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A Novel Study of Time Series Forecasting by Using Fuzzy Interval Separation Method

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Abstract:
In this paper, we introduce a novel approach for interval separation forecasting, specifically focusing on enhancing the accuracy of fuzzy time series forecasting. Our method builds upon the foundation of fuzzy time series models by interval separation technique, extending them to provide more precise forecasts.The core of our approach lies in utilizing a frequency distribution technique to determine the optimal length of intervals. By leveraging unequal interval obtained from k-means of algorithm with the help of Python, we construct delineating different fuzzy logical relationship groups (FLRGs) for each. These FLRGs serve as the basis for defining weight functions, crucial for computing forecasted outputs. Unlike conventional methods that rely on complex union and intersection operators of fuzzy sets, our method simplifies the process by employing a straightforward arithmetic mean.We validate our proposed method through empirical applications in forecasting university enrollments. Our results demonstrate superior accuracy, as measured by mean absolute percent error and root-mean-square error (RMSE), compared to alternative fuzzy time series approaches.In summary, our method offers a robust framework for interval separation forecasting, providing enhanced predictive capabilities particularly beneficial for domains characterized by fluctuating data patterns.

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I. Introduction

A "time series" denotes a collection of observations pertaining to any activity across different time intervals, such as hourly, daily, weekly, monthly, or yearly. It represents numerical data arranged sequentially and regularly, commonly utilized across various fields including signal processing, econometrics, mathematical finance, weather forecasting, and communications engineering.In 1965, Zadeh introduced a method aimed at enhancing human reasoning efficiency by popularizing fuzzy set theory. There are three distinct time series models involving exponential smoothing, regression, and moving average over a 7-year period of data from a sewing machine manufacturing goods group were investigated in the literature. The research concluded that for medium-term forecasting (less than six months), exponential smoothing and moving averages were suitable methods, while regression was more appropriate for longer-term forecasting (up to one year).

Song and Chissom (1993) pioneered FTS by utilizing fuzzy set theory to forecast enrollments based on data from the University of Alabama. This led to the development of various fuzzy forecasting techniques aimed at achieving superior forecasting results. Yolcu et al. (2009) proposed a novel interval length determination method using constrained optimization, while Aggarwal et al. (2017) employed fuzzy logic interval partitioning to investigate enrollment data intervals for better forecasting accuracyand also Nishad and Singh (2015) solve goal programming in fuzzy environment.

Abhishekh and Gautam (2019) demonstrated the effectiveness of FTS with a 3-year moving average method in forecasting enrollment and crop production. Chen (1996) introduced a new FTS method for enrollment forecasting at the University of Alabama, which utilized historical enrollment observations and simple arithmetic operations, proving to be efficient and less time-consuming compared to traditional methods.Huarng (2001) and Chen and Hsu (2004) introduced domain-specific and enrollment-based fuzzy forecasting methods respectively, with promising accuracy rates. Yu (2005) proposed a weighted technique for FTS modeling to forecast Taiwan Stock Index prices, comparing it with local regression models. Li and Cheng (2007) described FTS modeling as an approach to handling imprecise and incomplete datasets.

Jilani et al. (2008) proposed a FTS forecasting method based on frequency density-based partitioning of historical enrollment data, which was kth order and time-variant.

The study titled "A Study of Time Series Forecasting Using Fuzzy Interval Separation Method" introduces a novel approach to time series forecasting employing the Fuzzy Interval Separation (FIS) method. This method, rooted in fuzzy logic principles, aims to handle uncertainty and imprecision inherent in time series data by utilizing fuzzy intervals.The research begins with a comprehensive literature review, discussing existing forecasting techniques and their limitations, particularly in dealing with noisy and uncertain data. The theoretical background of fuzzy logic and fuzzy interval separation is presented, demonstrating their application in representing uncertain data and implementing the FIS method for forecasting.

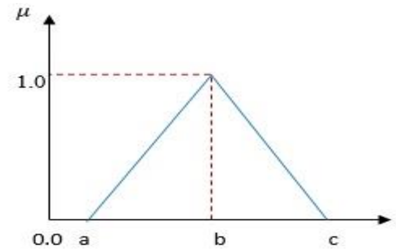

Fig. 1. The membership function of a TFN.

Empirical evaluation using real-world datasets across diverse domains compares the forecasting accuracy of the FIS method with traditional techniques like ARIMA and neural networks. Results indicate competitive performance and sometimes superiority of the FIS method, especially in scenarios with high data volatility and uncertainty.In conclusion, the study highlights the potential of fuzzy interval separation in improving the accuracy and robustness of TS forecasting models. The FIS method offers a promising alternative for analysts and researchers in domains where accurate predictions are crucial for decision-making.In this context, the study introduces the fuzzy interval separation method for addressing the unpredictability inherent in time series forecasting, particularly in enrollment data. It examines the efficacy of the Fuzzy Interval Separation (MAFIS) method for forecasting university applications, providing a comparative analysis with previous results for numerical interpretation.

II. Preliminaries:

In this section various type of basic definition are given to explain the basic need of the article.

2.1 Fuzzy Set: A fuzzy set $\tilde{F}$ in the discourse of the universe X, is a collection of ordered pairs such as:

$$\tilde{F} = \{(x, \mu_{\tilde{F}}(x)) | x \in X\}, \quad (1)$$

where, $\mu_{\tilde{F}}(x)$ is the membership grade (degree of truth) of x in $\tilde{F}$, which maps X to the membership space [0,1]. It may alternatively write fuzzy set $\tilde{F}$ as

$$\tilde{F} = \frac{\mu_{\tilde{F}}(x_1)}{x_1} + \frac{\mu_{\tilde{F}}(x_2)}{x_2} + \cdots + \frac{\mu_{\tilde{F}}(x_n)}{x_n}. \quad (2)$$

2.2 Triangular Fuzzy Number (TFN): If the membership function of a fuzzy set $\tilde{F} = (a,\ b,\ c)$, on a real line $\mathbb{R}$ is defined as (in Fig. 1.), then it is regarded as a TFN

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, if\ a \leq\ x < b, \\ 1, \quad if\quad x = b, \\ \frac{(c-x)}{(c-b)}, if\ \ b <\ x\ \leq c, \\ \quad 0, \quad \text{otherwise.} \end{cases} \quad (3)$$

2.3 Fuzzy Time Series (FTS): Let $X(t)$ $(t = 0, 1, 2, ...)$, be the universe of discourse (UoD) of real number subsets on which fuzzy sets$g_i(t)$ $(i = 1, 2, ...)$ are defined. If $G(t)$ is an accumulation of $g_i(t)$ $(i = 1, 2, ...)$, then $G(t)$ is regarded as a FTS on $X(t)$, $(t = 0, 1, 2, ...)$.

2.4 Fuzzy relation: Consider $X, Y \subseteq R$, be an universal sets, then, $R = \{((xy), \mu_R(xy)) | xy \in X \times Y\}$referred to as a fuzzy relation$X \times Y \subseteq R$or two universal sets are X and Y, and the fuzzy relation $R(x, y)$ is given as

$R(x, y) = \left\{\frac{\mu_R(xy)}{(xy)} : xy \in X \times Y\right\}$, Two-dimensional tables are a common format in which fuzzy relations are displayed. A $m \times n$ matrix represents, A pleasant method of interacting with fuzzy relation R.

$$R = \begin{matrix} & y_1 \ldots & y_n \\ x_1 & \mu_R(x_1, y_1) \ldots & \mu_R(x_1, y_n) \\ x_m & \mu_R(x_m, y_1) \ldots & \mu_R(x_m, y_n) \end{matrix}$$

2.5 Fuzzylogicalrelationships(FLR): Suppose that$F(t-1) = \widetilde{A}_i$and $F(t) = \widetilde{A}_j$, then FLR between $Fs(t-1)$ and $F(t)$ can be represented as $\widetilde{A}_i \longrightarrow \widetilde{A}_j$, where,$\widetilde{A}_i$is recognized as the present state and $\widetilde{A}_j$is the future state of FLR respectively.

2.6Fuzzy logical relationship groups (FLRG): Suppose that the subsequent fuzzy FLR as follows: $\widetilde{A}_i \longrightarrow \widetilde{A}_{j_1}$, $\widetilde{A}_i \longrightarrow \widetilde{A}_{j_1}, \widetilde{A}_i \longrightarrow \widetilde{A}_{j_1}, \ldots\ \widetilde{A}_i \longrightarrow \widetilde{A}_{j_n}$

These FLRs being in the same scenario right now, then they can group into a same FLRG. So, these FLRs can be arranged in the same FLRGs as:

$\widetilde{A}_i \longrightarrow \widetilde{A}_{j_1}, \widetilde{A}_{j_2}, \ldots, \widetilde{A}_{j_n}$.

Assume that$F(t-1) = \widetilde{A}_i$ and $F(t) = \widetilde{A}_j$, then FLR able to be portrayed as $\widetilde{A}_i \longrightarrow \widetilde{A}_j$ where,$\widetilde{A}_i$is recognized as the present state and $\widetilde{A}_j$ is the future state of FLR respectively. If $F(t)$is produced by additional fuzzy sets $F(t-n), F(t-n+1), \ldots, F(t-1)$then the FLRs can be represented as $\widetilde{A}_{i1}, \widetilde{A}_{i2}, \ldots, \widetilde{A}_{in} \longrightarrow \widetilde{A}_j$, where $F(t-n) = \widetilde{A}_{i1}, F(t-n+1) = \widetilde{A}_{i2}, \ldots, F(t-n) = \widetilde{A}_{in}$. The relationship is called n^{th}order (high-order) FTS model. Here $F(t-n), \ldots, F(t-2), F(t-1)$ and $F(t)$ areknown as the present state and the future state of FLR respectively.

III. A New Method for Time Series Forecasting:

In this segment, a new modified method for modeling the enrollment of the University of Alabama based on the separation of intervals method for forecasting has been presented. The steps of the proposed method are given as below

Step 1: In this step we calculate the unequal intervals $u_{eq1}, u_{eq2}, u_{eq3}, \ldots, u_{eqm}$, by using k-means centroid algorithms with the help of python.

Step 2: Employ the frequency statistical distribution method on historical time series data for each designated interval $u_{eq1}, u_{eq2}, u_{eq3}, \ldots, u_{eqm}$, Identify the interval containing the highest quantity of historical distribution data; this interval is subsequently subdivided into four equal-length subintervals, in the case of tie select any one of them. Proceed to locate the interval with the second highest quantity of historical distribution data; this is then divided into three equal-length subintervals. Next, determine the interval with the third highest amount of historical distribution data and bifurcate it into two equal-length subintervals. Lastly, identify the interval with the fourth highest volume of historical distribution data, leaving the length of this interval unaltered. Should any interval lack historical distribution data, it is to be excluded. The resulting subdivided intervals are denoted as $v_{eq1}, v_{eq2}, v_{eq3}, \ldots, v_{eqn}$.

Step 3: Construct n triangular fuzzy numbers (TFNs), denoted as $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \ldots, \tilde{A}_n$ corresponding to n intervals. For each piece of historical data, define the membership grades with respect to these TFNs.

Step 4: Fuzzify the historical time series data by choosing the fuzzy number having maximum grade of membership.

Step 5: *Establishment of fuzzy logical relationships (FLRs) and fuzzy logical groups (FLRGs)*

If $\tilde{A}_i$ denote the fuzzified value of historical TS data for the year n and $\tilde{A}_j$ is fuzzified value for the year n-1. Then the FLR is given as $\tilde{A}_j \rightarrow \tilde{A}_i$, here $\tilde{A}_j$ and $\tilde{A}_i$ are the current state and next state of fuzzified historical time series data respectively. Similarly define FLRGs, for example if FLRs are given as $\tilde{A}_j \rightarrow \tilde{A}_i$ and $\tilde{A}_j \rightarrow \tilde{A}_k$ then FLRG is defined as $\tilde{A}_j \rightarrow \tilde{A}_i, \tilde{A}_k$.

Step 6: *Weights assigning rule:*

Suppose that the fuzzy logical relationship at the time t is given as $\tilde{A}_i \rightarrow \tilde{A}_{i1}, \tilde{A}_{i2}, \tilde{A}_{i3}, \ldots, \tilde{A}_{ik}$. Then the corresponding weight for each fuzzy set are given as: $w_1, w_2, \ldots, w_k$, furthermore these weights normalized and subsequently the weighted matrix is obtained as given below:

Normalized weights $w_1' = \frac{w_1}{\sum_{r=1}^{k} w_r}, w_2' = \frac{w_2}{\sum_{r=1}^{k} w_r}, \ldots, w_k' = \frac{w_k}{\sum_{r=1}^{k} w_r}$,

And the weighted matrix is $W_m(t) = [w_1', w_2', \ldots, w_k']$ (4)

If $w_1, w_1, \ldots, w_k$, are the corresponding weights for the fuzzy set $\tilde{A}_{i1}, \tilde{A}_{i2}\tilde{A}_{i3}, \ldots, \tilde{A}_{ik}$, then the weight can be taken as $w_1 = 1, w_2 = 2, \ldots, w_k = k$,

Step 7: *Calculation of forecasted output*

Suppose we want to calculate forecasted output for the time t. Assume that the fuzzified data at that time is $\tilde{A}_j$ and fuzzified value at the time t-1 is $\tilde{A}_i$. Then from the fuzzy logical relationship groups if $\tilde{A}_i \rightarrow \tilde{A}_{i1}, \tilde{A}_{i2}\tilde{A}_{i3}, \ldots, \tilde{A}_{ik}$, then the forecasted output $F(t)$at the time t is given as

$$F(t) = M(t) \times W_m(t)^T, \quad (5)$$

where, $M(t)$ is the defuzzified matrix of mid points of the fuzzy numbers $\tilde{A}_{i1}, \tilde{A}_{i2}\tilde{A}_{i3}, \ldots, \tilde{A}_{ik}$ defined as $M(t) = [m_{i1}m_{i2}m_{i3} \ \ldots . m_{ik}]$

So that the forecasted output of each year can be calculated in similar way.

Step 8: The accuracy of a TS is evaluated using either the mean square error (MSE) or the average error (AE). A lower MSE or average error indicates a superior forecasting method. The MSE and forecasting error are defined as follows:

$$\text{Mean Square error } = \frac{\sum_{i=1}^{n}(actual\ value - forecasted\ value)^2}{n}$$

$$\text{Forecasting error (in percentage)} = \frac{|orecasted\ value - actual\ value|}{actual\ value} * 100$$

IV. Numerical Illustration:

In this section, we address the enrollment forecasting problem for the University of Alabama as an illustrative example of the algorithm presented in Section 3. The procedural steps are outlined as follows:

Step 1: The unequal intervals $u_{eq1}, u_{eq2}, u_{eq3}, \ \ldots, u_{eqm}$, by using k-means centroid algorithms with the help of python are given as:

$u_1 = [12547, 13563]$, u_2 =[13259, 13867],$u_3 = [13038, 14696]$, $u_4 = [14129, 15263]$,$u_5 = [15006, 15520]$, $u_6 = [15117.5, 15922.5]$, ,$u_7 = [15457, 16388]$,$u_8 = [15914.33, 6861.66]$,$u_9 = [15573.33, 18150]$, $u_{10} = [17377, 18923]$, $u_{11} = [18518, 19328]$, $u_{12} = [19033.35, 9631.64]$

Step 2: Utilizing the frequency statistical distribution method on historical TS data within each designated interval $u_1, u_2, u_3, \ \ldots, u_{12}$, it becomes evident that u_4, u_6 and u_8 exhibit the highest frequency distribution of time series data. Consequently, we divide these intervals into four, three, and two equal parts, respectively. So that frequency distribution table and new set of intervals are given as follows:

Table1. Frequency distribution data

S. No.	Intervals	Frequency
u_1	[10709.59, 15400.4]	7
u_2	[11217.59, 15908.4]	12
u_3	[11507.59, 16198.4]	13
u_4	[12350.59, 17041.4]	17
u_5	[12860.92, 17551.74]	17
u_6	[13152.84, 17843.65]	16
u_7	[13577.09, 18267.9]	16
u_8	[14042.59, 18733.4]	15
u_9	[14516.25, 19207.06]	17
u_{10}	[15804.59, 20495.4]	11
u_{11}	[16577.59, 21268.4]	8
u_{12}	[16987.09, 21677.9]	5

The new set of intervals are as follows:

v_1 =[10709.59, 15400.4], v_2 =[11217.59, 15908.4], v_3 =[11507.59, 16198.4],
v_4 =[12350.59, 13523.29], v_5 =[13523.29, 14695.98], v_6 =[14695.98, 15868.67],
v_7 =[15868.67, 17041.4], v_8 =[12860.92, 17551.74], v_9 =[13152.84, 14716.44],
v_{10} =[14716.44, 16280.08], v_{11} =[16280.08, 17843.65], v_{12} =[13577.09, 18267.9],
v_{13} =[14042.59, 16387.49], v_{14} =[16387.49, 18733.4], v_{15} =[14516.25, 19207.06],
v_{16} =[15804.59, 20495.4], v_{17} =[16577.59, 21268.4], v_{18} =[16987.09, 21677.9],

Step 3: Now we construct 18 fuzzy sets $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \ldots, \tilde{A}_{18}$ corresponding to the linguistic intervals $v_1, v_2, v_3, \ldots, v_{18}$ as follows:

$\tilde{A}_1 =< 10709.59, 13054.99, 15400.4 >, \tilde{A}_2 =< 11217.59, 13563.99, 15908.4 >, \tilde{A}_3 =< 11507.59, 13852.99, 16198.4 >, \tilde{A}_4 =< 12350.59, 12936.94, 13523.29 >$

,$\tilde{A}_5 =< 13523.29, 14109.63,14695.98 >$,
$\tilde{A}_6 =< 14695.98, 15282.32,15868.67 >$, $\tilde{A}_7 =< 15868.67, 16455.03,17041.4 >$, $\tilde{A}_8 =< 12860.92, 15206.33,17551.74 >$,
$\tilde{A}_9 =< 13152.84, 13934.64, 14716.44 >$,$\tilde{A}_{10} = 14716.44, 15498.26, 16280.08 >$,
$\tilde{A}_{11} =< 16280.08,17061.86, 17843.65 >$, $\tilde{A}_{12} =< 13577.09,15922.5, 18267.9 >$,
$\tilde{A}_{13} =< 14042.59, 15215.04,16387.49 >$, $\tilde{A}_{14} =< 16387.49,17560.4, 18733.4 >$,
$\tilde{A}_{15} =< 14516.25, 16861.65, 19207.06 >$, $\tilde{A}_{16} =< 15804.6,18149.99, 20495.4 >$, $\tilde{A}_{17} =< 16577.59,18923.99, 21268.4 >$, $\tilde{A}_{18} =< 16987.09,19332.49, 21677.9 >$

The grade of memberships for each data are given as:

$$\tilde{A}_1 = \langle \frac{1}{13055} + \frac{0.78}{13563} + \frac{0.65}{13867} + \frac{0.30}{14696} + \frac{0.03}{15311} + \frac{0.10}{15145} + \frac{0.10}{15163} \rangle$$

$$\tilde{A}_2 = \langle \frac{0.78}{13055} + \frac{0.99}{13563} + \frac{0.87}{13867} + \frac{0.51}{14696} + \frac{0.19}{15460} + \frac{0.25}{15311} + \frac{0.13}{15603} + \frac{0.02}{15861} + \frac{0.20}{15433} + \frac{0.17}{15497} + \frac{0.32}{15145} + \frac{0.31}{15163} \rangle$$

$$\tilde{A}_3 = \langle \frac{0.66}{13055} + \frac{0.87}{13563} + \frac{0.99}{13867} + \frac{0.64}{14696} + \frac{0.31}{15460} + \frac{0.37}{15311} + \frac{0.25}{15603} + \frac{0.14}{15861} + \frac{0.32}{15433} + \frac{0.29}{15497} + \frac{0.44}{15145} + \frac{0.44}{15163} \rangle$$

$$\tilde{A}_4 = \langle \frac{0.79}{13055} \rangle$$

$$\tilde{A}_5 = \langle \frac{0.06}{13563} + \frac{0.58}{13867} \rangle$$

$$\tilde{A}_6 = \langle \frac{0.69}{15460} + \frac{0.95}{15311} + \frac{0.45}{15603} + \frac{0.01}{15861} + \frac{0.74}{15433} + \frac{0.63}{15497} + \frac{0.76}{15145} + \frac{0.79}{15163} \rangle$$

$$\tilde{A}_7 = \langle \frac{0.40}{16807} + \frac{0.20}{16919} + \frac{0.88}{16388} + \frac{0.19}{15984} + \frac{0.31}{16859} \rangle$$

$$\tilde{A}_8 = \langle \frac{0.08}{13055} + \frac{0.29}{13563} + \frac{0.42}{13867} + \frac{0.78}{14696} + \frac{0.89}{15460} + \frac{0.95}{15311} + \frac{0.83}{15603} + \frac{0.72}{15861} + \frac{0.31}{16807} + \frac{0.27}{16919} + \frac{0.49}{16388} + \frac{0.90}{15433} + \frac{0.87}{15497} + \frac{0.97}{15145} + \frac{0.98}{15163} + \frac{0.66}{15984} + \frac{0.29}{16859} \rangle$$

$$\tilde{A}_9 = \langle \frac{0.52}{13563} + \frac{0.91}{13867} + \frac{0.02}{14696} \rangle$$

$$\tilde{A}_{10} = \langle \frac{0.95}{15460} + \frac{0.76}{15311} + \frac{0.86}{15603} + \frac{0.53}{15861} + \frac{0.91}{15433} + \frac{0.99}{15497} + \frac{0.54}{15145} + \frac{0.57}{15163} + \frac{0.37}{15984} \rangle$$

$$\tilde{A}_{11} = \langle \frac{0.67}{16807} + \frac{0.81}{16919} + \frac{0.13}{16388} + \frac{0.74}{16859} \rangle$$

$$\tilde{A}_{12} = \langle \frac{0.12}{13867} + \frac{0.47}{14696} + \frac{0.80}{15460} + \frac{0.73}{15311} + \frac{0.86}{15603} + \frac{0.97}{15861} + \frac{0.62}{16807} + \frac{0.57}{16919} + \frac{0.80}{16388} + \frac{0.79}{15433} + \frac{0.81}{15497} + \frac{0.66}{15145} + \frac{0.67}{15163} + \frac{0.97}{15984} + \frac{0.60}{16859} + \frac{0.05}{18150} \rangle$$

$$\tilde{A}_{13} = \langle \frac{0.55}{14696} + \frac{0.79}{15460} + \frac{0.91}{15311} + \frac{0.66}{15603} + \frac{0.44}{15861} + \frac{0.81}{15433} + \frac{0.76}{15497} + \frac{0.94}{15145} + \frac{0.95}{15163} + \frac{0.34}{15984} \rangle$$

$$\tilde{A}_{14} = \langle \frac{0.35}{16807} + \frac{0.45}{16919} + \frac{0.00}{16388} + \frac{0.40}{16859} + \frac{0.49}{18150} \rangle$$

$$\tilde{A}_{15} = \langle \frac{0.07}{14696} + \frac{0.40}{15460} + \frac{0.33}{15311} + \frac{0.46}{15603} + \frac{0.57}{15861} + \frac{0.97}{16807} + \frac{0.97}{16919} + \frac{0.79}{16388} + \frac{0.39}{15433} + \frac{0.41}{15497} + \frac{0.26}{15145} + \frac{0.27}{15163} + \frac{0.62}{15984} + \frac{0.99}{16859} + \frac{0.45}{18150} + \frac{0.10}{18970} + \frac{0.14}{18876} \rangle$$

$$\tilde{A}_{16} = \langle \frac{0.02}{15861} + \frac{0.42}{16807} + \frac{0.47}{16919} + \frac{0.24}{16388} + \frac{0.07}{15984} + \frac{0.4}{16859} + \frac{1}{18150} + \frac{0.60}{18970} + \frac{0.4}{19328} + \frac{0.4}{1937} + \frac{0.6}{18876} \rangle$$

$$\tilde{A}_{17} = \langle \frac{0.09}{16807} + \frac{0.14}{16919} + \frac{0.12}{16859} + \frac{.67}{18150} + \frac{0.98}{18970} + \frac{0.82}{19328} + \frac{0.82}{1937} + \frac{0.98}{18876} \rangle$$

$$\tilde{A}_{18} = \langle \frac{.49}{18150} + \frac{0.84}{18970} + \frac{0.99}{19328} + \frac{0.99}{1937} + \frac{0.80}{18876} \rangle$$

Step 4: Fuzzified historical TS data having maximum grade of membership are given as:

Table 2 Fuzzified historical data

Year	Actual enrollment	Fuzzified Number
1971	13055	$\tilde{A}_1$
1972	13563	$\tilde{A}_2$
1973	13867	$\tilde{A}_3$
1974	14696	$\tilde{A}_8$
1975	15460	$\tilde{A}_{10}$
1976	15311	$\tilde{A}_8$
1977	15603	$\tilde{A}_{10}$
1978	15861	$\tilde{A}_{12}$
1979	16807	$\tilde{A}_{15}$
1980	16919	$\tilde{A}_{15}$
1981	16388	$\tilde{A}_7$
1982	15433	$\tilde{A}_{10}$
1983	15497	$\tilde{A}_{10}$
1984	15145	$\tilde{A}_8$
1985	15163	$\tilde{A}_8$
1986	15984	$\tilde{A}_{12}$
1987	16859	$\tilde{A}_{15}$
1988	18150	$\tilde{A}_{16}$
1989	18970	$\tilde{A}_{17}$
1990	19328	$\tilde{A}_{18}$
1991	19337	$\tilde{A}_{18}$
1992	18876	$\tilde{A}_{17}$

Step 5: *Establishment of fuzzy logical relationships (FLRs) and fuzzy logical groups (FLRGs)*

Table 3. Fuzzy logical relationships

$\tilde{A}_1 \rightarrow \tilde{A}_2, \tilde{A}_2 \rightarrow \tilde{A}_3, \tilde{A}_3 \rightarrow \tilde{A}_8, \tilde{A}_8 \rightarrow \tilde{A}_{10}, \tilde{A}_{10} \rightarrow \tilde{A}_8, \tilde{A}_8 \rightarrow \tilde{A}_{10}, \tilde{A}_{10} \rightarrow \tilde{A}_{12}$, $\tilde{A}_{1\ 2}$ →Ã□□, $\tilde{A}_{1\ 5}$ → Ã□□, $\tilde{A}_{1\ 5}$ → Ã□, $\tilde{A}_7$ →Ã□□, $\tilde{A}_{1\ 0}$ →Ã□□, $\tilde{A}_{1\ 0}$ →Ã□, $\tilde{A}_8$ →Ã□, $\tilde{A}_8$ →Ã□□, $\tilde{A}_{1\ 2}$ →Ã□□, $\tilde{A}_{1\ 5}$ →Ã□□, $\tilde{A}_{1\ 6}$ →Ã□□, $\tilde{A}_{1\ 7}$ →Ã□□, $\tilde{A}_{1\ 8}$ →Ã□□, $\tilde{A}_{1\ 8}$ → $\tilde{A}_{1\ 7}$.

Table 4. Fuzzy logical relationship groups of historical TS data

Group 1	$\tilde{A}_1 \rightarrow \tilde{A}_2$
Group 2	$\tilde{A}_2 \rightarrow \tilde{A}_3$
Group 3	$\tilde{A}_3 \rightarrow \tilde{A}_8$
Group 4	$\tilde{A}_7 \rightarrow \tilde{A}_{10}$
Group 5	$\tilde{A}_8 \rightarrow \tilde{A}_8, \tilde{A}_{10}, \tilde{A}_{10}, \tilde{A}_{12}$
Group 6	$\tilde{A}_{10} \rightarrow \tilde{A}_8, \tilde{A}_8, \tilde{A}_{10}, \tilde{A}_{12}$
Group 7	$\tilde{A}_{12} \rightarrow \tilde{A}_{15}, \tilde{A}_{15}$
Group 8	$\tilde{A}_{15} \rightarrow \tilde{A}_7, \tilde{A}_{15}, \tilde{A}_{16}$
Group 9	$\tilde{A}_{16} \rightarrow \tilde{A}_{17}$
Group 10	$\tilde{A}_{17} \rightarrow \tilde{A}_{18}$
Group 11	$\tilde{A}_{18} \rightarrow \tilde{A}_{17}, \tilde{A}_{18}$

Step 6: In this step weight and normalized weight for each FLRGs has been calculated by applying weight function defined in the step 2.

Step 7& 8:*Calculation of forecasted output and accuracy measurement*

To compute a forecasted value for the year 1981, we employ a methodology exemplified by the following steps. Firstly, we consider the fuzzified enrollment of the year 1980 in observation context denoted as $\tilde{A}_{15}$, We then select from the table 4., the belonging FLG is 8. The normalized weights matrix is obtained from equation (1) is $W_m(1981) = [\frac{1}{6}\frac{2}{6}\frac{3}{6}]$ and the defuzzified matrix $M(1981) = [16455.04\ 168661.66\ 18150]$, so that the predicted output is given as: $F(1981) = 16455.04 \times \frac{1}{6} + 168661.66 \times \frac{2}{6} + 18150 \times \frac{3}{6} = 17438.0$for the year 1981.

Similarly, the forecasted output can be obtained with the help of the proposed method and the accuracy measurements are given in the table 5.

Table 5. Comparison of forecasted output with different method and proposed method forenrollmentdataset

Year	Actual enrollment	Song & Chissom (1993a)	Gangwar & Kumar (2014a)	Y. Wang, Lei, Lei, et al. (2016a)	Bisht & Kumar (2016a)	Abhishekh & Kumar (2019)	Proposed method
1971	13055	-	-	-	-	-	-
1972	13563	14,000	-	13500	13595.67	-	13563
1973	13867	14,000	13693	14155	13814.75	13950	13853
1974	14696	14,000	13693	14155	14929.79	14550	15206
1975	15460	15,500	14867	15539	15541.27	15150	15639
1976	15311	16,000	15287	15539	15540.62	15350	15580
1977	15603	16,000	15376	15502	15540.62	15550	15639
1978	15861	16,000	15376	15502	15540.62	15950	15580
1979	16807	16,000	15376	16667	16254.50	16650	16862
1980	16919	16833	16523	16667	17040.41	16550	17438
1981	16383	16833	16606	15669	17040.41	16150	17438
1982	15433	16833	17519	15564	16254.50	15750	15498
1983	15497	16,000	16606	15564	15540.62	15350	15580
1984	15145	16,000	15376	15564	15540.62	15550	15580
1985	15163	16,000	15376	15523	15541.27	15350	15639
1986	15984	16,000	15287	15523	15541.27	15550	15639
1987	16895	16,000	15287	16799	16254.50	16650	16862
1988	18150	16813	16523	18268	17040.41	17950	17438
1989	18970	19000	17519	18268	18902.30	18750	18923
1990	19328	19000	19500	18780	19357.30	19150	19196
1991	19337	19000	19000	19575	19168.56	19150	19196
1992	18876		19500	18825	19168.56	19150	19196

Table 6. Accuracymeasurescomparison

Evaluation criteria	Song and Chissom (1993)	Gangwar and Kumar (2014)	Wang et al. (2016)	Bisht and Kumar (2016)	Abhishekh and Gautam (2019)	Proposed method
MSE	423020.16	243601.47	123130.81	183723.677	56863.17	143585.1
RMSE	650.4	493.56	350.9	428.63	428.63	378.92
MFE	3.22	2.36	1.72	1.94	1.29	1.65

V. Conclusion and Result:

Conventional forecasting methods are utilized to disseminate precise information; however, there are instances where complete accuracy and correctness remain elusive. To achieve significant outcomes, the inherent unpredictability in time series data can be mitigated by introducing fuzzy concepts, this investigation implements with fuzzy interval separation. To evaluate the accuracy of the proposed method against the conventional model, enrollment data is forecasted using the fuzzy interval separating technique. The pragmatic results obtained through Mean Squared Error (MSE) facilitate forecasting. Various conventional methods for forecasting Root Mean Squared Error (RMSE), including the fuzzy moving average method, are considered.

Researchers can enhance overall forecasting performance in future studies by employing novel meta-heuristic optimization strategies in the defuzzification process. Additionally, weight functions can be utilized to establish fuzzy relationships, thus improving forecasting accuracy rates. Furthermore, the study explores artificial intelligence techniques such as genetic algorithms (GA), particle swarm optimization (PSO), and neural networks (NN) to refine fuzzy relations and the fuzzification process, thereby reducing forecasting errors. Consequently, this study suggests that the proposed approach can enhance forecasting performance and accuracy rates in FTS models, particularly in the presence of ambiguous information.

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