NUMERICAL SIMULATION OF THE MOTION OF A DROP IN PLANE POISEUILLE FLOW: DENSITY RATIO EFFECTS

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Abstract

The density ratio effects on the motion of a three-dimensional drop in Poiseuille flow are examined at finite Reynolds numbers using a finite difference front tracking method. The elliptic pressure equation is solved by a multi-grid method. For deformable drops, the wall repulsion increases and this effect moves the equilibrium position closer to the centerline of the channel. Results show that all drops with deferent density ratios migrate to an equilibrium position about halfway between the centerline and the wall. The drops move to an equilibrium position closer to the wall with increasing the density ratio. The axial velocities of the drops increase with decreasing the density ratio, because the drop with smaller density ratio moves to a lower final position. Also, the deformation of the drops is the same after an initial transient period. During the initial transient period, the deformation increases as the density ratio increases.

***Keywords*:** Poiseuille flow, finite difference method, front tracking method, density ratio, Reynolds number.

1. INTRODUCTION

Motion of drops through channels and tube has been always a matter of interest for many years. Segre and Silberberg (1962) [1] found that neutrally buoyant particles released off center in a pressure-driven flow, will not migrate to the pipe center-line, but instead will find an equilibrium position at about 0.6 radii off the center-line. Karnis et al. (1966) [2] showed that neutrally buoyant particles stabilized midway between the centerline and the wall. The equilibrium position was closer to the wall for larger flow rates and closer to the center for larger particles. The phenomenon of migration of liquid drops in Couette flow between concentric cylinder due to non-Newtonian fluid properties and shape deformation has been studied experimentally by Paul et al. (1981) [3]. Significant observations include the migration of a deformable Newtonian drop to an equilibrium position between the centerline and the inner rotor, and the competition between normal stresses and shape deformation effects for the case of a Newtonian drop in a non-Newtonian fluid. The stability of plane Poiseulle flow of two immiscible liquids of different viscoelastics and equal densities studied by Than et al. (1987) [4]. They found that regions of stability when there are three layers with one of the fluids centrally located. They showed the stability results depend on the viscosity and volume ratio in a fairly complicated way. Also, the flow with the high viscosity fluid centrally located is always stable. Feng, et al. (1994) [5] reported the results of a two-dimensional finite element simulation of the motion of a circular particle in a Couette & Poiseuille flow. They showed that a neutrally buoyant particle migrates to the centerline in a Couette flow and the stagnation pressure on the particle surface is particularly important in determining the direction of migration. Mortazavi & Tryggvasson (1999) [6] studied the motion of a drop in poiseuille flow. They simulated the motion of many drops at finite Reynolds numbers. Three-dimensional numerical simulation was presented on the motion of a deformable capsule undergoing large deformation in a plane Poiseulle flow in a channel by Doddi and Bagchi (2008) [7]. The numerical methodology was based on a mixed finite-difference/Fourier transform method for the flow solver and a front-tracking method for the deformable interface. Migration velocity and capsule deformation were observed to increase with increasing Capillary number. Bayareh and Mortazavi (2009, 2010) [8,9,10,11] studied the motion of a single drop and the interaction between two drops in simple shear flow at finite Reynolds numbers neglecting the gravity influence. They showed that the drops migrate to the centerline of the channel in shear flow. Also, they reported that the drop deformation depends strongly on the capillary number, so that; the proper non-dimensional number for the interfacial tension is the capillary number. Hudson (2010) [12] studied the drop circulation in microchannels. Two characteristic of transport of fluids in rectangular microchannels were addressed as a function of the cross-sectional aspect ratio.

2. FORMULATION

2.1 Governing Equations

The motion of a drop in Poiseuille flow at finite Reynolds numbers is governed by the Navier-Stokes equations. The Navier-Stokes equations are written in conservative form and variable physical properties. The surface tension is added to the equations by a delta function.

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla.\rho \mathbf{u}\mathbf{u} = -\nabla P + \nabla.\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \sigma \int \kappa \mathbf{n} \delta^{\beta}(\mathbf{x} - \mathbf{X}) ds.$$

This equation is valid for the whole flow field, even if the density field, ρ, and the viscosity field, μ, change discontinuously. Here $\boldsymbol{u}$ is the fluid velocity, p is the pressure, and σ is the surface tension coefficient. δ^{β} is a two- or three-dimensional delta function (for $\beta = 2$ and $\beta = 3$) respectively. κ is the curvature for two-dimensional flows and twice the mean curvature for three-dimensional flows. $\boldsymbol{n}$ is a unit vector normal to the drop surface pointing outside of the drop. $\boldsymbol{x}$ is the position in Eulerian coordinate and $\boldsymbol{X}$ is the position of front in Lagrangian coordinate. The integral is over the interface between the two fluids.

The fluids are incompressible and immiscible with constant material properties. Therefore:

$$\nabla.\mathbf{u} = 0.$$

$$\frac{D\rho}{Dt} = 0.$$

$$\frac{D\mu}{Dt} = 0.$$

he governing non-dimensional numbers of the flow are: the geometric ratio $\xi = R/H$, the ratio of the radius of the drop R to the height of the channel H. The viscosity ratio $\lambda = \mu_d / \mu_f$, the density ratio $\eta = \rho_d / \rho_f$. The viscosity and the density of the drop are denoted by μ_d and ρ_d, respectively, and the ambient fluid has viscosity μ_f and the density ρ_f. The bulk Reynolds number is defined in terms of the undistributed channel centerline velocity U_c and the channel height H, as $\mathrm{Re}_b = \rho_f U_c H / \mu_f$. The Reynolds number based on the centerline velocity and the drop diameter d is defined by $\mathrm{Re}_d = \rho_f U_c d / \mu_f$. A particle Reynolds number is defined as $\mathrm{Re}_p = \rho_f U_c R^2 / \mu_f H$. The Capillary number $Ca = \mu_f U_c / \sigma$ describes the ratio of the viscous stress to the interfacial tension.

2.2 NUMERICAL METHOD

In front tracking methods a separate front marks the interface but a fixed grid, is used for the fluid within each phase. In addition to front tracking methods that are applicable to the full Navier Stokes equations, specialized boundary integral methods have been used for both potential and Stokes flows. In general, the interface representation can be explicit (moving mesh) or implicit (fixed mesh) or a combination of both. The front-tracking method is combination of fixed and moving mesh method. Although an interface grid tracks the interface, the flow is solved on a fixed grid. The interface conditions are satisfied by smoothing the interface discontinuities and interpolating interface forces from the interface grid to the fixed grid. In this method, the governing equations are solved for whole flow field. Front capturing has two difficulties. The first is a sharp boundary between the fluids and the second is accurate computation of surface tension. Different methods have been made in overcoming these problems.

The front is resolved by discrete computational points that are moved by interpolating their velocities from the grid. These points are connected by triangular elements to form a front that is used to keep the density and viscosity stratification sharp and to calculate surface tension. At each time step information must be passed between the front and the stationary grid. This is done by a method that discussed by Unverdi & Tryggvason (1992), where the density jump is distributed to the grid points next to the front and a smooth density field that changes from one density to the other over two to three grid spaces generated by the solution of a Poisson equation. While this replaces the sharp interface by a slightly smoother grid interface, numerical diffusion of the density and the viscosity fields is eliminated, since the grid field is reconstructed at each step. The spatial differentiation is calculated by second order finite difference on a staggered Eulerian grid. We use an explicit second-order time integration method. Combining the incompressibility condition and momentum equations results in a non-separable elliptic equation for the pressure.

3. RESULTS

The geometry of the flow is shown in figure 1. The channel is bounded by two no-slip walls in the z-direction. The domain is periodic in the x- and y-directions. Tangential stresses are continuous on the surface of the drop and normal stresses show the jump across the interface by surface tension.

Figure 2 shows the trajectories of a neutrally buoyant drop released at different initial positions ($Re_b = 10$, Ca = 0.1, λ = η = 1, ξ = 0.125). The flow parameters in simulations of Feng et al. (1994) [4] are: $Re_b = 10$, $\lambda = \eta = 1$, ξ = 0.125. One can see that all drops with different initial positions migrate to an equilibrium position about halfway between the centerline and the wall (Segre-Silberberg effect). Feng et al. (1994) [4] reported that the rigid particles move to an equilibrium position a little outside the midpoint between the wall and the centre at z = 0.252 H. for deformable drops, the wall repulsion increases and this effect moves the equilibrium position closer to the centerline of the channel.

The effect of the density ratio on the lateral migration of a drop in plane Poiseuille flow was examined by carrying out three simulations with η = 1, 2 and 5. The other flow parameters are the same as the last simulation (Figure 2). Figure 3 shows the trajectories of drops versus their axial locations. It is observed that the drops migrate to an equilibrium position closer to the wall with increasing the density ratio. The axial velocity increases with increasing density ratio. This matter can be observed in figure 4 that shows the axial velocities of the drops versus the axial location. Because of similar final position of drops, it is expected that the deformation of the drops is the same after an initial transient period. The deformation of the drop is examined by considering the Taylor deformation parameter define by $D = (L-B) / (L+B)$, where L and B are, respectively, the major and minor axis of deformed drop (defined by the largest and smallest distance of the surface from the centre) (not shown).

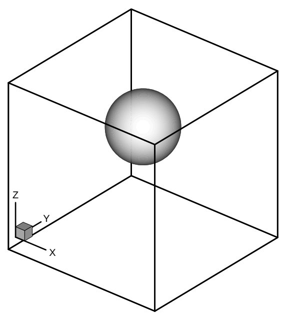

FIGURE 1. The geometry for the simulation of a drop in a channel.

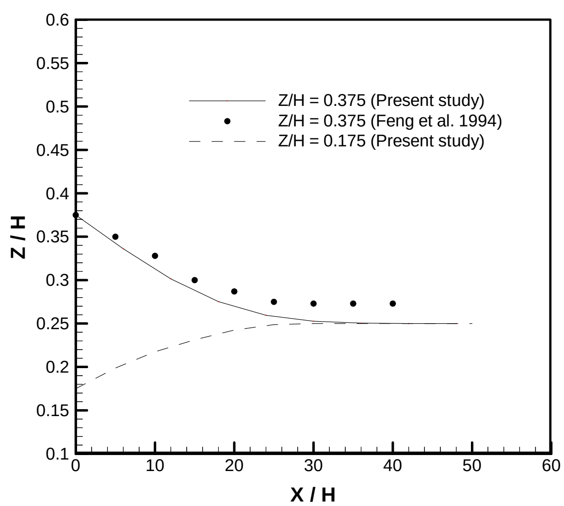

FIGURE 2. The Segre-Silberberg effect in a plane Poiseuille flow.

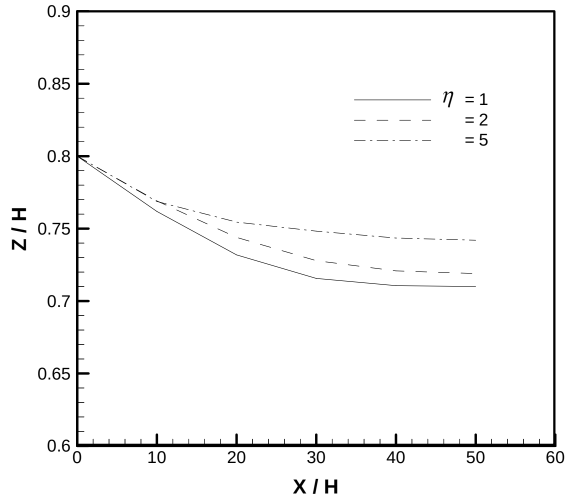

FIGURE 3. The lateral migration.

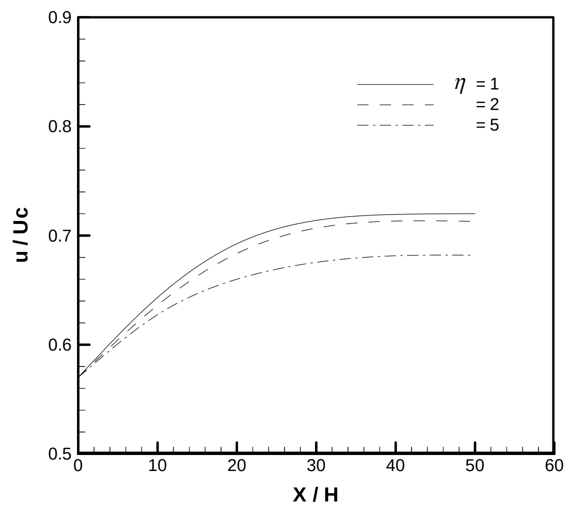

FIGURE 4. The axial velocity.

4. CONCLUSIONS

The density ratio effects on the motion of a three-dimensional drop in Poiseuille flow are examined at finite Reynolds numbers using a finite difference front tracking method. All drops with different initial positions migrate to an equilibrium position about halfway between the centerline and the wall (Segre-Silberberg effect). For deformable drops, the wall repulsion increases and this effect moves the equilibrium position closer to the centerline of the channel. The drops migrate to an equilibrium position closer to the wall with increasing the density ratio. The axial velocity increases with increasing density ratio. Results show that the deformation of the drops is the same after an initial transient period. During the initial transient period, the deformation increases as the density ratio increases.

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