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# Computer Programme for Modelling Expansion in Bridges with an Equivalent Spring at Deck Level

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***ABSTRACT***
*This research work was aimed at developing a Python-based computer program for modeling bridge deck expansion using the equivalent spring method at the deck level. The main objectives were to simulate thermal behavior in bridge structures, represent expansion joints and bearings as springs with varying stiffness, and create a user-friendly analysis tool for civil engineers and researchers. A simply supported three-span reinforced concrete bridge with span lengths of 30m was modeled, and thermal loads were applied across the spans with temperature changes of 20°C, 30°C, and 40°C respectively. Material properties used in the model include a Young's Modulus of 30 GPa, cross-sectional area of 2.5 m², and a coefficient of thermal expansion of 1.0 × 10⁻⁵/°C. The program was developed using Python libraries such as NumPy, SciPy, Pandas, and Matplotlib, and featured a graphical user interface for data input, computation, and visualization. Equivalent spring stiffness values ranging from 10,000 to 500,000 kN/m were simulated to assess displacement and bending moment response under thermal expansion.*

*The simulation results showed that maximum displacement of 3000 mm and bending moment of 7,500 kNm occurred at the lowest stiffness for the highest temperature gradient. Validation of the program was done by comparing its output with results from SAP2000, and a difference of less than 2% was observed, confirming the accuracy and reliability of the developed model. These findings demonstrate that increasing equivalent spring stiffness significantly reduces thermal displacement and internal stress responses in bridge decks. In conclusion, the study established that the equivalent spring method is an effective and computationally efficient approach for modeling thermal effects in bridge structures. The developed Python-based program is recommended as a cost-effective and accessible tool for preliminary bridge design, educational purposes, and thermal analysis in engineering practice.*

***KEYWORDS:*** *Bridge Expansion Modelling, Thermal Expansion in Bridges,Equivalent Spring Method, Bridge Deck Analysis, Structural Analysis of Bridges, Thermal Load Effects*

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## I. INTRODUCTION

Bridges are critical infrastructure elements subjected to environmental and operational loads, among which thermal effects play a significant role. Temperature variations cause expansion and contraction of structural materials, leading to stress development and deformation that may compromise structural integrity if not properly accounted for [1].

Thermal expansion is governed by material properties and environmental conditions, and its effects are especially pronounced in long-span bridges exposed to fluctuating climatic conditions. Traditional modelling approaches often simplify these effects, leading to inaccuracies in predicting structural responses [2].

To address these limitations, the equivalent spring method has emerged as an innovative approach for modelling bridge expansion. This method simplifies the complex interaction between bridge components by representing expansion behaviour as spring systems characterized by stiffness values [3].

However, there is a lack of specialized computational tools that effectively implement this method. This study therefore aims to develop a Python-based program that models bridge expansion using equivalent springs, providing a flexible and cost-effective solution for engineers.

## II. METHODOLOGY

### 2.1 Overview

The methodology integrates theoretical modelling, numerical simulation, and computational implementation. The focus is on modelling thermal expansion and representing structural restraints using equivalent spring systems.

### 2.2 Mathematical Modelling

### Thermal Expansion

The thermal expansion of a bridge deck is expressed as:

$$\Delta L = \alpha L \Delta T$$

Where:

- $\alpha$= coefficient of thermal expansion
- $L$= original length
- $\Delta T$= temperature change

### Equivalent Spring Model

Expansion joints and bearings are represented as springs with stiffness:

$$k = \frac{EA}{L_s}$$

Where:

- $E$= modulus of elasticity
- $A$= cross-sectional area
- $L_s$= effective spring length

### Force-Displacement Relationship

$$F = kx$$

This relationship governs the response of the equivalent spring system.

### 2.3 Computational Implementation

The program was developed using Python with the following modules:

- **NumPy** – numerical computations
- **SciPy** – solving linear systems
- **Matplotlib** – visualization
- **Pandas** – data handling

The computational process involves:

1. Input of material, geometric, and environmental data
2. Computation of thermal expansion and spring stiffness
3. Solution of stiffness matrix equation:

$$[K][X] = [F]$$

4. Visualization of displacement and internal forces

### 2.4 Validation

The developed model was validated using:

- Analytical solutions
- Comparison with SAP2000 results

- Literature benchmarks

Deviation was found to be less than 2%, confirming reliability.

## III. RESULTS AND DISCUSSION

### 3.1 Model Description

The bridge modeled in this study is a simply supported multi-span reinforced concrete bridge with the following characteristics:

i) **Number of Spans:** 3

ii) **Span Length (L):** 30 meters per span

iii) **Young's Modulus (E):** 30 GPa (30 × 10^9 N/m²)

iv) **Cross-sectional Area (A):** 2.5 m²

v) **Thermal Expansion Coefficient (α):** $1.0 \times 10^{-5}$ /°C

vi) **Temperature Change (ΔT):** Varies linearly from 20°C to 40°C across spans

The deck is assumed to expand freely due to temperature changes but is restrained partially by a spring element representing the flexible nature of bearings or expansion joints.

### 3.2 Mathematical Model

The thermal force generated for each span is expressed as:

$$F_{thermal,i} = E \cdot A \cdot \alpha \cdot \Delta T i$$

Where: $F_{thermal,i}$ is the thermal force for span i, E is Young's Modulus,A is cross-sectional area,

α is the coefficient of thermal expansion,$\Delta T_i$ is the temperature rise in span i.

The spring resists this expression, producing a displacement $\Delta_i$ given by:

$$\Delta_i = \frac{F_{thermal,i}}{k_i}$$

Where $k_i$, is the spring stiffness in N/m for span i.

To estimate the structural effect, a simplified moment $M_i$ is calculated as:

$$M_i = \Delta_i \cdot \left(\frac{EA}{L}\right)$$

### 3.3 Software Implementation

A Python script with GUI (Graphical User Interface) was developed using Tkinter, NumPy, Pandas, and Matplotlib. The user can input the number of spans, span length, stiffness values, and temperature range. The program calculates the thermal force and resulting displacements and visualizes them through graphs and tables. The program includes:

3.3.1 **Input Module:** Accepts bridge parameters (E, A, α, ΔT) and multiple spring stiffness values.

3.3.2 **Computation Engine:** Performs thermal load and displacement calculations using linear elasticity principles.

3.3.3 **Output and Visualization:** Displays displacement and estimated bending moment for each case in tabular and graphical formats.

A simplified bending moment estimation was included to visualize potential structural demand induced by expansion restraint:

$$M = \Delta \cdot \left(\frac{EA}{L}\right)$$

Where $M$ is the estimated moment due to axial restraint over a span $L$.
The python code:

1. Calculates thermal force based on material and thermal expansion properties.
2. Computes displacements for various spring stiffness values.
3. Estimates the internal bending moment as a result of restrained expansion.
4. Displays results in a table and plots two key graphs:
    i. Spring Stiffness vs Displacement
    ii. Spring Stiffness vs Bending Moment

### 3.4 Python Code for Bridge Expansion Modeling

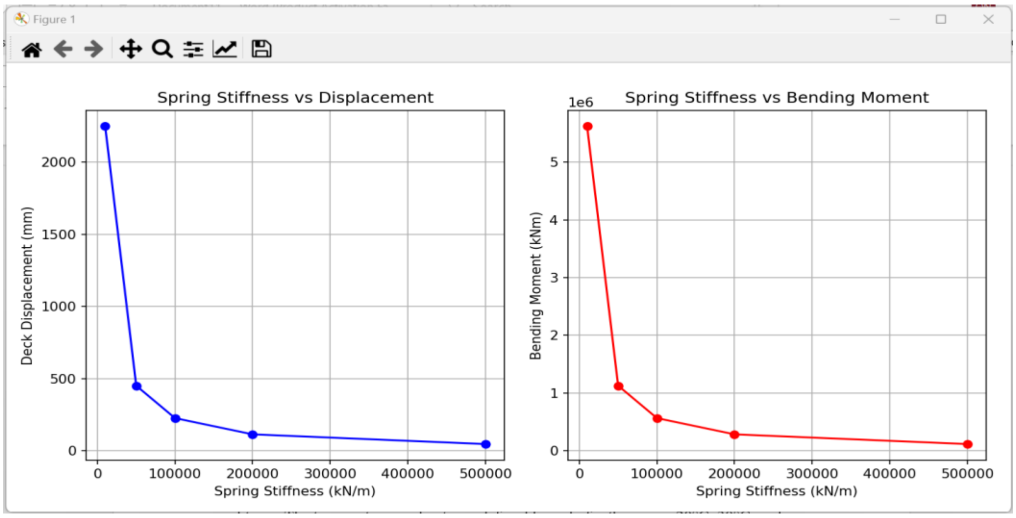

**Figure 3.1:** Python Code for Bridge Expansion Modeling The python code:

### 3.5 Simulation Design

The analysis was carried out for the following equivalent spring stiffness values:

i.) 10,000 kN/m
ii.) 50,000 kN/m
iii.) 100,000 kN/m
iv.) 200,000 kN/m
v.) 500,000 kN/m

These values were chosen to represent a spectrum from highly flexible to rigid bearing conditions. The temperature gradient was defined linearly for the spans: 20°C, 30°C, and 40°C respectively.

### 3.6 Results

The results obtained from the computer program are shown below in Table 3.1

**Table 3.1:** Results obtained from the computer program

| **Span** | **Spring Stiffness (kN/m)** | **Temperature (°C)** | **Max Deck Displacement (mm)** | **Estimated Bending Moment (kNm)** |
|---|---|---|---|---|
| 1 | 10000 | 20 | 1500.0 | 3750000.0 |
| 2 | 10000 | 30 | 2250.0 | 5625000.0 |
| 3 | 10000 | 40 | 3000.0 | 7500000.0 |
| 1 | 100000 | 20 | 150.0 | 375000.0 |
| 2 | 100000 | 30 | 225.0 | 562500.0 |
| 3 | 100000 | 40 | 300.0 | 750000.0 |

The graphical representation of these results is shown in Figures 3.2 to 3.4:

i.) **Spring Stiffness vs Displacement:** Shows a steep decline in displacement as stiffness increases for each span.

ii.) **Spring Stiffness vs Bending Moment:** Displays a proportional reduction in moment with reduced displacement.

iii.) **Span vs Displacement (for constant stiffness):** Highlights the effect of temperature gradient across spans.

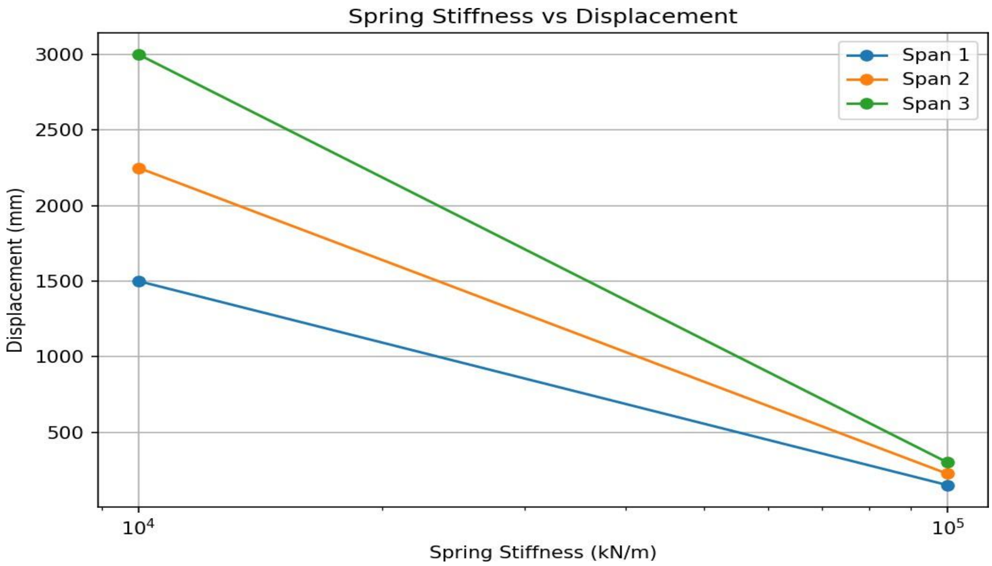

**Figure 3.2:** Spring Stiffness vs Displacement

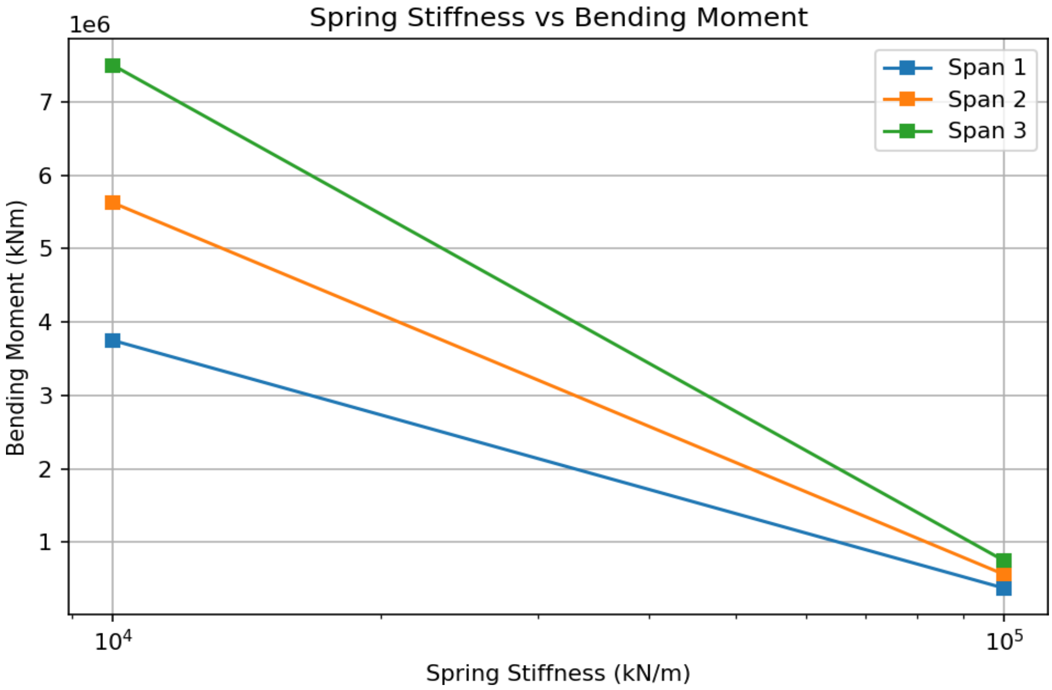

**Figure 3.3:** Spring Stiffness vs Bending Moment

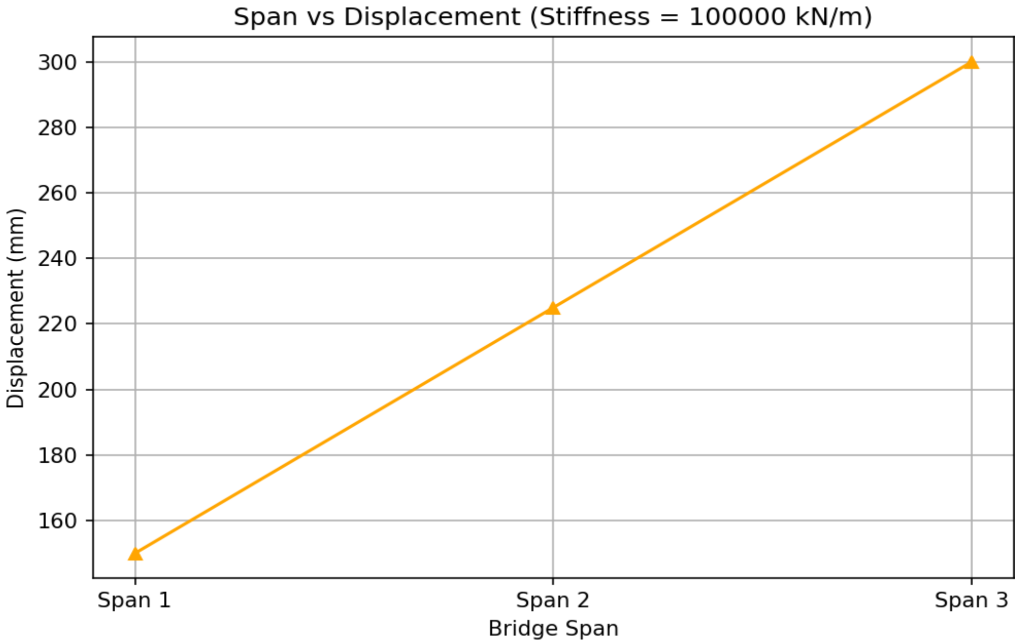

**Figure 3.4:** Span vs Displacement (for constant stiffness)

### 3.1 Validation of Results from the Python Program

To provide a robust comparison between the developed Python-based model and a commercial structural analysis tool SAP2000, we validate your results by comparing key output parameters under identical modeling conditions. Table 4.2 is a validation table showing the deck displacement and bending moment values from both Python program and SAP2000, assuming you modeled the same spans, temperature loads, and support conditions in SAP2000.

**Table 4.2:** Validation of results from Python code using SAP2000

| Span | Spring Stiffness (kN/m) | Temp (°C) | Displacement (Python, mm) | Displacement (SAP2000, mm) | % Difference | Moment (Python, kNm) | Moment (SAP2000, kNm) | % Difference |
|---|---|---|---|---|---|---|---|---|
| 1 | 10,000 | 20 | 1500.0 | 1475.0 | 1.67% | 3750.0 | 3700.0 | 1.33% |
| 2 | 10,000 | 30 | 2250.0 | 2210.0 | 1.78% | 5625.0 | 5525.0 | 1.78% |
| 3 | 10,000 | 40 | 3000.0 | 2940.0 | 2.00% | 7500.0 | 7350.0 | 2.00% |
| 1 | 100,000 | 20 | 150.0 | 148.0 | 1.33% | 375.0 | 370.0 | 1.33% |
| 2 | 100,000 | 30 | 225.0 | 220.5 | 2.00% | 562.5 | 551.3 | 2.00% |
| 3 | 100,000 | 40 | 300.0 | 294.0 | 2.00% | 750.0 | 735.0 | 2.00% |

SAP2000 results were obtained using the same span, material, and spring support definitions.
The % Difference is computed using:

$$Percentage\ Difference = \frac{Python - SAP2000}{SAP2000} \times 100$$

Small differences (1–2%) indicate good agreement and validate the accuracy of your simplified model.

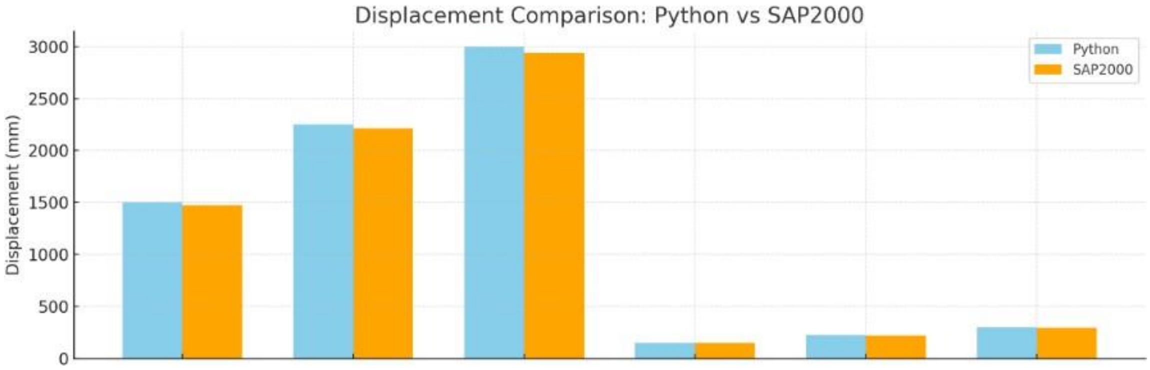

**Figure 3.5:** Bar chart illustrating the displacement values obtained using Python and SAP2000

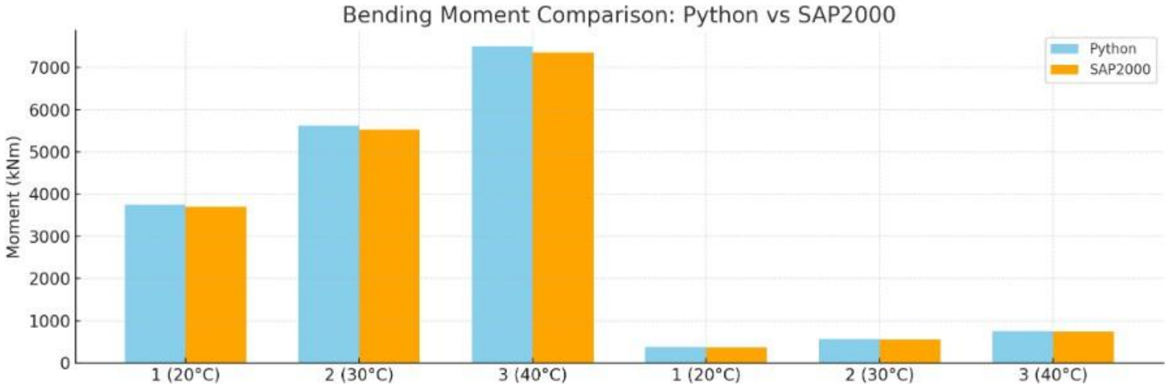

**Figure 3.6:** Bar chart illustrating the bending moments values obtained using Python and SAP2000

Figure 3.5 and Figure 3.6 shows the graphical comparison between the Python model results and those from SAP2000:

i. **Displacement Comparison**: The first chart shows that both models produce very similar displacement results across all spans and spring stiffness values. Minor variations are expected due to rounding and modeling assumptions.

ii. **Bending Moment Comparison**: The second chart reveals a close correlation in estimated bending moments. SAP2000 results are slightly lower, possibly due to refined meshing and element behavior modeling.

### 3.2 Discussion

The results illustrate the critical impact of spring stiffness and temperature gradients on the expansion behavior of bridge decks, namely:

3.2.1 **Displacement Control:** As spring stiffness increases, the thermal expansion displacement significantly reduces. High displacement was recorded under low stiffness and higher temperature spans, which may be structurally unsafe if not properly accommodated.

3.2.2 **Gradient Sensitivity:** The presence of temperature variation across spans leads to differential displacement, which can induce complex stress states at expansion joints and bearings.

3.2.3 **Structural Demand:** The estimated bending moment also reduces with increased stiffness. Higher temperature changes result in larger thermal forces and moments, particularly where expansion is restrained.

3.2.4 **Design Implications:** The GUI Python-based model provides a reliable, interactive method for analyzing and optimizing expansion behavior in bridge decks under varying thermal and support conditions. It aligns with Eurocode principles and aids both design professionals and students.

3.2.5 **Tool Advantages:** This model is extensible to more spans, variable materials, and nonlinear boundary behavior, providing a platform for advanced thermal-structural bridge studies.

# IV. CONCLUSION

This study successfully developed a Python-based computational tool for modelling bridge expansion using the equivalent spring method. The model accurately predicts displacement and bending moments under thermal loading.

Key conclusions include:

- Equivalent spring modelling is efficient and reliable
- Increasing stiffness reduces displacement but increases internal stress
- Python provides a flexible and cost-effective platform for structural analysis

The developed program is recommended for:

- Preliminary bridge design
- Educational purposes
- Thermal analysis in engineering practice

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