Solution of Fuzzy Games with Interval Data Using Approximate Method

Dr.C.Loganathan
Department of Mathematics
Maharaja Arts and Science College
Coimbatore – 641407, India

M.S.Annie Christi
Department of mathematics
Providence College for Women,
Coonoor-643104, India

Abstract
In this paper, we evaluate the value of the fuzzy game matrix with interval data using the approximate method. This method gives an approximate solution for the value of the game and the true value can be determined to any degree of accuracy. This method assumes that each player will play in such a manner so as to maximize the expected gain or to minimize the expected loss.

Keywords: Approximate method, Interval data, matrix game, value of the game.

1. Introduction
The solution methods of interval matrix games are studied by many authors. Most of the solution techniques are based on linear programming methods for interval numbers. We present, in this paper an approximate method for solving 3x3 or higher interval matrix games and illustrate this with a numerical example.The algebraic method is generally adopted to solve the game for which the graphical method cannot be applied, but the games with large payoff matrices are extremely tiresome to solve by algebraic method. For such large games the iterative method is very powerful to hand as well as machine computations.

1.1 Interval Numbers:
An interval number is of the form $\bar{a}$ = [L (a), U (a)] = {x $\in$ R: L (a) $\le$ x $\le$ U (a)}.
If L (a) = U (a) then $\bar{a}$ is a real number. Mid point of $\bar{a}$ m ($\bar{a}$) and range of $\bar{a}$ r ($\bar{a}$) is defined as

m ($\bar{a}$) = $\dfrac{L(a)+U(a)}{2}$ and r ($\bar{a}$) = $\dfrac{U(a)-L(a)}{2}$ respectively.

The basic operations are defined as follows:

Let $\bar{a}$ = [L (a), U (a)], $\bar{b}$ = [L (b), U (b)], be two intervals. Then,

(i) $\bar{a}+\bar{b}$ = [L (a) +L (b), U (a) +U (b)],

(ii) $\bar{a}$ _ $\bar{b}$ = [L (a)-L (b), U (a)-U (b)],

(iii) $\bar{a}$ $\bar{b}$ =[min{L(a).L(b), L(a).U(b), U(a).L(b), U(a).U(b), },
max{ L(a).L(b), L(a).U(b), U(a).L(b), U(a).U(b),}],

(iv) λ $\bar{a}$ = [λ L(a), λ U(a)] if λ > 0.
= [λ U (a), λ L (a)] if λ <0.
Similarly the other binary operations are defined.

1.2 Comparison Of Intervals:
Comparison of intervals is very important problem in interval analysis. Comparison on interval numbers is given by Nayak and Pal[8].

Disjoint Intervals:
Let $\bar{a}$ = [L (a), U (a)], $\bar{b}$ = [L (b), U (b)], be two intervals. Then $\bar{a}<\bar{b}$ if

U (a) < L (b).

Example: Let $\bar{a}$ = [2, 5] and $\bar{b}$ = [6, 8] then $\bar{a} < \bar{b}$.

Nested Sub-Intervals:

Let $\bar{a}$ = [L (a), U (a)], $\bar{b}$ = [L (b), U (b)], be two intervals. If L(a)≤L(b)≤U(b)≤U(a), then $\bar{b}$ is contained in $\bar{a}$,which is the concept of inclusion. The extension of the set inclusion here only describes the condition that the interval $\bar{b}$ is nested in $\bar{a}$ but it cannot order $\bar{a}$ and $\bar{b}$ in terms of its value.

2. Matrix Games:

Given a matrix game A= ($\overline{a_{ij}}$), the element $\overline{a_{ij}}$ is called a saddle interval of A if

$$\overline{a_{ij}} \leq \overline{a_{il}} \quad \forall \text{ l=1, 2…n}$$

$$\overline{a_{ij}} \geq \overline{a_{kj}} \quad \forall \text{ k= 1, 2…m.}$$

(i.e.) the element $\overline{a_{ij}}$ is simultaneously a minimum interval in its row and a maximum interval in its column.

When there is no saddle interval in a matrix game, an iterative method can be used to get an approximate solution

3. Procedure:

STEP: 1

Let the player A arbitrarily selects a row which will be the superior one of his other strategies and places it under the matrix.

STEP: 2

Player B examines this row and chooses a column corresponding to the smallest interval in the row and places to the right of the matrix.

STEP: 3

Player A now examines this column and chooses a row corresponding to the largest interval in this column. This row is added to the row last chosen and is placed under the previous row chosen.

STEP: 4

Player B then chooses the column corresponding to the smallest number in the new row and adds this column to the column chosen. In case of a tie, the player will select that row or column which is different from his last choice.

STEP: 5

The procedure is repeated for a finite number of iterations.

STEP: 6

The smallest elements in each succeeding row with the largest elements in each succeeding column are encircled.

STEP: 7

The approximate strategies after a certain number of iterations are found by dividing the number of encircled intervals in each row or column by total number of iterations.

STEP: 8

The upper bound for the value of the game can be determined by dividing the largest interval in the last column by the total number of iterations. Likewise the lower bound can be determined by dividing the smallest element in the last row by the total number of iterations. Thus the approximate value of game and the optimal strategies are evaluated.

4. Illustration:

$$\begin{bmatrix} [-4,-1] & [0,1] & [3,4] \\ [5,6] & [-2,0] & [1,2] \\ [0,1] & [3,6] & [-1,2] \end{bmatrix}$$

[0,1] [3,5] [6,9] [9,13] [5,12] [8,16] [4,15] [4,16] [4,17] [4,8] $q_1 = \frac{2}{10}$

[-2,0] [-1,2] [0,4] [1,6] [6,12] [7,14] [12,20] [10,20] [8,20] [6,20] $q_2 = \frac{3}{10}$

[3,6] [2,8] [1,10] [0,12] [0,13] [-1,15] [-1,16] [2,22] [5,28] [8,34]

$q3 = \frac{5}{10}$

[5, 6]	[-2, 0]	[1, 2]
[5, 7]	[1, 6]	[0, 4]
[5, 8]	[4, 12]	[-1, 6]
[5, 9]	[7, 18]	[-2, 8]
[1, 8]	[7, 19]	[1, 12]
6, 14]	[5, 19]	[2, 14]
[2, 13]	[5, 20]	[5, 18]
[7, 19]	[3, 20]	[6, 20]
[12, 25]	[1, 20]	[7, 22]
[12, 26]	[4, 26]	[6, 24]

$p_1 = \frac{2}{10}$ $p_2 = \frac{4}{10}$ $p_3 = \frac{4}{10}$

The strategies are approximate values. Hence they were taken as fuzzy strategies so that we have the expected pay-offs as

p_1[-4,-1]+ p_2[5, 6] + p_3[0, 1] = $[\frac{12}{10}, \frac{22}{10}]$

$$p_1[0, 1] + p_2[-2, 0] + p_3[3, 4] = [\frac{4}{10}, \frac{18}{10}]$$

$$p_1[3, 4] + p_2[1, 2] + p_3[-1, 2] = [\frac{6}{10}, \frac{24}{10}]$$

$$q_1[-4, -1] + q_2[0, 1] + q_3[3, 4] = [\frac{7}{10}, \frac{21}{10}]$$

$$q_1[5, 6] + q_2[-2, 0] + q_3[1, 2] = [\frac{9}{10}, \frac{22}{10}]$$

$$q_1[0, 1] + q_2[3, 6] + q_3[-1, 2] = [\frac{4}{10}, \frac{28}{10}]$$

These values lie around 4.5. Hence the value of the fuzzy game is approximately equal to $\bar{v}$ = [3, 6]. It lies between [-4, -1] and [5, 6] with R($\bar{v}$) = 4.5.

Conclusion:

An approximate method is applied to find out the value of the game matrix with interval data.Defuzzification is done to convert the fuzzy values to crisp values. These ideas can be applied to other fuzzy problems

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