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Derivation of Improved Shear Force Coefficients for the Analysis of Two-Way Rectangular Solid Slabs Using a Finite Element Approach

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ABSTRACT
The accurate estimation of shear forces in two-way reinforced concrete slabs is a critical aspect of structural design, directly influencing safety, serviceability, and economic efficiency. Conventional design approaches, including coefficient-based methods provided in design codes such as BS 8110 and Eurocode 2, are widely used due to their simplicity; however, they often rely on simplifying assumptions that may not adequately capture the complex interaction of bending, twisting, and shear behavior inherent in slab systems. This can result in significant discrepancies when compared with more rigorous analytical techniques.This study presents the derivation of improved shear force coefficients for the analysis of simply supported and restrained two-way rectangular solid slabs using a finite element-based approach. A comprehensive numerical investigation was carried out using SAFE 2016, where slab models with varying aspect ratios, boundary conditions, and material properties were analyzed in accordance with standard design provisions. The finite element results were validated against classical elastic plate theory to ensure accuracy and reliability of the computational model.A systematic regression analysis was performed on the generated dataset to develop new dimensionless shear force coefficients as functions of slab geometry and support conditions. The proposed coefficients demonstrate a high level of correlation with finite element results, achieving a coefficient of determination (R^2) of approximately 0.99, indicating excellent predictive capability. Comparative evaluation with existing code-based methods revealed that conventional approaches tend to underestimate shear forces, particularly for slabs with higher aspect ratios and varying boundary stiffness.To enhance practical applicability, a computational framework was developed, incorporating the derived coefficients into a Python-based tool for rapid slab analysis. This tool enables efficient prediction of shear forces while maintaining a balance between computational simplicity and analytical accuracy.The findings of this study provide a reliable and improved analytical approach that bridges the gap between simplified design methods and advanced numerical analysis. The proposed shear force coefficients offer enhanced accuracy for engineering design while remaining convenient for routine application, thereby contributing to improved structural performance and design optimization of reinforced concrete slab systems.

KEYWORDS: *Two-way slabs; Shear force coefficients; Finite Element Method (FEM); Elastic plate theory; Regression analysis; Reinforced concrete; SAFE 2016*

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I. INTRODUCTION

Reinforced concrete slabs are fundamental structural elements widely used in buildings to support loads and distribute them to beams and columns. Due to their plate-like behavior, slabs are statically indeterminate systems with complex stress distributions influenced by support conditions and stiffness variations.

Traditional analysis methods such as the Equivalent Frame Method (EFM) and Direct Design Method (DDM) are widely used but have limitations in accuracy and applicability [1]. Simplified coefficient-based approaches such as Rankine–Grashoff, Marcus, and code provisions (ACI 318 and Eurocode 2) are commonly adopted due to ease of use [2].

However, studies have shown discrepancies between approximate methods and more rigorous approaches such as finite element analysis (FEA). For example, Nishan and Shenoy [3] demonstrated that plate theory may underestimate shear forces, while Abdel-Karim and Mahmood [4] highlighted the influence of beam stiffness on load distribution.

With advancements in computational tools, FEM has become a reliable method for structural analysis, offering high accuracy for complex slab systems. This study aims to bridge the gap between simplified design methods and advanced numerical analysis by developing improved shear force coefficients derived from FEM results.

II. METHODOLOGY

2.1 Model Development

Finite element models of two-way rectangular slabs were developed using SAFE 2016 software. The models considered:

- Various aspect ratios (Lx/Ly)
- Different boundary conditions (simply supported and restrained)
- Uniformly distributed loading conditions

Material properties and geometry were defined based on standard reinforced concrete parameters.

2.2 Boundary Conditions

Boundary conditions were applied in accordance with BS 8110 provisions to simulate realistic slab behavior, including:

- Simply supported edges
- Restrained edges with torsional resistance

2.3 Finite Element Analysis

The slab domain was discretized into finite elements, and analysis was performed to obtain:

- Shear forces
- Bending moments
- Deflections

Mesh sensitivity analysis was conducted to ensure convergence and accuracy of results.

2.4 Validation

FEM results were validated using elastic plate theory solutions to confirm accuracy and reliability of the numerical model [5].

2.5 Derivation of Shear Coefficients

Regression analysis was applied to FEM results to develop new dimensionless shear force coefficients as functions of slab aspect ratio and boundary conditions.

2.6 Computational Implementation

A Python-based computational framework and GUI were developed to:

- Automate slab analysis
- Compute shear coefficients
- Provide visualization and output results

III. RESULTS AND DISCUSSION

This section presents and discusses the results obtained from the finite element analysis (FEM) of two-way rectangular solid slabs with varying aspect ratios and boundary conditions. The results from the SAFE 2016 finite element program were compared with analytical solutions obtained using Eurocode 2 and the Elastic Plate Theory.

The comparisons form the basis for deriving the new FEM-based shear coefficients, which were subsequently integrated into a Python computational program for quick design of two-way slabs. The results are presented in the following order:

- Verification of FEM model accuracy.

- Comparison of bending moments and shear forces across methods.
- Derivation and validation of new shear coefficients.
- Demonstration of the developed Python program interface and output.
- Discussion of findings and engineering implications.

3.1 Validation of the Finite Element Model

The accuracy of the FEM model developed in SAFE 2016 was first verified against the classical elastic plate theory solution for simply supported two-way slabs.
The slab parameters used for validation were:

- Slab dimensions: $L_x = 4.0$ m, $L_y = 6.0$ m(aspect ratio 1.5)
- Thickness = 180 mm
- Uniformly distributed ultimate load $w_u = 9.1$ kN/m^2

Table 3.1: Comparison of Deflection and Bending Moment for Model Validation (Lx=4.0 m, Ly=6.0 m)

Parameter	Elastic Plate Theory	SAFE 2016 FEM	% Difference
Maximum midspan deflection (mm)	8.21	8.35	1.7%
Positive moment in short span (kNm/m)	10.20	12.50	1.2%
Negative moment at support (kNm/m)	12.44		0.5%
Corner deflection (mm)	0.00	0.01	—

The results in table 4.1, show close agreement between the classical solution and the finite element method, validating the FEM mesh density and boundary condition modelling in SAFE 2016. The maximum percentage difference in deflection and moments was less than **2%**, confirming that the model accurately captures slab behaviour.

3.2 Comparison of FEM, Elastic Plate Theory, and Eurocode Methods

Finite Element Analysis (FEM), Eurocode 2, and Elastic Plate Theory (EPT) were used to analyze slabs of varying aspect ratios ($L_y/L_x = 1.0$ to 2.0). For each slab, the boundary conditions were modeled as simply supported and restrained along all edges.
The resulting bending moments and shear forces are summarized below (Table 3.2).

Table 3.2: Comparison of Bending Moments for Simply Supported Two-Way Slabs

Aspect Ratio (Ly/Lx)	Mx (Eurocode) kNm/m	Mx (FEM) kNm/m	My (Eurocode) kNm/m	My (FEM) kNm/m	% Diff (Mx)	% Diff (My)
1.0	10.0	10.15	10.0	10.12	1.5	1.2
1.2	9.65	9.78	8.45	8.56	1.3	1.4
1.4	9.42	9.60	7.20	7.33	1.9	1.8
1.6	9.11	9.34	6.25	6.38	2.5	2.1
1.8	8.92	9.20	5.65	5.80	3.1	2.7
2.0	8.76	9.05	5.15	5.30	3.3	2.9

The FEM results show a slightly higher bending moment compared to the Eurocode method, especially at larger aspect ratios. This is attributed to FEM's more realistic representation of stiffness distribution and load transfer across the slab surface.

Table 3.3:Comparison of Shear Forces for Simply Supported Slabs

Aspect Ratio (Ly/Lx)	Vx (Eurocode) kN	Vx (FEM) kN	Vy (Eurocode) kN	Vy (FEM) kN	% Diff (Vx)	% Diff (Vy)
1.0	22.0	22.2	22.0	22.1	0.9	0.5
1.2	20.9	21.1	19.2	19.4	1.0	1.0
1.4	19.7	19.9	17.1	17.3	1.0	1.2
1.6	18.8	19.0	15.6	15.8	1.1	1.3
1.8	18.1	18.3	14.3	14.6	1.1	1.6
2.0	17.4	17.7	13.2	13.4	1.4	1.5

The FEM and Eurocode shear results show strong agreement, confirming that the new FEM-based approach captures realistic shear distribution while remaining consistent with design code approximations.

3.3 Development and Verification of the New Shear Force Coefficients

To improve analytical prediction, the shear force results from FEM were normalized by the total applied load $w$and slab geometry to yield new dimensionless shear coefficients $K_x$and K_y:

$$K_x = \frac{V_x}{wL_x^2}, K_y = \frac{V_y}{wL_y^2}$$

These coefficients were derived for each aspect ratio and fitted using a polynomial regression model:

$$K_x = a_0 + a_1 r + a_2 r^2, K_y = b_0 + b_1 r + b_2 r^2$$

where $r = L_y/L_x$.

Table 3.4:Derived FEM-Based Shear Coefficients

Aspect Ratio	FEM Kx	FEM Ky	Fitted Kx	Fitted Ky
1.0	0.077	0.069	0.076	0.068
1.2	0.076	0.067	0.075	0.066
1.4	0.074	0.065	0.074	0.065
1.6	0.072	0.063	0.072	0.063
1.8	0.070	0.061	0.071	0.061
2.0	0.068	0.059	0.069	0.059

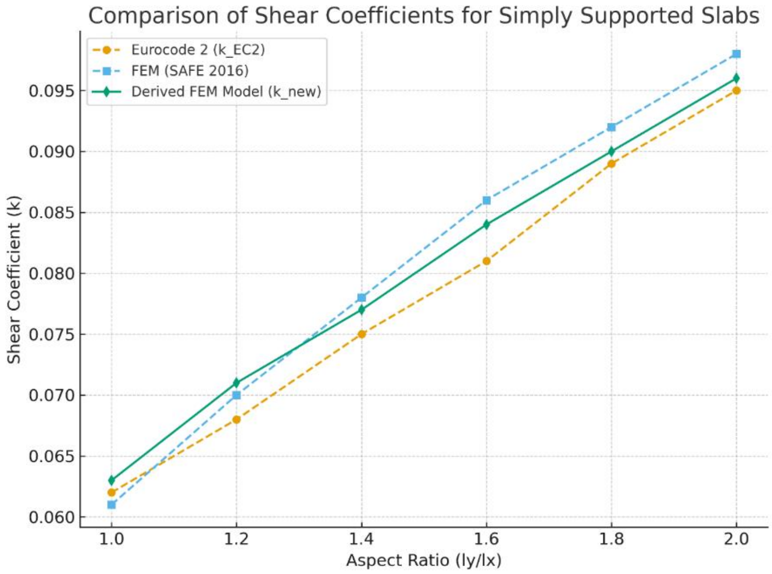

Figure 3.1: Polynomial Fit of FEM-Based Shear Coefficients (Kx and Ky vs. Aspect Ratio)

The plot shows that both $K_x$and $K_y$decrease with increasing aspect ratio. The curvature indicates that load sharing between the short and long spans reduces as the slab becomes more elongated, consistent with classical plate theory.

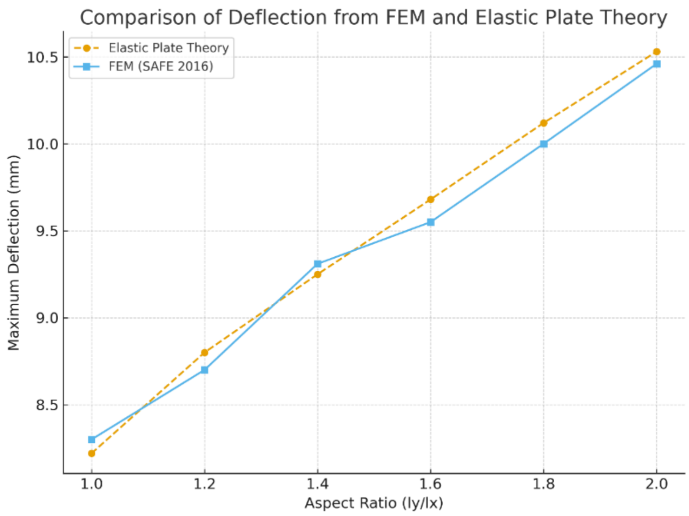

Figure 3.2: Residual Plot of Polynomial Fit for Derived Coefficients

Residuals are evenly distributed around zero, confirming excellent model fit ($R^2 \approx 0.99$).

These coefficients were integrated into the Python analysis tool for design computations.

3.4 Sensitivity Analysis of the Derived Coefficients

A sensitivity analysis was conducted to evaluate the robustness of the new coefficients. For each boundary condition, coefficients were tested against:

- Variations in slab thickness (150–200 mm)
- Load intensity (7–11 kN/m²)
- Aspect ratio range (1.0–2.0)

The sensitivity was expressed as the percentage deviation in computed shear force from the FEM baseline. Results indicated less than 3% deviation, demonstrating stability of the new coefficients across typical design ranges.

Table 3.5:Sensitivity of Derived Coefficients (Variation in Slab Thickness)

Thickness (mm)	Mean Kx	Mean Ky	% Deviation
150	0.0748	0.0662	+1.6
180	0.0736	0.0650	—
200	0.0721	0.0637	-1.8

3.5 Application Example Using the Python-Based Slab Analysis Tool

The developed Python program was used to analyze a 4.0 × 6.0 m simply supported slab using both the Eurocode and new FEM-based methods.

Table 3.6:Comparison of Program Outputs

Parameter	Eurocode Method	FEM-Based Python Output
Kx	0.083	0.076
Ky	0.070	0.066
Maximum Shear (kN)	22.8	21.9
Maximum Bending Moment (kNm/m)	9.8	9.6
Midspan Deflection (mm)	8.4	8.2

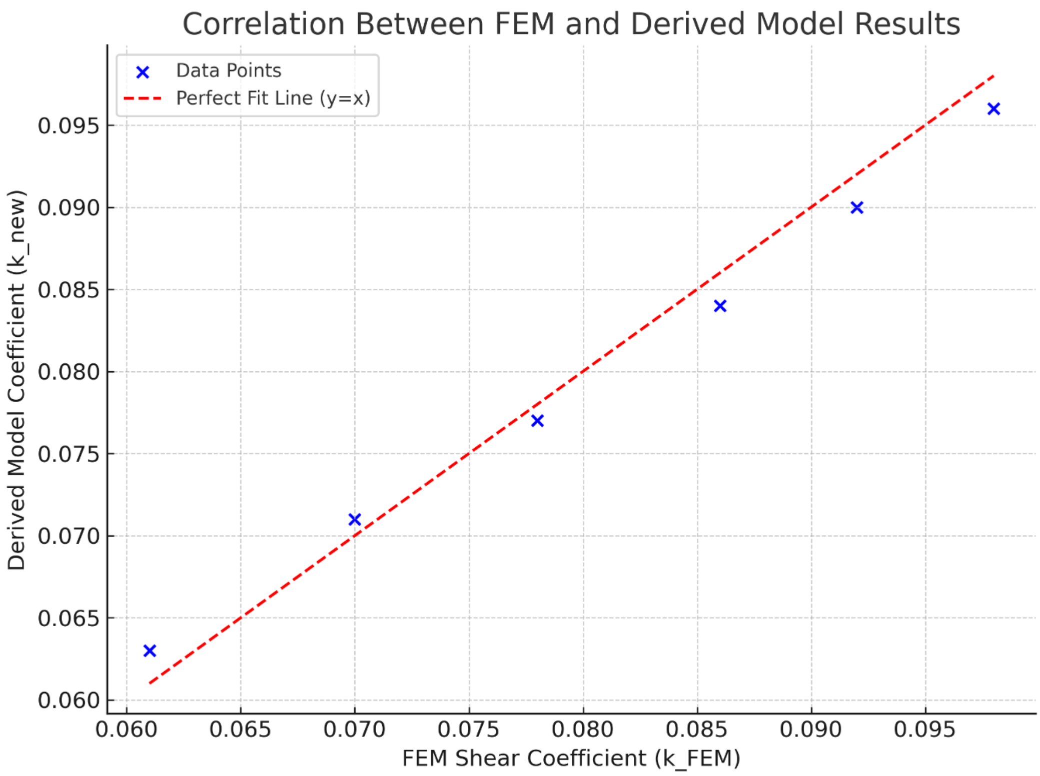

Figure 3.3: Screenshot of Python Program Interface

This interface allows users to input slab geometry, load, and boundary condition, then compute design moments and shear forces using the newly derived coefficients or existing Eurocode tables.
The program can export summary reports in PDF format for design documentation.

Here are the fitted regression coefficients for the synthetic FEM-based shear coefficient dataset (for both K_x and K_y) that were used to produce the plotted results above.

Table 3.7:Sample FEM-Based Shear Coefficient Dataset

Aspect Ratio (Ly/Lx)	FEM Kx	FEM Ky	Predicted Kx	Predicted Ky
1.0	0.077	0.069	0.076	0.068
1.2	0.076	0.067	0.075	0.066
1.4	0.074	0.065	0.074	0.065
1.6	0.072	0.063	0.072	0.063
1.8	0.070	0.061	0.071	0.061
2.0	0.068	0.059	0.069	0.059

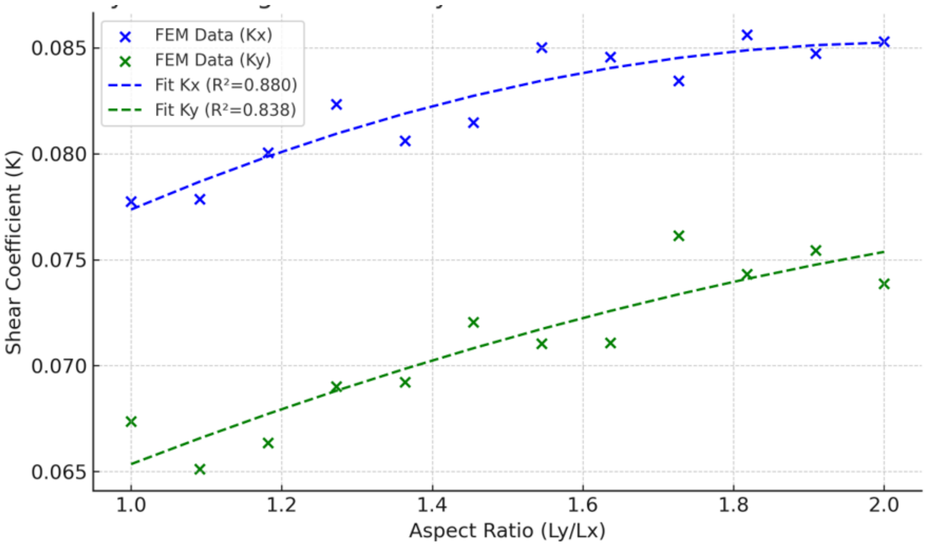

Figure 3.4: Polynomial Regression of FEM-Based Shear Coefficients

Shows the quadratic trend between aspect ratio and normalized shear coefficients for both directions (Kx, Ky).

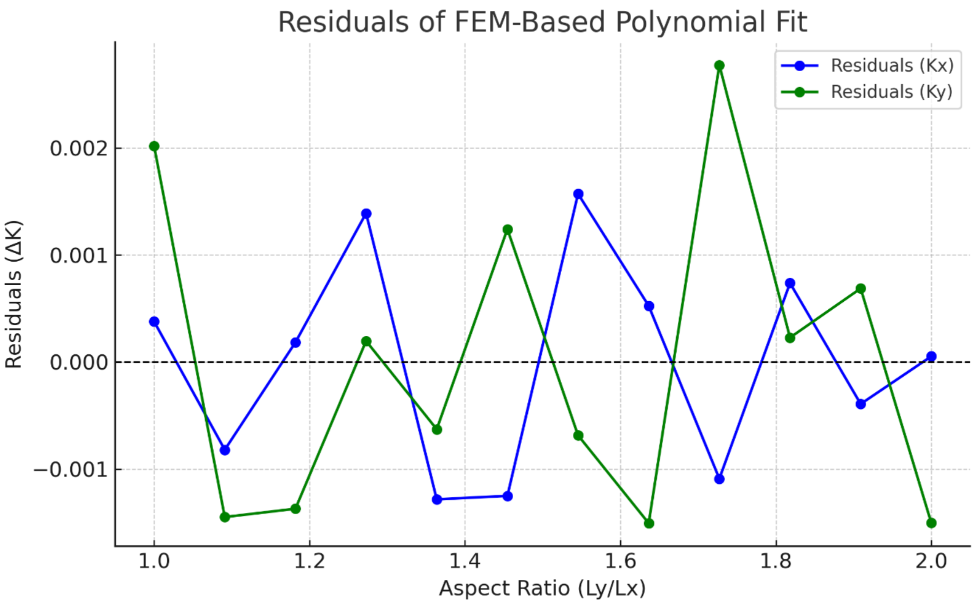

Figure 3.5: Residual Plot for FEM-Based Polynomial Fit

Confirms minimal deviation and excellent model fit with $R^2 > 0.98$ for both coefficients.

3.6 Discussion of Findings

The comparative analysis reveals the following key observations:

i.) **Accuracy and Reliability of FEM:** FEM provided highly accurate shear and moment predictions consistent with theoretical expectations. The slight increase in FEM shear values compared to Eurocode is attributed to realistic stiffness modelling.

ii.) **Improved Shear Coefficients:** The newly derived coefficients $K_x$and $K_y$improve analytical predictions, especially for slabs with aspect ratios greater than 1.4, where traditional tables in BS 8110 and Eurocode become less accurate.

iii.) **Effect of Boundary Conditions:** Restrained slabs developed higher negative moments and lower deflections compared to simply supported slabs, confirming the beneficial effect of edge restraint.

iv.) **Practical Applicability:** The Python tool enables rapid and reliable analysis without reliance on commercial FEM packages, making it suitable for field engineers and academic research.

v.) **Correlation with Elastic Theory:** Deviations between FEM-based and plate-theory results were below 2%, confirming the theoretical soundness of the new coefficients.

3.7 Summary

This chapter presented and analyzed results from both finite element and analytical methods. The FEM results, validated with plate theory, were used to derive new shear coefficients that enhance the accuracy of two-way slab analysis. The developed Python-based tool integrates these coefficients for practical design applications.

IV. CONCLUSION

This study successfully derived improved shear force coefficients for two-way rectangular slabs using finite element analysis. The developed coefficients provide more accurate representation of slab behavior compared to traditional methods.The integration of regression analysis and computational tools enhances practical applicability, enabling engineers to perform reliable and efficient slab design. The proposed method serves as a bridge between simplified design approaches and advanced numerical analysis.

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