

A Meta Heuristic Approach to Solve Knapsack Problem

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Abstract

A novel fuzzy approach to solve Knapsack Problem (KP). This problem was first formulated as a multilevel programming problem. When there are multiples decision makers (DMs). Then we established the satisfaction level of each DM. Dynamic recursive formulation Programming was used to solve decisions in interrelated steps. overall satisfaction Decisions were achieved through this step-by-step manipulation of the hierarchical structure. Capacity Assignments were developed and step by step solution steps were provided. detailed comparison Coordination was also done between multi-purpose and multi-level KPs.

Keywords--Knapsack problem, Fuzzy, Multilevel programming, DP, Resource allocation.

I. INTRODUCTION

The well-known knapsack problem (KP) can be summarized as follows: a hiker must decide, among the $i = 1, 2, \dots, N$ objects, which objects to include in her or his knapsack on a forthcoming trip. Objective i has weight w_i and utility c_i to the hiker. The objective is to maximize the total utility of the hiker's trip subject to a weight limitation, W [1]. This problem has been studied extensively and has been applied to various areas such as capital budgeting, cargo loading, cutting stock, flyaway kit, project selection, etc. [2,3]. During the past decades, there has been an increasing realization on the practical needs to identify and to consider simultaneously several conflicting objectives [4]. This multi-objective knapsack problem (MOKP) can be expressed as [5]. Note that the above expression is also classified as the unbounded knapsack problem [6] with one knapsack. If $x_i \in \{0, 1\}$ (i.e., $x_i = 1$, if the object i is selected; and $x_i = 0$, otherwise), the problem becomes a 0-1 KP [2]. Furthermore, the decision of these multiple objectives can be made by a team or by two groups of individual decision makers (DMs) [6]. In a hierarchical organization with multiple DMs, a multilevel knapsack problem (MLKP) can be formulated with decentralized planning. The simplest case of MLKP is the bilevel programming problem, where the top level DM has control over the vector x_1 while the bottom level DM controls the vector x_2 . Let the performance functions of z_1 and z_2 for the two planners be linear and bounded, then the new bilevel knapsack problem can be represented as [7]. Equation (2) is a nested optimization model involving two problems, an upper one and a lower one [9]. Notice that if there exists no hierarchical control feature, equation (2) simplifies to the MOKP of equation (1). Thus, MOKP and MLKP are closely related. And the constraint regions in two equations are the same, i.e., $X = \{x_i, i = 1, \dots, N\} = \{x_1, x_2\}$. The usual solution techniques dealing with KPs are dynamic programming (DP) and integer programming [3]. We will focus on DP in this study for the ease of extension to the fuzzy environment. For crisp MOKPs, recent approaches are concentrated on how to find an efficient solution in the DP structure. Cho and Kim [10] developed an improved interactive hybrid method to adjust DM preference information through a scaling constant among objectives. Klamroth and Wiecek [11] proposed DP-based approaches to obtain all the nondominated solutions. Most of other studies can be categorized as integer programming-based approaches. One example is the work of Salman et al. [12]. Because of the similarity between MOKP and MLKP, these two problems will be imbedded in a common DP structure. We will first review the related literature of fuzzy MODP. Bellman and Zadeh [13] suggested that fuzzy decisions could be considered as the confluence or intersection of goals and constraints. Esogbue and Bellman [14] made some extensions and introduced many applications. At the same time, Kacprzyk offered a general view about multistage decision-making under fuzziness and derived a general structure for solving fuzzy DP problems. Kacprzyk and Esogbue made a fairly comprehensive survey of the major developments and applications of fuzzy DP. However, the breadth of theory and applications of fuzzy MODP is still limited. This is especially true in the field of decentralized planning of hierarchical systems. Due to the complexity of the multilevel programming (MLP) problems, there exist no efficient traditional techniques for obtaining the numerical solutions of a reasonable sized problem. Shih et al. suggest a fuzzy approach for MLP to simplify the complex structure, and it was proved to be feasible and efficient. In addition, the suggested supervised search procedure can be easily extended to a k -level hierarchical system, and it is also a flexible structure for further expansion. Consequently, we would like to examine the possibility of unifying the level-wise (hierarchical) operation and stage-wise operation for multilevel DP in a fuzzy environment. Following an extension of the structure of Kacprzyk, the new structure with interrelation among stages and objectives/levels can be simplified [8,9]. In the remaining sections, we will introduce the concept of capacity allocation for KP. The equations for fuzzy DP and MODM are introduced used to solve the

MOKP and MLKP. A discussion on the well-known turnpike theorem with fuzzy approach is also carried out. The final section contains some conclusions and remarks.

II. DYNAMIC PROGRAMMING FOR KNAPSACK PROBLEMS

Dynamic programming is an effective algorithm for solving multistage decision problems. It utilizes a functional equation to circumvent the dimensional explosion for multistage processes [10] and has been applied to solve many real world problems such as optimal control, inventory control, advertising campaign, production planning, equipment replacement, resource allocation, etc. [11]. In this study, we only focus on the knapsack problem.

III. Proposed Algorithm

STEP 1. According to the basic information in these tables, we will set up the degree of satisfaction by fuzzifying the objectives through their PIS and NIS of the possible capacity allocated at each stage.

STEP 2. The satisfactory solution of the imbedded bi-objective returns at each stage can be obtained through max-min operation, which means minimizing the satisfactory degree of two objectives, and then go to the next step.

STEP 3. The process is for maximizing the satisfactory degree among all possible cases of combinations under different inputs and states, and yet the given budget remains a constant constraint.

STEP 4. After all individual problems are solved, i.e., the single stage problems, we will establish the backward relationship among stages through DP structure. To trace the capacity allocated, we start from the last stage, Stage 4, with the capacity remaining after the previous stage, Stage 3, then go back to the second stage, and to the first stage.

IV. CONCLUSIONS

In this study, we have investigated multi-objective and multilevel knapsack problems in a fuzzy environment. An efficient algorithm is developed, and a solution procedure has been proposed through Microsoft Excel as well. Although the size of the examples under scrutinizing is small, large size problems are expected to solve through the same procedure. Since different decision variables can be controlled by separated different decision units (or DMs) in a multilevel system, its decision space will be more restricted than that of the MODM problems'. In general, its solution will be no better than that of the MODM's. However, both are efficient solutions. In a general resource allocation problem, the number of decision variables will not always equal the number of the stages, but the situation will happen in solving KPs. Consequently, we could think that the KP is a special case of the resource allocation problems. Notwithstanding only bi-level problems are illustrated; multiple-level dynamic programming problems can be solved through the same algorithm. Please see the details of the search algorithm in [13] for simplifying multilevel structures. The efficiency of our fuzzy approach is dependent on the scheme of dynamic programming with a loose structure. We have not discussed the computational problem in this study, and interested readers might check some algorithms, e.g., [10], in the literature. And looking for a short-cut algorithm in DP structure will be the future direction. In this study we have not involved the integer programming-based algorithms for MOKPs and MLKPs. Readers can go through the contents of Shih and Lee seeing a case of multilevel minimum-cost flow problem. This would be another direction for further investigation.

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