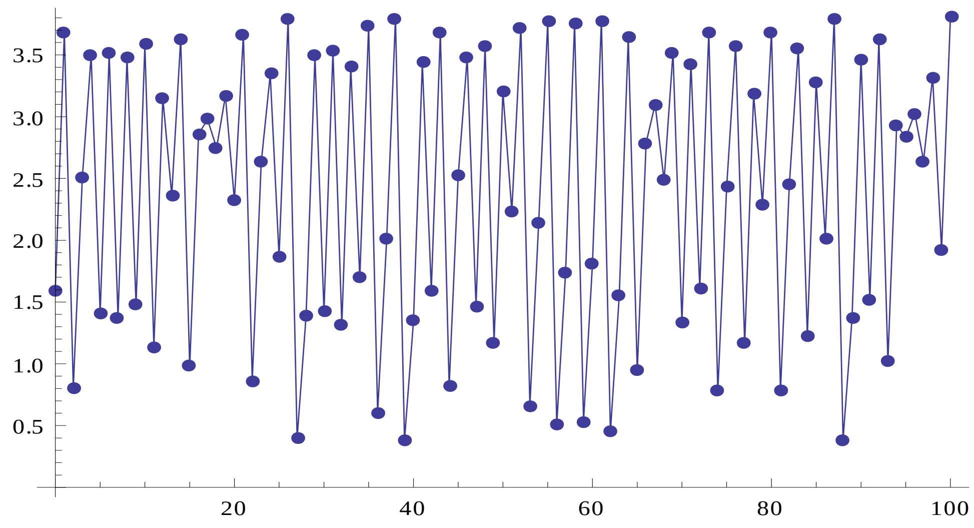

Fig. 1.1 Time series graph for parameter $b = -3.9$ ***and initial point*** $x_0 = 1.6$

For $q = 1$ and $b = -3.9$ the control curve $H1(x)$ is drawn in figure 1.2. The range is restricted to $-1 < Hq(x^*) < 1,\ q = 1$ and in this interval we can stabilize orbits of period one at every point x^* in the range about (0, 2.6).

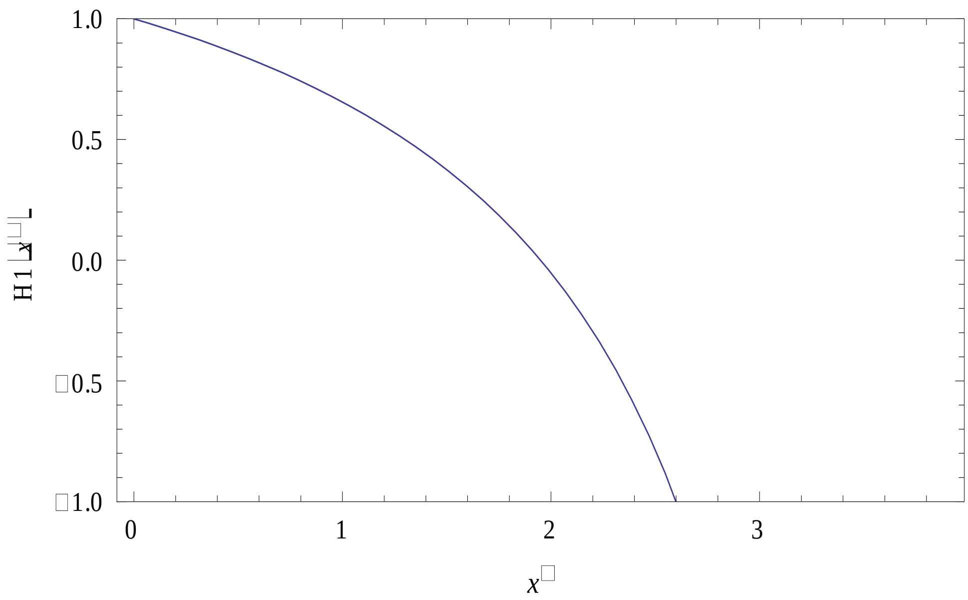

Fig. 1.2 Control curve for parameter $b = -3.9$

Taking $x^* = 1.9$ in the above stated range, the value of the kicking factor $\lambda = 0.5$ is calculated and the control procedure stabilizes the dynamics at a periodic orbit of period-one, passing through the given point as shown in the figure 1.3.

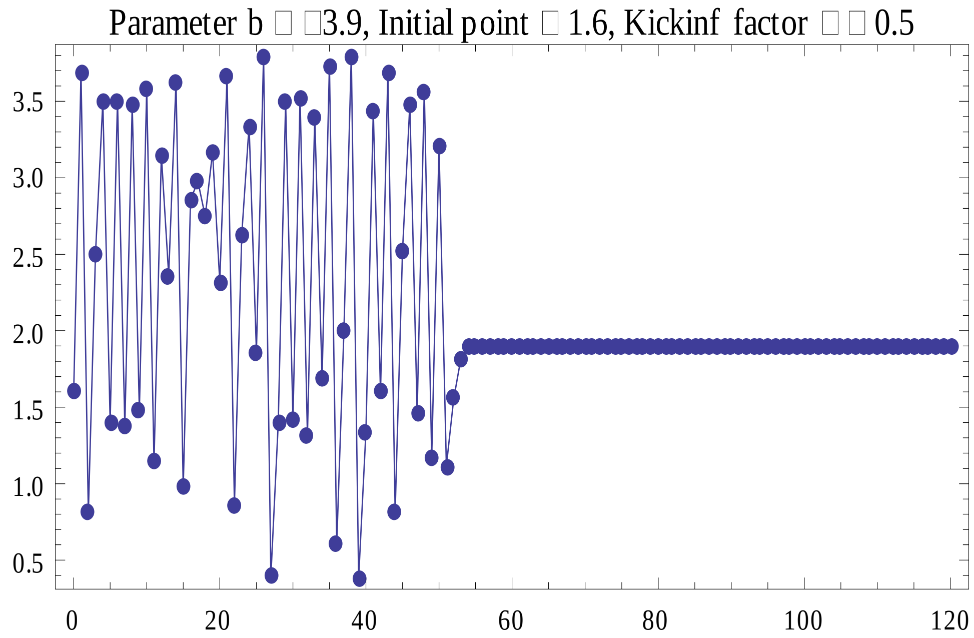

Fig. 1.3 Time series graph for parameter $b = -3.9$

Again for $q = 2$ and $b = -3.9$ the control curve $H2(x)$ is drawn in figures 1.4. Here also the range is restricted to $-1 < Hq(x^*) < 1,\ q = 2$ and the figure 1.5 shows that we can stabilize orbits of period-two at point x^* only in three ranges. For this purpose the kicking factor is found as $\lambda \approx 0.868056$, taking the given point as $x^* = 3.3$.

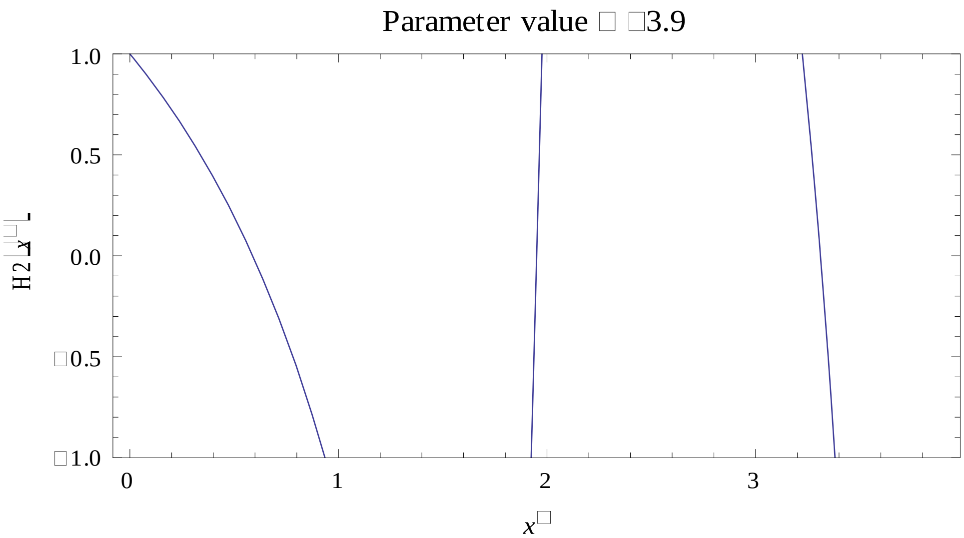

Fig. 1.4 Control curves for parameter $b = -3.9$

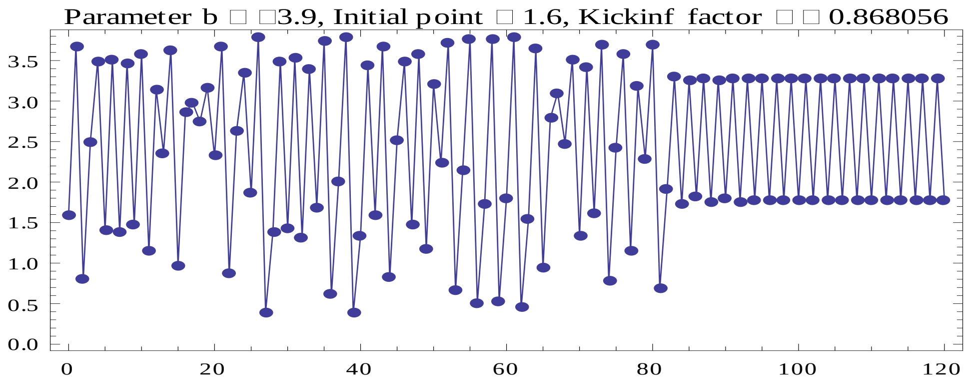

Fig. 1.5 Time series graph for parameter $b = \text{-}3.9$

Similarly for $q = 3{,}4$ and $m = 0.815$ the control curves $H3(x)$, $H4(x)$ are drawn in figures 1.6 and 1.7.

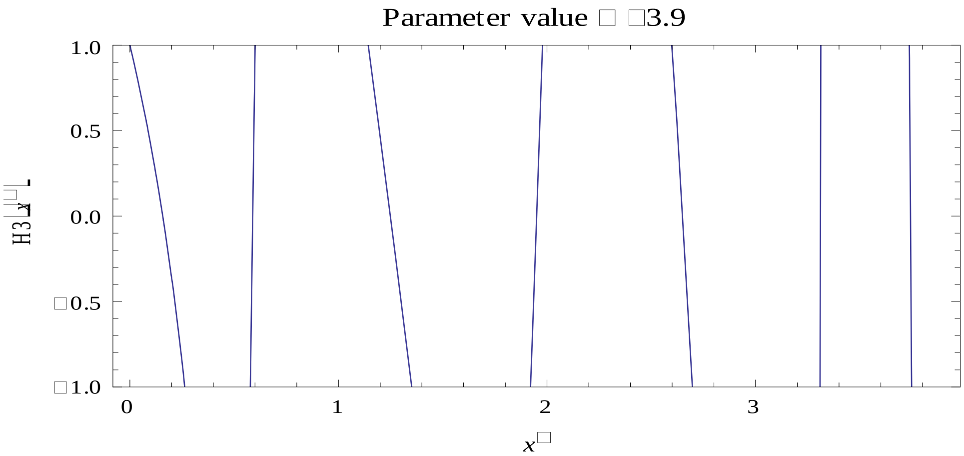

Fig. 1.6 Control curves for parameter $b = \text{-}3.9$

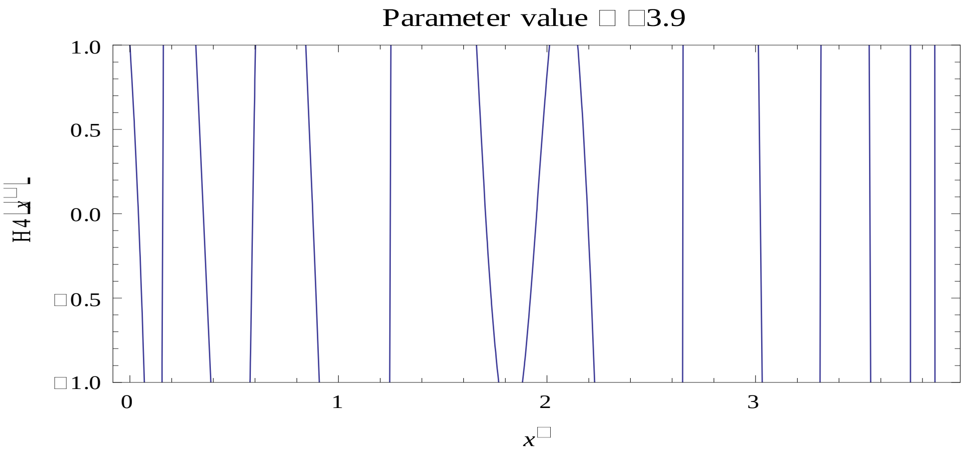

Fig. 8.7 Control curves for parameter $b = \text{-}3.9$

These figures indicate that there are 7 and 15 narrow ranges of x^* values of periods 3 and 4 respectively. We note that the control ranges are getting smaller and smaller as the periodicity increase. Lastly, $q = 4$ and $x^* = 1.11892.$ stabilize orbits of period-4 with the kicking factor as shown in the figure 1.8.

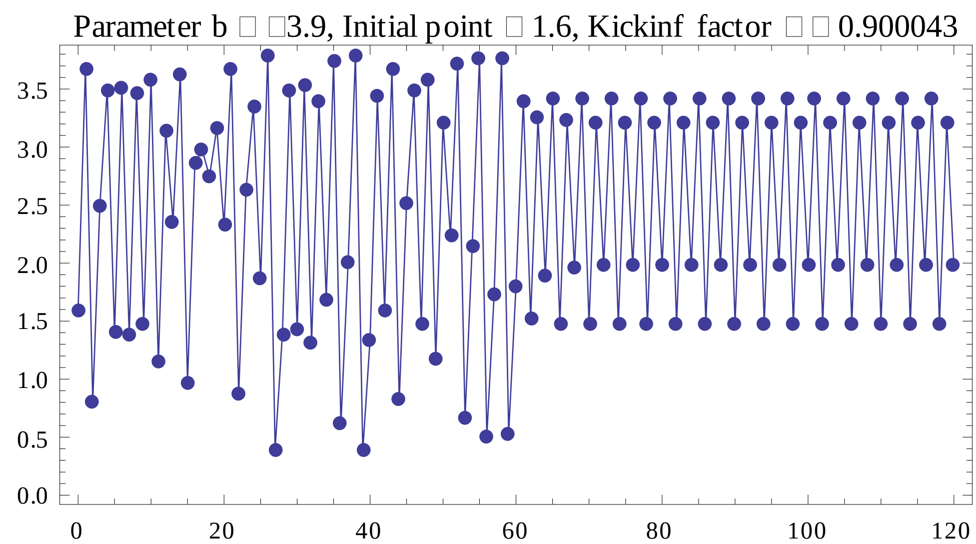

Fig. 1.8 Time series graph for parameter $b = -3.9$

4. Conclusion

By the above technique, we can conclude that an irregular orbit of any period can be controlled by the above technique. But in practice, chaos control always deals with periodic orbits of low periods, say $q = 1,2,3,4,5$.

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Bibliography

[1] E. Ott, C. Grebogi, J. A. Yorke, Controlling Chaos, Phys. Rev. Lett. 64 (1990) 1196-1199
[2] G. Chen and X. Dong, From Chaos to Order: Methodologies, Perspectives and Applications, World Scientific, 1998
[3] J. M. Gutierrez and A. Iglesias, Mathematica package for analysis and control of chaos in nonlinear systems, American Institute of Physics, 1998
[4] J. Güémez and M.A. Matias, Phys. Lett. A 181 (1993) 29
[5] M.A. Matias and J. Güémez, Phys. Rev. Lett. 72 (1994) 1455
[6] Marco Sandri, Numerical calculation of Lyapunov Exponents, University of Verona, Italy.
[7] N.P. Chau, Controlling Chaos by period proportional pulses, Phys. Lett. A, 234 (1997), 193-197.
[8] N. Das, R. Sharmah and N. Dutta, Period doubling bifurcation and associated universal properties in the Verhulst population odel, International J. of Math. Sci. & Engg. Appls. , Vol. 4 No. I (March 2010), pp. 1-14
[9] T. Kapitaniak, Controlling Chaos – Theoretical and Practical Methods in Nonlinear Dynamics, Academic Press, New York, 1996.
[10] T Basar "Systems & Control: Foundations & Applications ",University of Illinois at Urbana-Champaign.
[11] T. K. Dutta and N. Das, Period Doubling Route to Chaos in a Two-Dimensional Discrete Map, Global Journal of Dynamical Systems and Applications, Vol.1, No. 1, pp. 61-72, 2001