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On Pre Generalized Regular α-Continuous and Irresolute Mappings in Intuitionistic Fuzzy Topological Space

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ABSTRACT
In this paper, we introduce and study the notions of intuitionistic fuzzy pre generalized regular α-continuous mappings and intuitionistic fuzzy pre generalized regular α-irresolute mappings and study some of its properties in intuitionistic fuzzy topological spaces.

KEYWORDS: *Intuitionistic fuzzy topology, Intuitionistic fuzzy point, Intuitionistic fuzzy pre generalized regular α-losed sets, Intuitionistic fuzzy pre generalized regular α-continuous mappings and Intuitionistic fuzzy pre generalized regular α-irresolute mappings.*

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I. INTRODUCTION

The concept of fuzzy set[FS] was introduced by Zadeh [14] and later fuzzy topology was introduced by Chang [2] in 1967. By adding the degree of non membership to FS,Atanassov [1] proposed intuitionistic fuzzy set[IFS] using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduced intuitionistic fuzzy pre generalized regular α-continuous mappings and intuitionistic fuzzy pre generalized regular α-irresolute mappings and studied some of their basic properties.

II. PRELIMINARIES

Throughout this paper, (X, τ), (Y, σ) and (Z, γ) (or simply X, Y and Z) denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1: [1] Let X be a nonempty set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership(namely $\nu_A(x)$) of each element x∈ X to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each x∈ X. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B =\{\langle x, \mu_B(x), \nu_B(x)\rangle / x\in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(ii) A = B if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ \langle x, \nu_A(x), \mu_A(x)\rangle / x \in X \}$,
(iv) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$,
(v) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation$A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A,$

$B/\mu_B)$, $(A/\nu_A, B/\nu_B)\rangle$. The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3:[3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short)in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then

(i) $int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A \}$,

(ii) $cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K \}$,

(iii) $cl(A^c) = (int(A))^c$,

(iv) $int(A^c) = (cl(A))^c$.

Definition 2.5: [4] An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy regular closed set (IFRCS in short) if A = cl(int(A)),

(ii) intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)),

(iii) intuitionistic fuzzy semiclosed set (IFSCS in short) if int(cl(A)) ⊆ A,

(iv) intuitionistic fuzzy semiopen set (IFSOS in short) if A ⊆ cl(int(A)),

(v) intuitionistic fuzzy preclosed set (IFPCS in short) if cl(int(A)) ⊆ A,

(vi) intuitionistic fuzzy preopen set (IFPOS in short) if A ⊆int(cl(A)),

(vii) intuitionistic fuzzy α-closed set (IFαCS) if cl(int(cl(A))) ⊆ A,

(viii) intuitionistic fuzzy α-open set (IFαOS in short) if A ⊆int(cl(int(A)))

Definition 2.6: [4] Let $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then

(i) pint(A) = ∪{G : G is an IF P OS in X and G ⊆ A},

(ii) pcl(A) = ∩{K : K is an IF P CS in X and A ⊆ K}.

Definition 2.7: [12] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is called an intuitionistic fuzzy regular α- open set (IFRαOSfor short) if there exist an IFROS U such that U ⊆A ⊆αcl(U).

Definition 2.8: An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is called an

(i) intuitionistic fuzzy weakly closed set (IFWCSfor short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFSOS[11]

(ii) intuitionistic fuzzy generalized pre regular closed set (IFGPRCSfor short) if pcl(A) ⊆ U whenever A ⊆ U and U is an IFROS[13]

An IFS A is said to be an intuitionistic fuzzy weakly open set (IFWOSfor short) and intuitionistic fuzzy generalized pre regular open set (IFGPROSfor short) if the complement of A is an IFWCS and IFGPRCS respectively.

Definition 2.9: [12] An IFS $A= \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy regular generalized α-closed set (IFRGαCSfor short) if αcl(A) ⊆ U whenever A ⊆ U, U is IFRαOS in X.

An IFS A is said to be an intuitionistic fuzzy regular generalized α-open set (IFRGαOS for short) in (X, τ) if the complement of A is an IFRGαCS in X.

Every IFWCS is an IFRGαCS in X.

Definition 2.10: [8] An IFS $A= \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy regular weakly generalized closed set (IFRWGCSfor short) if cl(int(A)) ⊆ U whenever A ⊆ U, U is IFROS in X.

An IFS A is said to be an intuitionistic fuzzy regular weakly generalized open set (IFRWGOS for short) in (X, τ) if the complement of A is an IFRWGCS in X.

Definition 2.11: [7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy pre generalized regular α-closed set (IFPGRαCS for short) if pcl(A) ⊆ U whenever A ⊆ U and U is an IFRαOS in (X, τ).

The family of all IFPGRαCSs of an IFTS (X, τ) is denoted by IFPGRαC(X). Every IFCS, IFRCS, IFαCS, IFPCS, IFWCS, IFRGαCS are an IFPGRαCS and every IFPGRαCS is an IFRWGCS and IFGPRCS but the converses are not true in general.

Definition 2.12: [7] The complement A^c of an IFPGRαCS A in an IFTS (X, τ) is called an intuitionistic fuzzy pre generalized regular α-open set (IFPGRαOS for short) in X.

Every IFOS, IFROS, IFαOS, IFPOS, IFWOS, IFRGαOS are an IFPGRαOS and every IFPGRαOS is an IFRWGOS and IFGPROS but the converses are not true in general.

Definition 2.13: [10] Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP for short) $p_{(\alpha, \beta)}$ of X is an IFS of X defined by

$$p_{(\alpha, \beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if} \quad y = p \\ (0, 1) & \text{if} \quad y \neq p \end{cases}$$

Definition 2.14: [7] An IFTS (X, τ) is called an intuitionistic fuzzy pre generalized regular α-$T_{1\backslash 2}$ space (IFPGRα-$T_{1\backslash 2}$ space for short) if every IFPGRαCS is an IFPCS.

Definition 2.15: [7] An IFTS (X, τ) is called an intuitionistic fuzzy pre generalized regular -$T_{1\backslash 2}$ space (IFPGR-$T_{1\backslash 2}$ space for short) if every IFPGRαCS is an IFCS.

Definition 2.16: [4] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous(IF continuous in short) if $f^{-1}(B) \in$ IFO(X) for every $B \in \sigma$.

Definition 2.17: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be

(i) intuitionistic fuzzy α- continuous(IFα continuous in short) if $f^{-1}(B) \in$IFαO(X) for every $B \in \sigma$,

(ii) intuitionistic fuzzy pre continuous(IFP continuous in short) if $f^{-1}(B) \in$IFPO(X) for every $B \in \sigma$.

Definition 2.18: [11] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy weakly continuous (IFW continuous for short) mappings if $f^{-1}(V)$ is an IFWCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.19: [6] A mapping f: (X, τ) →(Y, σ) is called an intuitionistic fuzzy regular generalized α-continuous (IFRGα continuous for short) mapping if $f^{-1}(V)$ is an IFRGαCS in (X, τ) for every IFCS V of (Y, σ). Every IFW continuous mapping is an IFRGα continuous mapping but not conversely.

Definition 2.20: [13] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy generalized pre regular continuous (IFGPR continuous for short) mappings if $f^{-1}(V)$ is an IFGPRCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.21:[9] A mapping f: (X, τ) →(Y,σ) is called an intuitionistic fuzzy regular weakly generalized continuous (IFRWG continuous for short) mapping if $f^{-1}(V)$ is an IFRWGCS in (X, τ) for every IFCS V of (Y, σ).

Result 2.22: [7] For any IFS A in (X, τ) where X is an IFPGRα$T_{1/2}$ space, A ∈IFPGRαO(X) if and only if for every IFP $p_{(\alpha, \beta)} \in A$, there exists an IFPGRαOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

III. INTUITIONISTIC FUZZY PRE GENERALIZED REGULAR α-CONTINUOUS MAPPINGS

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzypre generalized regular α-continuous (IFPGRαcontinuous for short) mappings if $f^{-1}(V)$ is an IFPGRαCS in (X, τ) for every IFCS V of (Y, σ).

For the sake of simplicity, we shall use the notation A= ⟨x, (μ, μ), (ν, ν)⟩ instead of A= ⟨x,(a/μ_a, b/μ_b), (a/ν_a, b/ν_b)⟩ in all the examples used in this paper. Similarly we shall use the notation B = ⟨x, (μ, μ), (ν, ν)⟩instead of B = ⟨x, (u/μ_u, v/μ_v), (u/ν_u, v/ν_v)⟩ in the following examples.

Example 3.2: Let X= {a, b}, Y= {u, v} and G_1 = ⟨x, (0.3, 0.3), (0.7, 0.7)⟩, G_2 = ⟨y, (0.8, 0.7), (0.2, 0.2)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} andσ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFPGRα continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFPGRαcontinuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFPGRαCS,$f^{-1}(V)$ is an IFPGRαCS in X. Hence f is an IFPGRα continuous mapping.

Example 3.4: In Example 3.2, f: (X, τ) → (Y, σ) is an IFPGRαcontinuous mapping but not an IF continuous mapping. Since G_2 = ⟨y, (0.8, 0.7),(0.2, 0.2)⟩ is an IFOS in Y but $f^{-1}(G_2)$ = ⟨x, (0.8, 0.7), (0.2, 0.2)⟩ is not an IFOS in X.

Theorem 3.5: Every IFα continuous mapping is an IFPGRα continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFPGRαCS,$f^{-1}(V)$ is an IFPGRαCS in X. Hence f is an IFPGRα continuous mapping.

Example 3.6: In Example 3.2, f: (X, τ) → (Y, σ) is an IFPGRα continuous mapping but not an IFα continuous mapping. Since G_2 = ⟨y, (0.8, 0.7), (0.2, 0.2)⟩ is an IFOS in Y but $f^{-1}(G_2)$ =⟨x, (0.8, 0.7), (0.2, 0.2)⟩ is not an IFαOS in X.

Theorem 3.7: Every IFP continuous mapping is an IFPGRαcontinuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFPGRαCS, $f^{-1}(V)$ is an IFPGRαCS in X. Hence f is an IFPGRα continuous mapping.

Example 3.8: Let X = {a, b}, Y = {u, v} and G_1 =⟨x, (0.7, 0.9), (0.3, 0.1)⟩, G_2 = ⟨y,(0.3, 0.1), (0.7, 0.9)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFPGRα continuous mapping but not an IFP continuous mapping.

Theorem 3.9: Every IFRGα continuous mapping is an IFPGRα continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFRGα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFRGαCS in X. Since every IFRGαCS is an IFPGRαCS, $f^{-1}(V)$ is an IFPGRαCS in X. Hence f is an IFPGRα continuous mapping.

Example 3.10: In Example 3.2, f: (X, τ) → (Y, σ) is an IFPGRα continuous mapping but not an IFRGα continuous mapping. Since G_2 = ⟨y, (0.8, 0.7), (0.2, 0.2)⟩ is an IFOS in Y but $f^{-1}(G_2)$ = ⟨x, (0.8, 0.7), (0.2, 0.2)⟩ is not an IFRGαOS in X.

Theorem 3.11: Every IFW continuous mapping is an IFPGRα continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFW continuous mapping. Since every IFW continuous mapping is an IFRGα continuous mapping. By Theorem 3.9., f is an IFPGRα continuous mapping.

Example 3.12: In Example 3.2, f: (X, τ) → (Y, σ) is an IFPGRα continuous mapping but not an IFW continuous mapping. Since G_2 = ⟨y, (0.8, 0.7), (0.2, 0.2)⟩ is an IFOS in Y but $f^{-1}(G_2)$ = ⟨x, (0.8, 0.7), (0.2, 0.2)⟩ is not an IFWOS in X.

Theorem 3.13: Every IFPGRα continuous mapping is an IFGPR continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFPGRα continuous mapping. Let V be an IFCS in Y. Thenf $^{-1}(V)$ is an IFPGRαCS in X. Since every IFPGRαCS is an IFGPRCS, $f^{-1}(V)$ is an IFGPRCS in X. Hence f is an IFGPR continuous mapping.

Example 3.14: Let X = {a, b}, Y = {u, v} and G_1 =⟨x, (0.2, 0.3), (0.5, 0.6)⟩, G_2 =⟨y, (0.5, 0.4), (0.4, 0.4)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGPR continuous mapping but not an IFPGRα continuous mapping.

Theorem 3.15: Every IFPGRα continuous mapping is an IFRWG continuous mapping but not conversely.

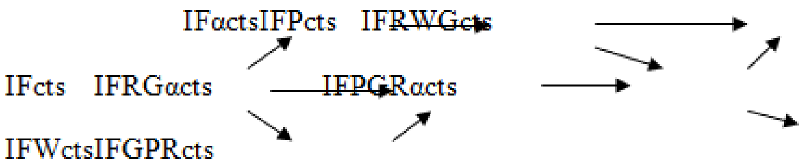
Proof: Let f: (X, τ) → (Y, σ) be an IFPGRα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPGRαCS in X. Since every IFPGRαCS is an IFRWGCS, $f^{-1}(V)$ is an IFRWGCS in X. Hence f is an IFRWG continuous mapping.

Example 3.16: In Example 3.14, f: (X, τ) → (Y, σ) is an IFRWG continuous mapping but not an IFPGRα continuous mapping.

Theorem 3.17: Let f: (X, τ) → (Y, σ) be a mapping wheref $^{-1}(V)$ is an IFRCS in X for every IFCS in Y. Then f is an IFPGRα continuous mapping but not conversely.

Proof: Let A be an IFCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFPGRαCS, $f^{-1}(V)$ is an IFPGRαCS in X. Hence f is an IFPGRα continuous mapping.

Example 3.18: In Example 3.2, f: (X, τ) → (Y, σ) is an IFPGRα continuous mapping but not a mapping defined in Theorem 3.17. The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts' means continuous.



Theorem 3.19: If f: (X, τ) → (Y, σ) is an IFPGRα continuous mapping, then for each IFP $p_{(\alpha, \beta)}$ of X and each A ∈ σ such that f($p_{(\alpha, \beta)}$) ∈ A there exists an IFPGRαOS B of X such that$p_{(\alpha, \beta)}$∈ B and f(B) ⊆ A.

Proof: Let $p_{(\alpha, \beta)}$ be an IFP of X and A ∈ σ such that f($p_{(\alpha, \beta)}$) ∈ A. Put B = $f^{-1}(A)$. Then by hypothesis B is an IFPGRαOS in X such that $p_{(\alpha, \beta)}$∈ B and f(B) = f($f^{-1}(A)$) ⊆ A.

Theorem 3.20: Let f: (X, τ) → (Y, σ) be an IFPGRα continuous mapping. Then f is an IFP continuous mapping if X is an IFPGRα$T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPGRαCS in X, by hypothesis. Since X is an IFPGRα$T_{1/2}$ space, $f^{-1}(V)$ is an IFPCS in X. Hence f is an IFP continuous mapping.

Theorem 3.21: Let f: (X, τ) → (Y, σ) be an IFPGRα continuous mapping and g: (Y, σ) → (Z, γ) bean IF continuous mapping, then g ∘ f: (X, τ) →(Z, γ) is an IFPGRαcontinuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y, by hypothesis. Since f is an IFPGRα continuous mapping,$f^{-1}(g^{-1}(V))$ is an IFPGRαCS in X. Hence g ∘ f is an IFPGRαcontinuous mapping.

Theorem 3.22: Let (Y, σ) be an $IFPGRT_{1/2}$ space, f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, γ) be two IFPGRα continuous mapping, then g ∘ f: (X, τ) →(Z, γ) is an IFPGRα continuous mapping.
Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFPGRαCS in Y. By hypothesis (Y, σ) is an $IFPGRT_{1/2}$ space implies that $g^{-1}(V)$ is an IFCS in Y. Since f is an IFPGRα continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFPGRαCS in X. Hence g ∘ f is an IFPGRα continuous mapping.
Theorem 3.23: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IFPGR\alpha T_{1/2}$ space:
(i) f is an IFPGRα continuous mapping,
(ii) $f^{-1}(B)$ is an IFPGRαOS in X for each IFOS B in Y,
(iii) for every IFP $p_{(\alpha, \beta)}$ in X and for every IFOS B in Y such that $f(p_{(\alpha, \beta)}) \in B$, there exists an IFPGRαOS A in X such that $p_{(\alpha, \beta)} \in A$ and f(A) ⊆ B.
Proof: (i) ⟺ (ii) is obvious, since $f^{-1}(B^c) = (f^{-1}(B))^c$.
(ii) ⇒ (iii) Let B be any IFOS in Y and let $p_{(\alpha, \beta)} \in X$. Given $f(p_{(\alpha, \beta)}) \in B$. By hypothesis $f^{-1}(B)$ is an IFPGRαOS in X. Take A= $f^{-1}(B)$. Now $p_{(\alpha, \beta)} \in f^{-1}(f(p_{(\alpha, \beta)}))$. Therefore $f^{-1}(f(p_{(\alpha, \beta)})) \in f^{-1}(B) = A$. This implies $p_{(\alpha, \beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.
(iii) ⇒ (i) Let A be an IFCS in Y. Then its complement, say B = A^c, is an IFOS in Y. Let $p_{(\alpha, \beta)} \in C$ and f(C) ⊆ B.Now $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. Thus $p_{(\alpha, \beta)} \in f^{-1}(B)$. Therefore $f^{-1}(B)$ is an IFPGRαOS in X by Result 2.24. That is $f^{-1}(A^c)$ is an IFPGRαOS in X and hence $f^{-1}(A)$ is an IFPGRαCS in X. Thus f is an IFPGRα continuous mapping.
Theorem 3.24: Let f : (X, τ) → (Y, σ) be an IFPGRα continuous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.
Proof: Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(cl(A^c)) = f^{-1}(A^c)$, since $A^c$is an IFCS. Now $(int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(A^c) = (f^{-1}(A))^c$. This implies $f^{-1}(A) \subseteq int(cl(int(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFαOS in X and hence it is an IFPGRαOS in X. Therefore f is an IFPGRα continuous mapping, by Theorem 3.23.

IV. INTUITIONISTIC FUZZY PRE GENERALIZED REGULAR α-IRRESOLUTE MAPPINGS

Definition 4.1: A mapping f: (X, τ) → (Y, σ) be an intuitionistic fuzzypre generalized regular α-irresolute (IFPGRα irresolute) mapping if $f^{-1}(V)$ is an IFPGRαCS in(X, τ) for every IFPGRαCS V of(Y, σ).
Example 3.2: Let X= {a, b}, Y= {u, v} and G_1 = ⟨x, (0.4, 0.5), (0.6, 0.5)⟩, G_2 = ⟨y, (0.2, 0.3), (0.8, 0.7)⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFPGRα irresolute mapping.
Theorem 4.2: Let f: (X, τ) → (Y, σ) be an IFPGRα irresolute mapping, then f is an IFPGRα continuous mapping but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be an IFPGRα irresolute mapping and V be any IFCS in Y. Then V is an IFPGRαCS in Y and by hypothesis, $f^{-1}(V)$ is an IFPGRαCS in X. Hence f is an IFPGRα continuous mapping.
The converse need not be true which can be seen from the following example.
Example 4.3: Let X= {a, b}, Y= {u, v} and G_1 = ⟨x, (0.3, 0.4), (0.7, 0.6)⟩, G_2 = ⟨y, (0.8, 0.7), (0.2, 0.2)⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Define a mappingf: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFPGRα continuous mapping but not an IFPGRα irresolute mapping.
Theorem 4.4: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, γ) be IFPGRα irresolute mapping. Theng ∘ f: (X, τ) →(Z, γ) is an IFPGRαirresolute mapping.
Proof: Let V be an IFPGRαCS in Z. Then $g^{-1}(V)$ is an IFPGRαCS in Y. Since f is an IFPGRα irresolute, $f^{-1}(g^{-1}(V))$ is an IFPGRαCS in X. Hence g ∘ f is an IFPGRα irresolute mapping.

Theorem 4.5: Let f: (X, τ) → (Y, σ) be an IFPGRα irresolute mapping and g: (Y, σ) → (Z, γ) be IFPGRα continuous mapping, the g ∘ f: (X, τ) →(Z, γ) is an IFPGRα continuous mapping.
Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFPGRαCS in Y. Since f is an IFPGRα irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFPGRαCS in X. Hence g ∘ f is an IFPGRα continuous mapping.
Theorem 4.6: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IFPGR\alpha T_{1/2}$ space:
(i) f is an IFPGRα irresolute mapping,
(ii) $f^{-1}(B)$ is an IFPGRαOS in X for each IFPGRαOS B in Y,
(iii) $f^{-1}(pint(B)) \subseteq pint(f^{-1}(B))$ for each IFS B of Y,
(iv) $pcl(f^{-1}(B)) \subseteq f^{-1}(pcl(B))$ for each IFS B of Y.

Proof: (i) ⟺ (ii) is obvious, since $f^{-1}(A^c) = (f^{-1}(A))^c$.
(ii) ⇒ (iii) Let B be any IFS in Y and pint(B) ⊆ B. Also f^{-1}(pint(B)) ⊆ f^{-1}(B). Since pint(B) is an IFPOS in Y, it is an IFPGRαOS in Y. Therefore f^{-1}(pint(B)) is an IFPGRαOS in X, by hypothesis. Since X is an IFPGRα$T_{1/2}$ space, f^{-1}(pint(B)) is an IFPOS in X. Hence f^{-1}(pint(B)) = pint(f^{-1}(pint(B))) ⊆pint(f^{-1}(B)).
(iii) ⇒ (iv) is obvious by taking complement in (iii).
(iv) ⇒ (i) Let B be an IFPGRαCS in Y. Since Y is an IFPGRα$T_{1/2}$ space, B is an IFPCS in Y and pcl(B) = B. Hence f^{-1}(B) = f^{-1}(pcl(B)) ⊇pcl(f^{-1}(B)), by hypothesis. But f^{-1}(B) ⊆pcl(f^{-1}(B)). Therefore pcl(f^{-1}(B)) = f^{-1}(B). This implies f^{-1}(B) is an IFPCS and hence it is an IFPGRαCS in X. Thus f is an IFPGRα irresolute mapping.
Theorem 4.7: Let f: (X, τ) → (Y, σ) be an IFPGRα irresolute mapping from an IFTS X into an IFTS Y. Then f^{-1}(B) ⊆pint(f^{-1}(int(cl(B)))) for every IFPGRαOS B in Y, if X and Y are IFPGRα$T_{1/2}$ spaces.
Proof: Let B be an IFPGRαOS in Y. Then by hypothesis f^{-1}(B) is an IFPGRαOS in X. Since X is an IFPGRα$T_{1/2}$ space, f^{-1}(B) is an IFPOS in X. Therefore pint(f^{-1}(B)) = f^{-1}(B). Since Y is an IFPGRα$T_{1/2}$ space, B is an IFPOS in Y and B ⊆int(cl(B)). Now f^{-1}(B) = pint(f^{-1}(B)) ⊆ pint(f^{-1}(int(cl(B))) implies, f^{-1}(B) ⊆ pint(f^{-1}(int(cl(B))).
Theorem 4.8: Let f: (X, τ) → (Y, σ) be an IFPGRα irresolute mapping from an IFTS X into an IFTS Y. Then f^{-1}(B) ⊆pint(int(cl(f^{-1}(B)))) for every IFPGRαOS B in Y, if X and Y are IFPGRα$T_{1/2}$ spaces.
Proof: Let B be an IFPGRαOS in Y. Then by hypothesis f^{-1}(B) is an IFPGRαOS in X. Since X is an IFPGRα$T_{1/2}$ space, f^{-1}(B) is an IFPOS in X. Therefore pint(f^{-1}(B)) = f^{-1}(B) ⊆int(cl(f^{-1}(B))). Hence f^{-1}(B) ⊆pint(int(cl(f^{-1}(B)))).
Theorem 4.9: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an IFPGRα$T_{1/2}$ space:

(i) f is an IFPGRα irresolute mapping.
(ii) f is an IFPGRα continuous mapping.

Proof: (i) ⇒(ii) Let V be an IFCS in (Y, σ). Since every IFCS is an IFPGRαCS and f is an IFPGRα irresolute, f^{-1}(V) is an IFPGRαCS in (X, τ). Thus f isIFPGRα continuous.
(ii) ⇒(i) Let F be an IFPGRαCS in (Y, σ). Since Y is an IFPGRα$T_{1/2}$ space, F is an IFCS in Y. By hypothesis f^{-1}(F) is an IFPGRαCS in X. Therefore f is an IFPGRα irresolute.

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