Analysis of Skew Bridges Using Computational Methods

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Abstract

In spite of increases in computing power, analysis of skew bridge deck has not changed to the same extent. Therefore, there is a need for more research to study the skew bridges using different computational methods. Grillage analyze is a fast and simpler approach compared to the finite element method, and has been used by engineers to analyses bridge deck over a long time. On the other hand the finite element method is thought to be better method for the slab analysis because of its capability to represent the complex geometry of the structure more realistically. In this present study, a bridge deck consists of beam and slab is defined and modeled using grillage and finite element method. The effect of grid spacing on different skew angles on same-span of reinforced concrete bridges using the finite-element method and grillage analogy method is compared. Maximum reactions force, deflection, bending and torsional moments is calculated and compared for both analysis methods. A total of nine different grid sizes (4 divisions to 12 divisions) have been studied on skew angles 30°, 45° and 60° to determine the most appropriate and efficient grid size. It is observed that finite element method (FEM) and Grillage method results are always not similar for every grid size. Bending moment calculated by using FEM overestimates the results obtained by grillage analysis for larger grid sizes. Torsion moment behavior shows reverse of bending moment and difference between reaction values of grid sizes between two methods decreases as skew angle increases. FEM gives lesser variations of bending and torsional moment with the change of grid sizes than Grillage one. Deflection doesn't vary much on the change of the grid sizes. The appropriate grid size is estimated for this narrow and long bridge is seven divisions whose ratio of transverse to longitudinal grid spacing is about 2.

Keywords: Skew slab, FEM, Grillage analysis, Grid size

1. Introduction

Generally, grillage analysis [14] is the most common method used in bridge analysis. In this method the deck is represented by an equivalent grillage of beams. The finer grillage mesh, provide more accurate results. It was found that the results obtained from grillage analysis compared with experiments and more rigorous methods are accurate enough for design purposes. The other method used in modeling the bridges is the finite element method. The finite element method is a well known tool for the solution of complicated structural engineering problems, as it is capable of accommodating many complexities in the solution. In this method, the actual continuum is replaced by an equivalent idealized structure composed of discrete elements, referred to as finite elements, connected together at a number of nodes.

In the skew bridges, the effects of skew on the response of completed structures have been well documented [1, 7, 9], with effects being shown to be more significant for skew angles greater than 30◦. Critical values for vertical deflections and bending moments within in-service skewed bridges have been shown to be lower when compared against those in similar right bridges. Conversely, torsional rotations, shears and moments have been shown to be larger for skewed bridges. In addition, studies have also demonstrated that interaction between main support girders and transverse bracing members (diaphragms and cross frames) influences skewed bridge load distribution due to an increase in torsional rotations at certain sections of the longitudinal girders. Additional work has shown that the magnitude of torsional shear rotations at skewed bridge supports is largest at the obtuse corners [2, 3].

While a number of studies [4, 5] dedicated to the response of in-service skewed bridges have been completed, as presented above, there are few studies that focus on the behavior of skewed bridges during construction. There has been a lack of research studying the effects of the disproportionate distribution of dead loads on the superstructure during construction. The reactions at the obtuse angled end of slab support are larger than the other end, the increase in value over the average value ranging from 0 to 50 per cent for skew angles 20 to 50 degrees. The bearing reactions tend to change to uplift in the acute angle corners with increase in skew angle.

For skew angle lesser than 15°, an approximate method [12] of design may be adopted as, it is reasonably correct. According to this method, bending moments are calculated as for a right bridge of span centre to centre of supports measured parallel to the centre line of the roadway. The main reinforcement is provided in a direction parallel to traffic. Cross reinforcement, which is usually taken as 0.2per cent of effective cross section of the slab is placed parallel to the supports. For skew angles greater than 15°, a more rigorous analysis is desirable, but it is complicated. Analytical and experimental methods have been attempted. A number of methods like FEM, grillage analogy method are used to analyses bridge decks. However, grillage analogy method seems to be general, simple, sufficiently accurate, easy to comprehend and convenient to work even on easily available Personal Computers [8]. For the analysis, using STAAD PRO 2007, in each case the skew bridge deck is discretized in grid of interconnected beams in case of Grillage analogy and grid of interconnected plates in FEM [13].

2. Analysis of Deck Slab Bridge

The effect of different grid spacing on the behavior of skewed bridges under dead & live load (70R) [6] using grillage analogy method and FEM are analyzed and the appropriate grid size for the various skewed slab bridge were found.

The skew slab bridges having 30°,45° and 60° skew angle of carriage way 7.5m and skew span 12.11 m is considered for present analysis i.e. reactions, bending moment, deflection under dead and live load with different cases of grid spacing (4 to 12 divisions). Slab thickness is of uniform depth 750 mm.

A skew slab bridge is supported on five isolated bearings at each end, adoption of neoprene bearings of spring stiffness of 40t/mm at each support point. The flexibility of support is considered in analysis.

For the analysis, using STAAD PRO 2007, in each case the whole slab is discretized in grid of interconnected beams in case of Grillage analogy and grid of interconnected plates in FEM .For sake of convenience in analysis and also comparison of results, the spacing of transverse grid lines are kept constant and only longitudinal grid lines changes to study the variation of deflection, reaction, bending moment [10, 11] for different mesh size. Rectangle sections of all grillage beams are considered with constant thickness of 750 mm.

2.1 Grid Pattern

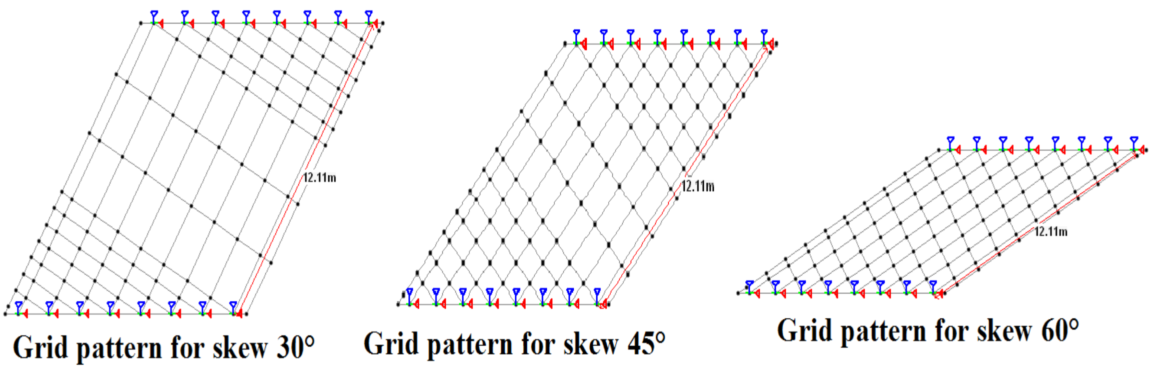

Figure 1: Grid pattern for different skew

All different skewed angle bridges have same grid pattern and same span. STAAD PRO 2007 is the software used for analysis.

a) Grillage analogy i.e. given decking system is converted into series of interconnected beams such that given prototype bridge bridge deck and the equivalent grillage of beams are subjected to identical deformations under loading.

b) Finite Element Method consists of solving the mathematical model which is obtained by idealizing a structure as an assemblage of various discrete two or three dimensional elements connected to each other at their nodal points, possessing an appropriate number of degrees of freedom.

3. Results and discussions

3.1 Analysis of skew bridge for Skew angle 30^{0}

A skew slab bridge having 30° skew angle supported on five isolated bearings at each end is analyzed for different grid spacing (i.e., 4 to 12 divisions) is analyzed and presented in the fig 2 and 3 and Table 1.

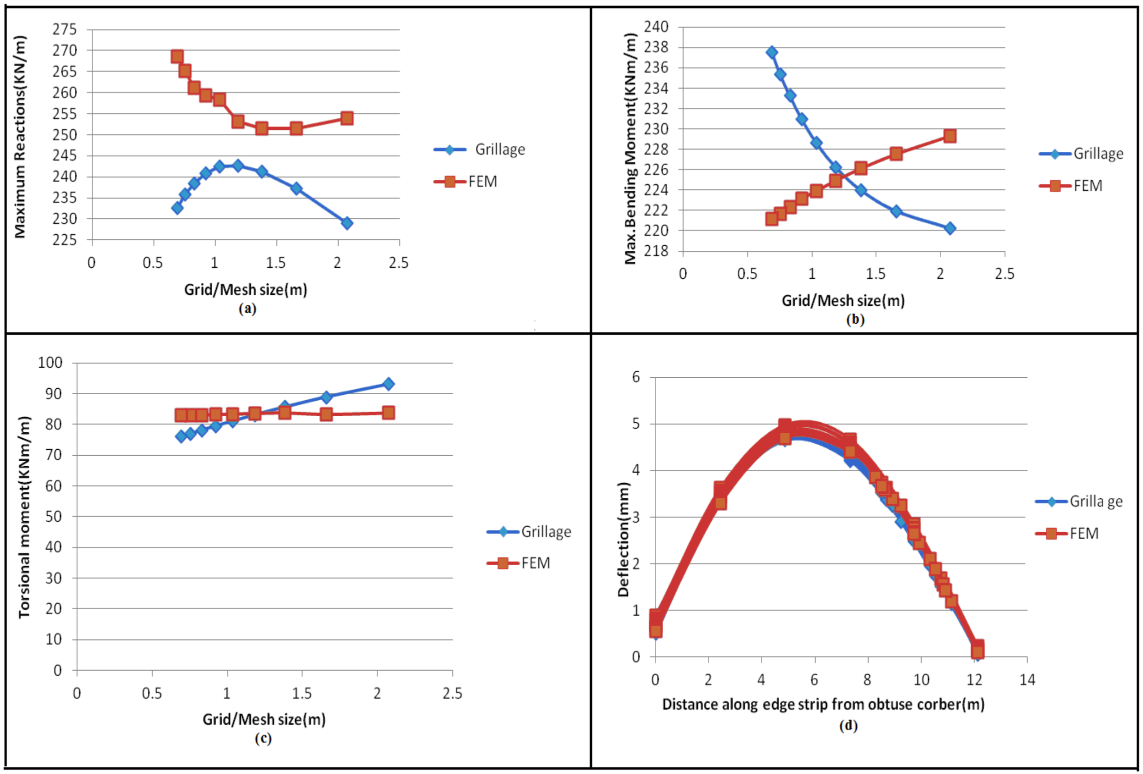

Figure 2: Skew Slab Bridge having 30° skew angle, considering only dead load

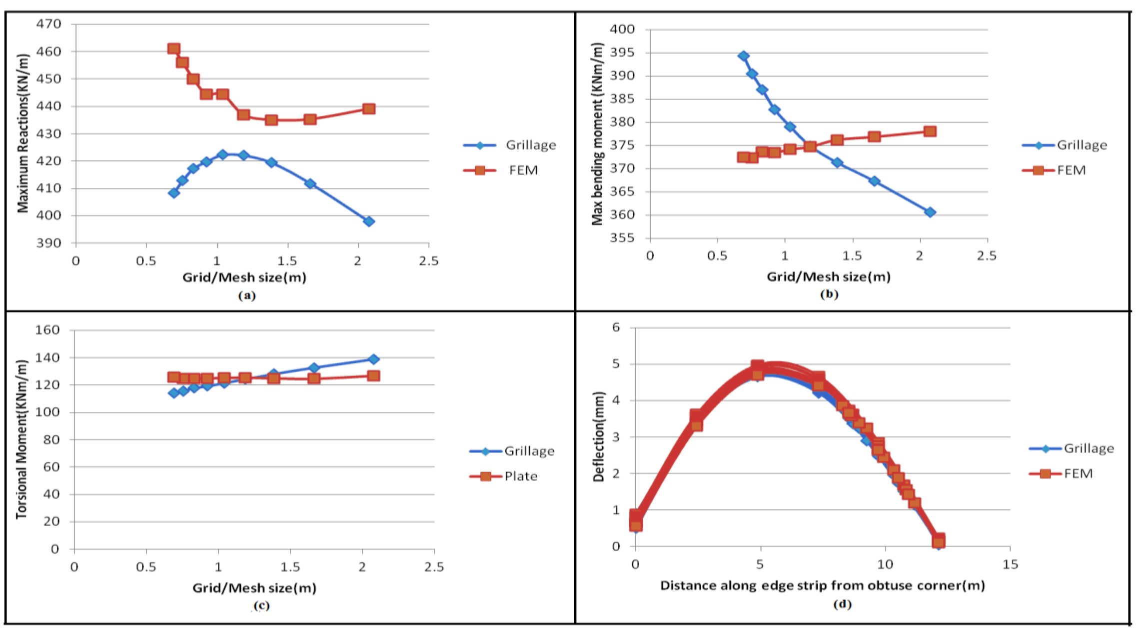

Figure 3: Skew Slab Bridge having 30° skew angle, considering dead load plus live load

It is observed from analysis that, maximum reaction results of FEM overestimate the Grillage results. Grid corresponding to 5, 6 and 7 divisions gives same approx results (<5%). Maximum bending moment results of FEM overestimates grillage results in 4, 5 and 6 divisions. Later, grillage overestimates FEM results and they intersect in between 6 and 7 divisions. So, grid size corresponding to 6 and 7 divisions gives almost same results. Unlike maximum bending moment results, torsional moment results were more in Grillage in 4, 5 and 6 divisions and later it become more in FEM results. Grid corresponding to 6, 7 and 8 divisions gives same (0-2%) results and grid corresponding to 7 divisions gives more accurate result.

Deflection results are almost same in any of grid spacing. i.e., about only 0.2mm changed from 4 divisions to 12 divisions. Although grid corresponding 9 and 10 division gives same results in both methods, but 0.2mm difference corresponding to 6 and 7 divisions also accepted, and it comes closer at 6 and 7 divisions.

As in dead load, in combination of both dead and live load, grid corresponding to 6 and 7 divisions mainly gives closer maximum reactions results in both methods. Grid corresponding to 6, 7 and 8 divisions gives lesser difference in bending moment between two methods. As in dead load case, combination of both dead and live load case grid corresponding to 7 divisions gives same torsional moment in both methods. Deflection results are almost same in any of grid spacing. i.e., about only 0.2mm changed from 4 divisions to 12 divisions. Deflection change is negligible in both methods as in dead load case.

Table 1: Skew Slab Bridge having 30° skew angle

30°Skew Angle		Division	4	5	6	7	8	9	10	11	12
		Grid size(m)	2.074	1.659	1.382	1.185	1.037	0.921	0.829	0.754	0.691
Dead Load Only	Reactions (kN/m)	Grillage	228.9	237.2	241.3	242.7	242.3	240.8	238.5	235.7	232.6
		FEM	253.9	251.4	251.6	253	259.2	258.3	261.2	265.1	268.6
		% diff.	**9.84**	**5.67**	**4.09**	**4.08**	**6.51**	**6.77**	**8.7**	**11.07**	**13.39**
	Bending Moment (kNm/m)	Grillage	220.2	221.9	223.9	226.2	228.6	230.9	233.2	235.3	237.5
		FEM	229.3	227.5	226.1	224.9	223.9	223.1	222.3	221.6	221.1
		% diff.	**3.95**	**2.47**	**0.95**	**-0.58**	**-2.09**	**-3.51**	**-4.93**	**-6.17**	**-7.41**
	Torsional Moment (kNm/m)	Grillage	93	88.8	85.6	83	81	79.4	78	76.9	75.9
		FEM	83.7	83.2	83.8	83.6	83.3	83.2	83.1	83	82.9
		% diff.	**-11.1**	**-6.72**	**-2.14**	**0.62**	**2.77**	**4.53**	**6.03**	**7.25**	**8.4**
	Deflection (mm)	Grillage	4.68	4.65	4.65	4.66	4.68	4.71	4.745	4.77	4.80
		FEM	4.97	4.94	4.88	4.84	4.80	4.72	4.75	4.72	4.70
		% diff.	**6.23**	**5.06**	**4.78**	**3.57**	**2.31**	**0.12**	**0.02**	**-1.03**	**-2.14**
Dead Load plus Live Load	Reactions (kN/m)	Grillage	397.8	411.7	419.4	422	422.2	419.7	417.2	412.8	408.3
		FEM	439	435.2	435	436.8	444.4	444.3	449.9	455.9	461.1
		% diff.	**9.37**	**5.39**	**3.58**	**3.38**	**4.99**	**5.54**	**7.27**	**9.44**	**11.46**
	Bending Moment (kNm/m)	Grillage	360.5	367.2	371.2	374.8	379	382.7	387	390.4	394.2
		FEM	378	376.8	376.1	374.6	374.1	373.4	373.5	372.3	372.3
		% diff.	**4.61**	**2.54**	**1.3**	**-0.04**	**-1.31**	**-2.47**	**-3.59**	**-4.87**	**-5.88**
	Torsional Moment (kNm/m)	Grillage	138.5	132.4	127.8	124.2	121.4	119.1	117.8	115.7	114.3
		FEM	126.4	124.5	124.8	125.3	125	124.7	124.6	124.7	125.7
		% diff.	**-9.55**	**-6.35**	**-2.44**	**0.869**	**2.93**	**4.51**	**5.48**	**7.26**	**9.12**
	Deflection (mm)	Grillage	7.62	7.57	7.57	7.57	7.61	7.64	7.69	7.73	7.79
		FEM	8.02	8.07	7.98	7.89	7.82	7.7	7.71	7.67	7.63
		% diff.	**5.05**	**6.22**	**5.2**	**3.98**	**2.72**	**0.73**	**0.22**	**-0.09**	**-2.05**

From the analysis of 30^0 skew angle bridge (both dead load and combine load cases), we can conclude from the discussions that, mostly in 6 and 7 divisions, the difference of analysis results between them as well as among two methods (Grillage and FEM) are less, and most accurate and closer results given by grid corresponding to 7 grid. Ratios of transverse to longitudinal grid spacing in 7 and 6 divisions are 2.05 & 1.765 respectively.

3.2 Analysis of skew bridge for Skew angle 45^0

A skew slab bridge having 45° skew angle supported on five isolated bearings at each end is analyzed for different grid spacing (i.e., 4 to 12 divisions). Ratio of transverse grid spacing to longitudinal grid spacing = 1.83.

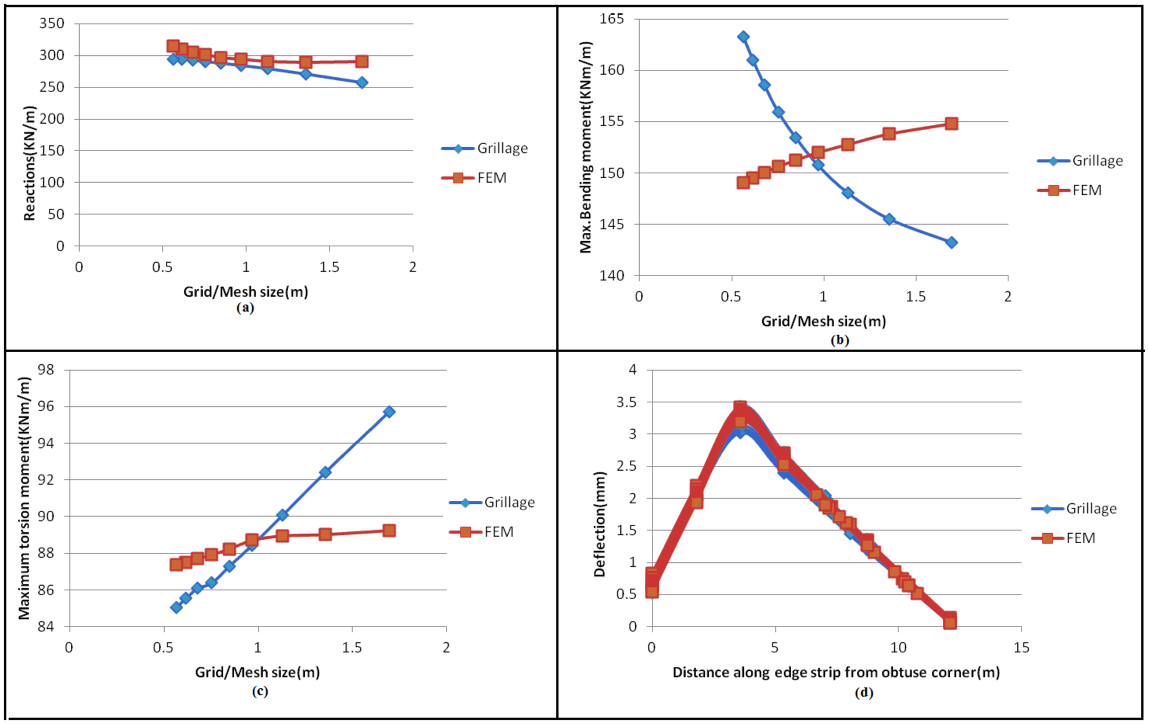

Figure 4: skew Slab Bridge having 45° skew angle, considering only dead load

It is to be noted that difference in maximum reaction between two methods, corresponding to 6, 7, 8 and 9 divisions is less, i.e. about 2-3 %. Deflection results are almost same in any of grid spacing. i.e., deflection change is negligible in both methods. A grid corresponding to 7 and 8 gives only 0.2 mm difference between grillage and FEM results.

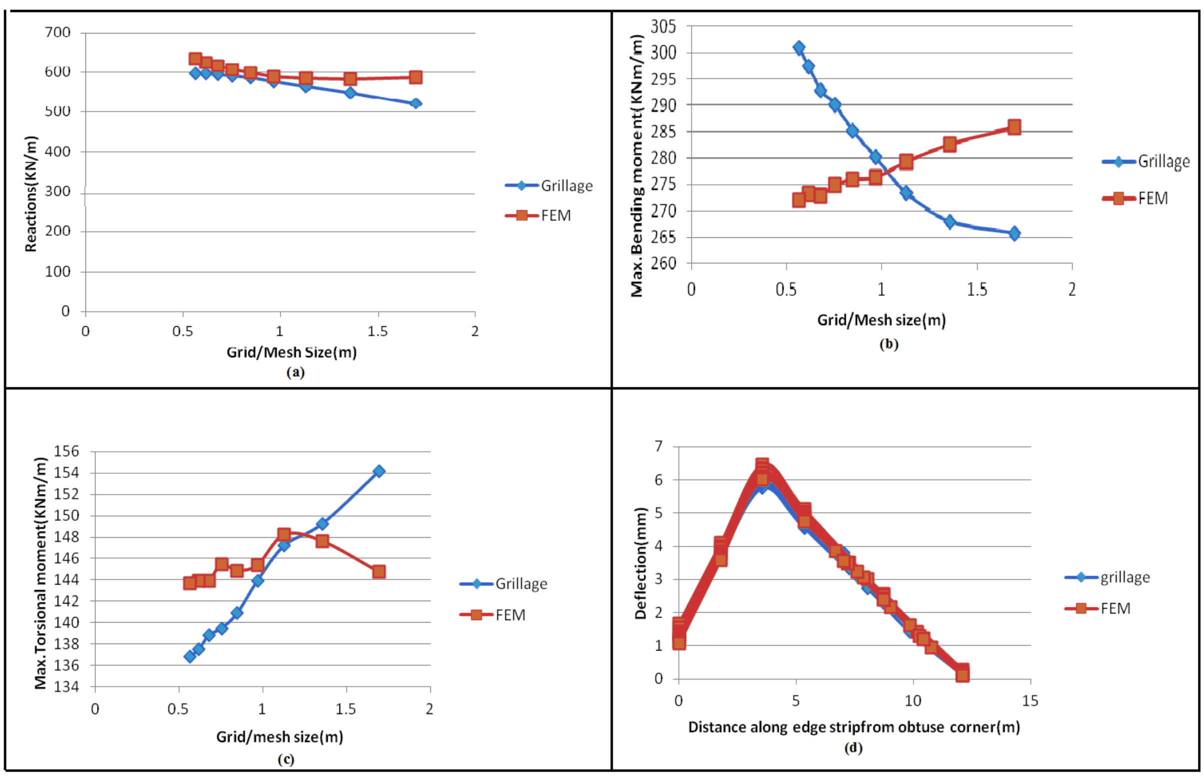

Figure 5: skew Slab Bridge having 45° skew angle, considering dead load plus live load

As in dead load case, combination of dead and live loads, 7, 8 and 9 divisions give closer maximum reaction results between Grillage & FEM method. In dead load plus live load case, FEM overestimates maximum bending moment results over grillage in 4 and 5 divisions, later it underestimates grillage method. It is to be noted that grid corresponding to 6 and 7 gives approximately same results in both method. In combination of dead and live load, grillage over estimates FEM results in 4 and 5 divisions, later it underestimates FEM results. Grid corresponding to 5, 6 and 7 divisions gives lesser difference in results of two methods. We can mark that there is not such variations in torsional moment, in FEM it appears to be constant.

Deflection change due to live load in skew angle 45^0 is not negligible as that of in dead load case. There is some difference in between two methods compared to other angles. Maximum deflection in both methods becomes same in 9 and 10 divisions. However, previous discussions finds 7 and 8 division appropriate, mainly 7 divisions, which gives 0.4 mm of deflection change between two methods due to combination of dead and live load, it is acceptable. Most of analysis results are same in 6, 7 and 8 but 7 division always gives same results. So, here also 7 divisions we found appropriate. Ratio of transverse grid spacing to longitudinal grid spacing = 1.83

Table 2: skew Slab Bridge having 45° skew angle

45°Skew Angle	Division		4	5	6	7	8	9	10	11	12
	Grid size(m)		1.693	1.354	1.129	0.967	0.846	0.752	0.677	0.615	0.564
Dead Load Only	Reactions (kN/m)	Grillage	257.9	270.7	279.1	284.6	288.4	290.8	292.4	293.4	293.5
		FEM	290.5	289.3	290.2	293.5	296.8	301.1	305.4	310.2	314.4
		% diff.	**11.20**	**6.443**	**3.85**	**3.01**	**2.84**	**3.44**	**4.27**	**5.39**	**6.64**
	Bending Moment (kNm/m)	Grillage	143.2	145.5	148.0	150.7	153.4	155.9	158.5	160.9	163.2
		FEM	154.8	153.8	152.7	151.9	151.2	150.6	150.0	149.5	149.0
		% diff.	**7.49**	**5.40**	**3.08**	**0.80**	**-1.46**	**-3.53**	**-5.71**	**-7.66**	**-9.52**
	Torsional Moment (kNm/m)	Grillage	95.7	92.4	90.0	88.4	87.3	86.3	86.0	85.5	85.0
		FEM	89.2	89.0	88.9	88.7	88.2	87.9	87.7	87.5	87.3
		% diff.	**-7.25**	**-3.81**	**-1.26**	**0.29**	**1.05**	**1.75**	**1.86**	**2.25**	**2.66**
	Deflection (mm)	Grillage	3.02	3.03	3.07	3.13	3.19	3.24	3.31	3.37	3.43
		FEM	3.42	3.36	3.33	3.29	3.27	3.24	3.23	3.21	3.19
		% diff.	**11.64**	**9.82**	**7.62**	**5.12**	**2.50**	**0.03**	**-2.63**	**-5.10**	**-7.47**
Dead Load plus Live Load	Reactions (kN/m)	Grillage	522.3	548.3	565.5	575.9	586.2	590.5	594.7	597.0	596.2
		FEM	585.5	581.9	583.8	588.7	597.8	605.3	615.3	624.6	632.8
		% diff.	**10.80**	**5.78**	**3.12**	**2.17**	**1.94**	**2.43**	**3.33**	**4.42**	**5.78**
	Bending Moment (kNm/m)	Grillage	265.6	267.9	273.3	280.1	285.2	290.2	292.8	297.4	300.9
		FEM	285.8	282.6	279.3	276.3	275.9	276.8	272.8	273.1	271.9
		% diff.	**7.06**	**5.19**	**2.14**	**-1.35**	**-3.36**	**-5.59**	**-7.33**	**-8.89**	**-10.6**
	Torsional Moment (kNm/m)	Grillage	154.1	149.2	147.2	143.9	140.8	139.4	138.8	137.4	136.8
		FEM	144.7	147.6	148.2	145.4	144.8	145.4	143.9	143.9	143.7
		% diff.	**-6.49**	**-1.09**	**0.69**	**1.00**	**2.74**	**4.09**	**3.53**	**4.45**	**4.78**
	Deflection (mm)	Grillage	5.75	5.75	5.85	5.89	5.99	6.05	6.16	6.23	6.40
		FEM	6.45	6.34	6.31	6.20	6.17	6.09	6.07	6.01	6.00
		% diff.	**10.85**	**9.16**	**7.39**	**5.08**	**2.89**	**0.73**	**-1.56**	**-3.75**	**-6.76**

3.3 Analysis of skew bridge for Skew angle 60^0

A skew slab bridge having 60° skew angle supported on five isolated bearings at each end is analyzed for different grid spacing (i.e., 4 to 12 divisions). It can be marked that as skew angle increases, differences in maximum reaction values between two methods becomes less. In bending moment case, FEM overestimates grillage results in 4, 5 and 6 divisions. Later, it underestimates grillage results and 7, 8 and 9 divisions give almost same results.

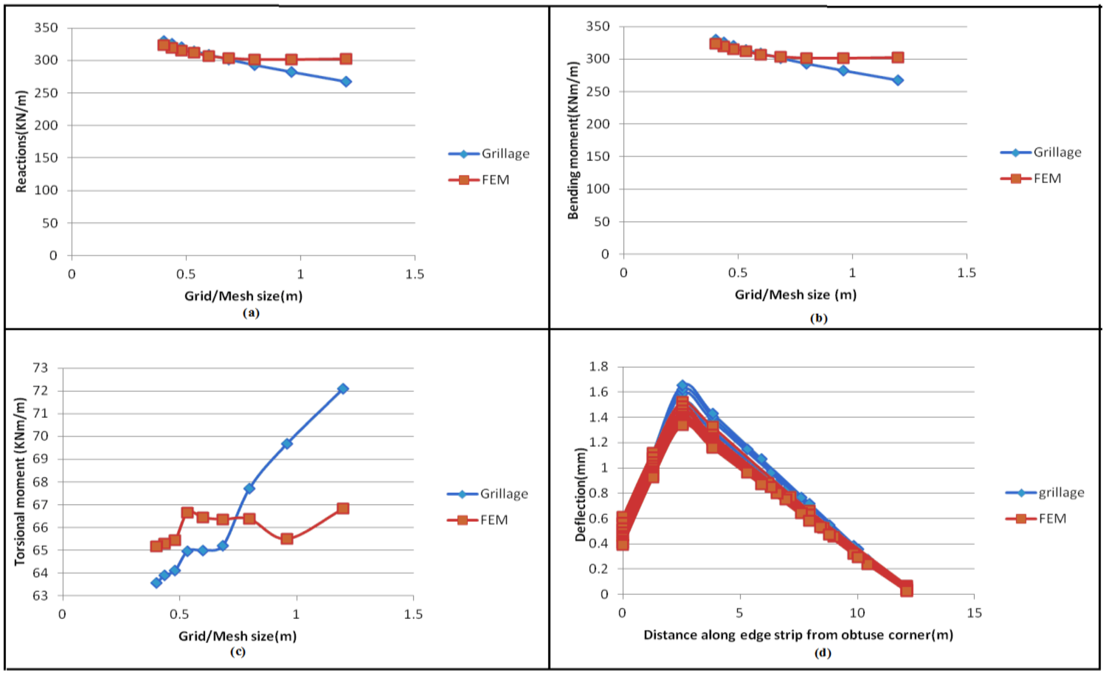

Figure 6: skew Slab Bridge having 60° skew angle, considering only dead load

In bending moment case FEM overestimates grillage results in larger grid sizes and vice versa happens in smaller grid sizes. Grid corresponding to 6 and 7 divisions gives closer results in both methods. It can also be noted that as skew angle increases, more closer is the difference as in reactions above. From above table and graph, we can see that there is not so difference in torsion moment between two methods in any of grid spacing and grillage overestimates FEM results and from 7 divisions it underestimates FEM result. As per above results, 6, 7, 8, 9 and 10 divisions any one can be taken as difference is negligible and also gives almost same value. As similar to other cases, deflection is same everywhere, its change is negligible. Grids corresponding to 6 and 7 divisions give same value in both methods. As in dead load case, same in combinations of dead and live load case, maximum reactions have very less difference between two methods as skew angle increases. Grid corresponding to 7, 8, 9, 10 and 11 divisions gives lesser difference of results between two methods. As in dead load case, same happens in combinations of dead and live load also and grids corresponding to 6 and 7 divisions give almost same value in both methods.

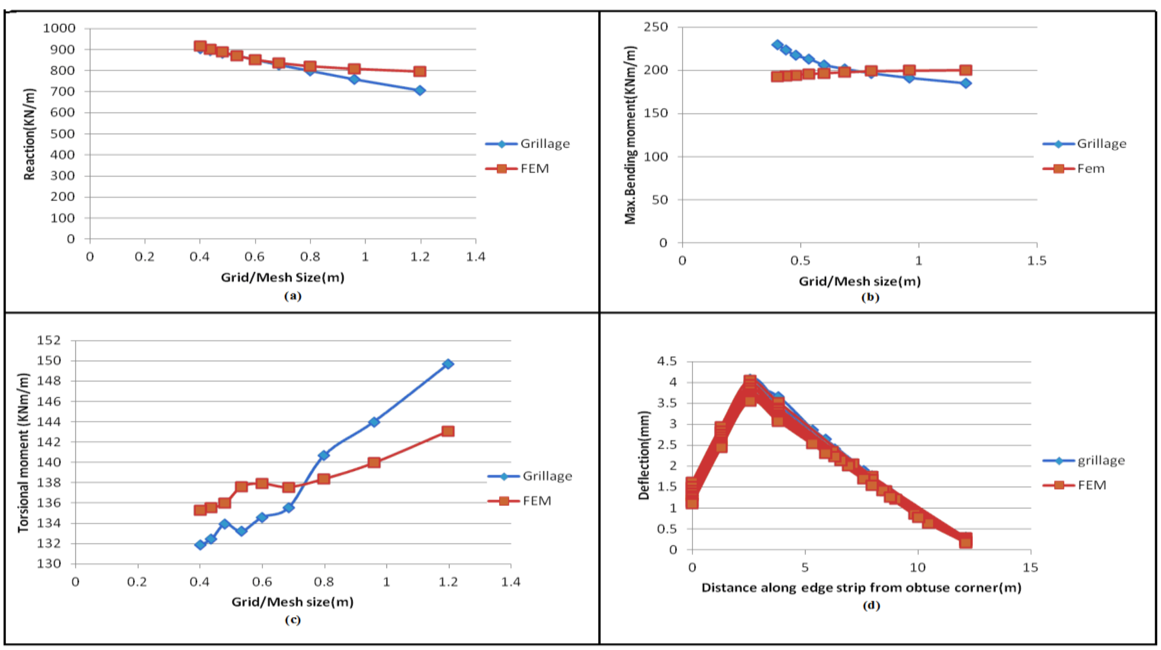

Figure 7: skew Slab Bridge having 60° skew angle, considering dead load plus live load

In torsional moment case grillage overestimates FEM results in larger grid sizes, vice versa happens in lesser grid sizes. Unlike, dead load case, in combination of dead and live load case, considerable difference between results is there. Grids corresponding to 5, 6 and 7 division give lesser difference between two methods.
In this case also deflection change is not so significant. There is not so much difference in any grid size between two methods. Deflection pattern is consistent with bending moment pattern, first FEM overestimates grillage results. Later FEM underestimates grillage results. Grid corresponding to 7 and 8 gives same results.
In skew angle 60^0 also, all the analysis results are approximately same corresponding to 7 divisions. Ratio of transverse to longitudinal grid spacing =1.858

Table 3: skew Slab Bridge having 60° skew angle

60°Skew Angle		Division	4	5	6	7	8	9	10	11	12
		Grid size(m)	1.197	0.958	0.798	0.684	0.598	0.532	0.479	0.435	0.399
Dead Load Only	Reactions (kN/m)	Grillage	267.0	282.0	293.0	301.7	309.0	314.4	320.8	325.6	330.2
		FEM	302.6	301.4	301.9	303.6	307.0	311.9	314.8	319.2	323.7
		% diff.	**11.73**	**6.45**	**2.95**	**0.63**	**-0.63**	**-0.82**	**-1.89**	**-2.00**	**-2.00**
	Bending Moment (kNm/m)	Grillage	77.6	80.2	82.5	84.5	86.4	91.5	93.7	98.4	102.7
		FEM	85.1	84.2	83.6	83.0	82.6	82.1	81.7	81.4	81.0
		% diff.	**8.85**	**4.82**	**1.40**	**-1.79**	**-4.63**	**-11.4**	**-14.5**	**-20.9**	**-26.8**
	Torsional Moment (kNm/m)	Grillage	72.0	69.6	67.7	65.2	64.9	64.9	64.0	63.8	63.5
		FEM	66.8	65.4	66.3	66.3	66.4	66.6	65.4	65.2	65.1
		% diff.	**-7.88**	**-6.39**	**-2.04**	**1.73**	**2.16**	**2.56**	**2.04**	**2.13**	**2.46**
	Deflection (mm)	Grillage	1.42	1.43	1.45	1.48	1.51	1.50	1.58	1.61	1.65
		FEM	1.52	1.48	1.45	1.42	1.40	1.38	1.36	1.35	1.33
		% diff.	**6.30**	**3.36**	**-0.02**	**-4.13**	**-8.19**	**-8.59**	**-15.92**	**-19.74**	**-23.37**
Dead Load plus Live Load	Reactions (kN/m)	Grillage	706.6	760.2	799.3	828.3	851.2	869.5	883.6	895.3	905.3
		FEM	795.9	808.9	821.1	836.3	851.7	870.0	888.8	901.9	915.7
		% diff.	**11.22**	**6.025**	**2.65**	**0.959**	**0.066**	**0.063**	**0.581**	**0.722**	**1.13**
	Bending Moment (kNm/m)	Grillage	184.9	191.3	196.7	201.7	206.3	213.3	217.3	223.6	229.9
		FEM	200.4	199.8	199.1	197.8	196.6	195.7	194.4	193.5	192.4
		% diff.	**7.70**	**4.25**	**1.22**	**-1.96**	**-4.89**	**-8.99**	**-11.7**	**-15.5**	**-19.5**
	Torsional Moment (kNm/m)	Grillage	149.6	143.9	140.6	135.5	134.5	133.2	133.9	132.4	131.8
		FEM	143.0	139.9	138.3	137.5	137.9	137.6	135.9	135.5	135.2
		% diff.	**-4.40**	**-2.79**	**-1.64**	**1.49**	**2.46**	**3.29**	**1.52**	**2.34**	**2.58**
	Deflection (mm)	Grillage	3.79	3.79	3.80	3.84	3.88	3.88	3.98	4.02	4.07
		FEM	4.05	3.93	3.86	3.79	3.72	3.67	3.62	3.58	3.54
		% diff.	**6.26**	**3.65**	**1.39**	**-1.34**	**-4.29**	**-5.71**	**-9.85**	**-12.51**	**-15.03**

4. Conclusion

In the present study comparison of different skew angle bridges is performed using Finite element and Grillage method and results are presented. It is observed from the analysis that mostly seven divisions on gridding is appropriate i.e., ratio of transverse grid lines to longitudinal grid lines is 1.8-2.0. However, six and eight divisions also gives approx similar results as that of seven, so seven divisions can be preferred with ratio within 1.5-2.0 As skew angle increases, difference between reaction values (Grillage and FEM) of grid sizes decreases. In skew angle 30°, there is lot of difference but it decreases, as skew angle increases. In the case of bending moment, FEM results overestimated Grillage results, and after the point of intersection FEM results underestimated Grillage. Deflection is consistent with longitudinal bending moment, it follows the same pattern. Torsion moment follows the reverse pattern of longitudinal moment. Grillage results overestimated the FEM results in larger grid sizes and after intersection it underestimated FEM results. Variation of grid sizes analysis results predicts that, variation in reaction value is same in FEM and Grillage method but variation of bending and torsion moment in FEM is lower than grillage results. So, FEM may be preferred for analysis of skew bridges efficiently with certain limitation.

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