Fuzzy Reliability Analysis in Interconnection Networks

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Abstract
Multiprocessors are quite suitable for task-based applications for the fact that they can support graceful degradation. However the uncertainty associated with the failure ,needs to be addressed for these systems. This paper proposes the fuzzy reliability model for multiprocessor interconnection systems, which takes into an account of uncertainty process. The effect of processor failure rate and coverage on fuzzy reliability is investigated.

Keywords: Reliability, Coverage, Failure rate, Fuzzy reliability, Interconnection network,

I. Introduction
Advances in computer technology and the need to have the computers communicating with each other have led to an increased demand for reliable parallel computing systems. The multiprocessing systems typically consist of an ensemble of processors. They have found wide applications in real time environments and are becoming increasingly popular for large commercial application as well. Researchers in the past have developed techniques for the combined analysis of performance and reliability [2][3]. Reliability of an interconnection system depends upon the reliability of its components. However, for large parallel processing systems, it is very difficult to evaluate the probability of many failures since they might have never occurred before. Also for a system working in an ever-changing environment, feedback does not help much in estimating system reliability using classical statistics. The conventional statistical methods fail to give correct measures of reliability of a parallel computer network. The reason may be, due to insufficiency of failure data, variations due to different reporting sources, variations in application, fluctuations in environmental conditions etc**.,** if they are not due to erroneous modeling.

The multiprocessor systems are quite suitable for task-based applications because of their ability to support graceful degradation. Graceful degradation in multiprocessors can be accomplished through automatic reconfiguration and recovery capabilities provided by the on-line maintenance units [1]. However, the system designer must be certain that faults and errors are detected promptly, so that the redundant processing modules can be utilized. Models that can predict the reliability of systems incorporate a parameter called Coverage(C).

Definition 1. The coverage (C) = Pr{System recover | fault occurs} reflects the ability of the system to automatically recover from occurrence of a fault during normal system operation.
In a practical situation, the error handling naturally depends on the detection of the error, but can range from error correction or masking, to instruction retry, to complete reconfiguration of the system, and thus coverage can be a very difficult parameter to predict in a real life parallel computing environment. Because of system dependencies and the attribute of graceful degradation it is hard to interpret the meaning of success (failure) in a multiprocessor interconnection system. These systems often react to a detected failure by reconfiguring to a state, which may have a decreased level of performance. In these situations, subsystem or component failures may result in the performance degradation.

All the previous analysis on parallel computers has addressed its various aspects using the conventional statistical procedures. However, the uncertainty associated with the system functioning has not received enough attention. The conventional analysis using probability concepts alone is not adequate to treat such imprecise nature of systems performance data. Thus, it has become necessary to develop a new kind of formalism to capture the subjectivity and the impression of failure data for use in reliability analysis. Fuzzy set theory provides a powerful approach for solving these kinds of problems involving uncertainties [4][5]. By resorting to this concept a degree of uncertainty needs to be allocated to each value of the probability of failure and thus different aspects of uncertainty i.e. possibility and probability are treated simultaneously. The concept of fuzzy reliability has been proposed and developed by several authors[7][8].Nahman [6] presented a method for assessment of reliability of non-series parallel network using fuzzy logic. Tripathy et al. [5] have proposed a method to evaluate fuzzy reliability of MIN's. Narasimhan et al. [7] presented a method for evaluating fuzzy reliability of a communication with fuzzy elements capabilities and probabilities. Bastani et. al [8] considered the reliability modeling continuous processor control systems. But none of the methods discussed above suggested a general method of evaluating fuzzy reliability of a multiprocessor interconnection systems where there lies a large degree of uncertainty in system failure. In this paper we present a general and efficient fuzzy probability method for computing reliability of multiprocessor interconnection networks viz. hupercube and star based network..

The rest of the paper is organizes as follows. Section II of this paper presents a topological details of Hypercube and Star graph multiprocessor interconnection networks. In section III Reliability model of multiprocessor system is discussed. Fuzzy reliability analysis has been carried out in section IV. Results are discussed in Section V. Section VI concludes the paper.

II. Topological details

In order to examine the fuzzy reliability of cube-based and a permutation graph based multiprocessor one must consider their topological features in details. The multiprocessors considered here are Hypercube [9], and Star graph[10].

Hypercube

The n-dimensional hypercube can be modeled as a graph $H_n(V, E)$ with the node set V_n and the edge set E_n, where $|V_n| = 2^n$ and $|E_n| = n2^{n-1}$ nodes. The n-dimensional hypercube have 2^n nodes and $n2^{n-1}$ links. The 2^n nodes are distinctly addressed by n-bit binary numbers with values from 0 to 2^n-1. Each node has link at n-dimensions ranging from (lowest dimension) to n (highest dimension) , connecting to n neighbours. An edge connecting nodes $X = x_n x_{n-1}$---------------x_1 and $Y = y_n y_{n-1}$------------y_1 is said to be the dimension j or to the j[th] dimensional edges if their binary address $x_n x_{n-1}$--------x_1 and $y_n y_{n-1}$--------y_1 different bit position j only , $x_j \neq y_j$. An edge in H_n can also be represented by n character string with one hypen (-) and n-1 binary symbols {0,1} . For example, in a H_4, the string 00-1 denotes the edge connecting nodes 0001and 0011. The degree of the n-dimensional hypercube is n. Fig. 1 shows the 4-dimentional hypercube.

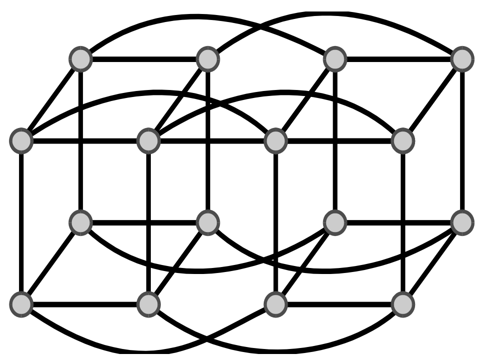


Figure 1. 4-dimensional hypercube (N=16,n=4)

Star graph

Star graph has more complex structure rather than binary n-cube. The n-dimensional star graph S_n is an edge and node symmetric graph containing n! nodes and $(n-1)n!/2$ links. Each node is labeled by a distinct permutation set of integers{1-----n} .Two joints are linked with a link i if and only if the label of the one can be obtained from the label of the other by swapping the first digit (conventionally the leftmost) and the i[th] digit where $1<i<n$. *For example:* in S_5 two nodes 12345 and 42315 are neighbors and joined via a link labeled 4. 4-dimentional star graph is shown in fig.2.

Star Graph has been extensively studied for its applications and structured properties . We summarize some of its properties that are relevant to our discussions.

i. The star graphs are members of Cayley group graphs. A star graphs S_n has n-1 generators g_1, g_2-------------g_n where g_i swaps the 1[st] symbol with the i[th] symbol of any permutation . Each generator has its own inverse i.e the star graph is symmetric. Also the star graph S_n is an $(n-1)$ regular graph with n! nodes $n!(n-1)/2$ edges.
ii. Since Star graphs are vertex symmetric, one can always view the distance between any two arbitrary nodes as the distance between the source node and the identity permutations by suitably renaming the symbols representing the permutations.
iii. It is easy to see that only permutations of n elements can also be specified in terms of its unique cycle structure with respect to the identity permutation. For example 345216 ={135,24,6}. The maximum number of cycles in a permutation of n elements is n and minimum number is 1 .The length of the cycle is defined to be number of symbols present in the cycle.
iv. The dimension of S_n is given by $D_n=[3(n-1)/2]$.

The star graph is an attractive alternative to the hypercube and compares favorably with it in several aspects. For example, the degree of S_n is n-1 i.e sub-logarithmic in the number of nodes of S_n while a hypercube with $\Theta(n!)$ nodes has degree $\Theta(\log n!) = \Theta(n\log n)$, i.e logarithmic in the number of nodes. The same can be said about the diameter of S_n .

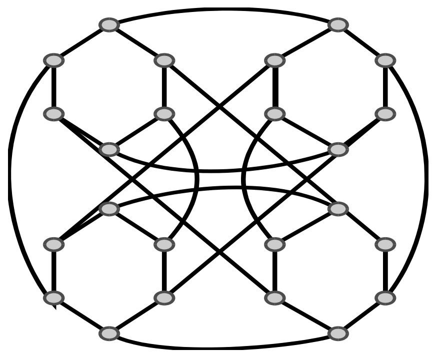


Figure 2. 4-dimensional Star graph

III. Reliability Model of Multiprocessor System

The evaluation of reliability over a systems life cycle is characterized by the notions of stability, growth and decrease that can be stated for the various attributes of reliability. These notions are illustrated by failure intensity, i.e number of failures per unit time. For building a model for multiprocessor, we consider a general case, where, the system consists of N identical processing nodes.

Initially, all the N processors are in fully operational state. Let this state be denoted as S_N. In the event of occurrence of a fault, the fault is detected and the system is reconfigured, or else if the fault is of permanent type, then the failed processors are logically isolated from the system, whereas other processors continue to perform their job without interruption. Let

S_i = The state of the system with *i* operational processors, and there are N+1 states in total.

C = System coverage factor,

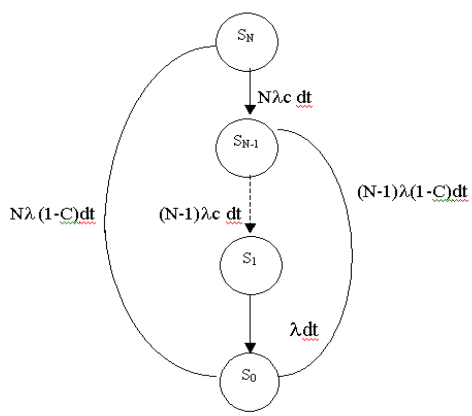


Figure 3. Markov diagram for multiprocessor system.

The Markov Transition diagram for the multiprocessor system can be easily constructed as shown in Figure 1. Now, mathematically, this system can be represented by the following equations.

$$\frac{dp_N}{dt} = -N\lambda_P P_N \quad (1)$$

$$\frac{dp_i}{dt} = i\lambda_p p_i + (i+1)\lambda_p c p_{i+1} \quad (2)$$

i =1,2,3………..N-1

$$\frac{dp_0}{dt} = ip_1 + \lambda_P(1-c)\, j = \sum_{j=2}^{N} jp_j \quad (3)$$

where $p_i(t)$, i=0,1,2,3……N represents the probability that the system remains in the state S_i at time t. Now, imposing the boundary conditions and assuming

$P_N(t) = 1$, for t=0, we have,

$$P_i(t) = C^{N-1} e^{-i\lambda_p t} \binom{N}{i} (1 - e^{-i\lambda_p t})^{N-1} \qquad (4)$$

i= 1,2,3……..N

and the system reliability is given by

$$R(t) = \sum P_i(t) \qquad (5)$$

where i belongs to successive states.

The above analysis considers the states of a multiprocessor as distinct. However, in a practical situation, because of the system's ability to work in a degraded mode, it is difficult to interpret a state of a multicomputer as success (failure). For the purpose, we consider the states of a multiprocessor as fuzzy state. Consequently, the transition from a fuzzy success state to a failure state can also be considered as a fuzzy event.

IV. Fuzzy Reliability Analysis

Uncertainty and imprecision of information is expected to prevail as the network becomes complex, and even the degree of dependence between the components may not be described deterministically. Under such circumstances, it is difficult to obtain precise reliability measures for a system. Fuzzy set is one of the best tool to model and evaluate such situation. Reliability of a network or node may be evaluated as high, medium and low with the reliability grade scale ranging from 0 to 100% and the membership function of grades ρ in the sets of high, medium, low may overlap partially which corresponds to the nature of the linguistic attributes, i.e same grades may be both medium & low or high & medium. This section presents fuzzy reliability measures of multiprocessor interconnection networks. The following assumptions are made for the development of fuzzy reliability in multiprocessor interconnection network.

Assumptions

i) Initially, all components of the system are in good conditions.
ii) The link failure and link success probability is assumed to be fuzzy numbers.
iii) Failures cannot be determined with certainty
iv) Repair facility is not available
v) Processing modules may fail with a constant exponential failure rate λp

A multiprocessor system in general, consists of much valuable units. Therefore, naturally all failure events incur some economic loss in addition to bringing performance degradation to the overall system. The economic aspect though important, is often ignored while modeling the behavior of a system. In what follows, we build a model for multiprocessor incorporating all the above measures in equation 5.

Let A be a fuzzy set defined on the universe of U. It can be expressed as a set of a ordered pairs $\{x, \mu_A(x)\}$, denoted by

$$A=\{x, \mu_A(x)\} \qquad (6)$$

where, x= fuzzy number

$\mu_A(x)$ = membership function bounded by a value between the interval [0,1]

= degree of membership of x in A

Let P denote the probability measure over the universe U, then the probability of A denoted by Pr(A), can be expressed as

$$Pr(A) = \int_A dP \qquad (7)$$

Or equivalently, where a finite or countable number of elements $\{x_i\}$, i={1,2,….n} constitutes the universe of discourse, the probability of a fuzzy event A with membership function $\mu_A(x)$ becomes

$$\Pr(A) = \sum_{i}^{n'} \mu_A(x_i) \qquad (8)$$

where, P_i is the probability measure corresponding to x_i and n is the number of points constituted by the universe.

Let U= $\{S_1, S_2, \ldots S'_n\}$ denote the universe of discourse. On this universe, we define a fuzzy success state S and a fuzzy failure state F,

$$S = \{S_i, \mu_S(Si), i=1,2\ldots n'\}$$

$F = \{S_i, \mu_F(Si), i=1,2,\ldots\ldots n'\}$

The transition from fuzzy success state to the fuzzy failure state is denoted as T_{SF}. Since a multiprocessor system behaves stochastically in the time domain.

Defination 2.

Fuzzy reliability (FR) of the system in $[t_0, t_{0+1}]$ as: $R(t_0, t_{0+t}) = Pr\{T_{SF}$ does not occur in the time interval $[t_0, t_{0+t}]\}$.

Since both S and F are fuzzy states, the transitions between them are consecutively fuzzy events. This justifies our approach to assume T_{SF} as fuzzy event. We can define T_{SF} on the universe U_T

$U_T = \{m_{ij}\}$; i, j = 1,2,….n',

Where, m_{ij} = transition from state S_i to S_j with membership function:

$\{\mu_{TF}(m_{ij})\}$; i, j = 1,2,….n

i.e. $T_{SF} = \{m_{ij}, \mu T_{SF}(m_{ij})\}$; i.j=1,2,…..n

$$\beta_{F/S}(S_i) = \frac{\mu_F(S_i)}{\mu_F(S_i) + \mu_S(S_i)}$$

Then, $\beta_{F/S}(S_i)$ can be viewed as the grade of membership of S_i. It is reasonable to say that the transition from S_i to S_j makes the transition from S to F possible to some extent if and only if

$\beta_{F/S}(S_j) > \beta_{F/S}(S_i)$ holds.

Therefore ,

$$[\mu T_{SF}(m_{ij})] = \begin{cases} \beta_{F/S}(S_j) - \beta_{F/S}(Si); & \text{when } \beta_{F/S}(S_j) > \beta_{F/S}(S_i) \\ 0; & \text{when } \beta_{F/S}(S_j) \le \beta_{F/S}(S_i) \end{cases}$$

Based on the above discussions, the general expression of fuzzy reliability process is developed. Incorporating the concept of fuzzy success/ failure state in equation 5. The Fuzzy reliability can be evaluated as

$$FR(t) = \sum_{i=1}^{N} \{1 - \mu T_{SF}(m_{ij})\} P_i(t)$$

Considering the economic loss to nonlinearly increase by a factor x', with increase in the number of faulty modules , we define

$$\mu_S(S_i) = \frac{i}{N}$$

$$\mu_F(S_i) = 1 - \left(\frac{i}{N}\right)^{x'}, \; i=0,1,2,\ldots,N$$

It can be noted that here, the conventional concept $\mu_F(S_i) = 1 - \mu_S(S_i)$ does not hold good. It may also be noted that $Pr(F) \neq Pr(T_{SF}).\,Pr(S)$

Therefore, the degree of the membership function becomes

$$\mu_{T_{SF}}(m_{N_i}) = \frac{\mu_F(S_i)}{\mu_F(S_i) + \mu_S(S_i)} = \frac{1 - (\frac{i}{N})^{x'}}{1 - (\frac{i}{N})^{x'} + \frac{i}{N}} \qquad \text{where, } i = 0,1,\ldots, N$$

Now, Fuzzy Reliability of a system is expressed as

$$FR(t) = \sum_{i=t}^{N} \{1 - \mu T_{SF}(m_{ij})\}.Pr\{S_i \mid t_0\} = \sum_{i=1}^{N} \frac{(\frac{i}{N})^{x'}}{1 - (\frac{i}{N})^{x'} + \frac{i}{N}} C^{N-i}.e^{-i\lambda_p t} \binom{N}{i} (1 - e^{-\lambda_p t})^{N-i}$$

V. Results and Discussions

In this section, results on various fuzzy reliability measures of multiprocessor interconnection networks are presented. Fig. 4 and fig. 5 shows the effect of processor failure rate (**λ_p**) on the fuzzy reliability of star based and hypercube networks. From these figures, it is observed that at time t=500 hrs, coverage c=1.0 and economic loss factor x'=2, the effect of variation of **λ_p** on the fuzzy reliability of a 5- dimensional star (SC_5) is more, compared to 5- dimensional(HC_5). Further, with the increase in processor failure rate, the fuzzy reliability of both star and hypercube multiprocessor networks decreases. For example, when the processor failure rate increases from 0 to 0.0004, the fuzzy reliability of SC_5 and HC_5 gradually decreases.

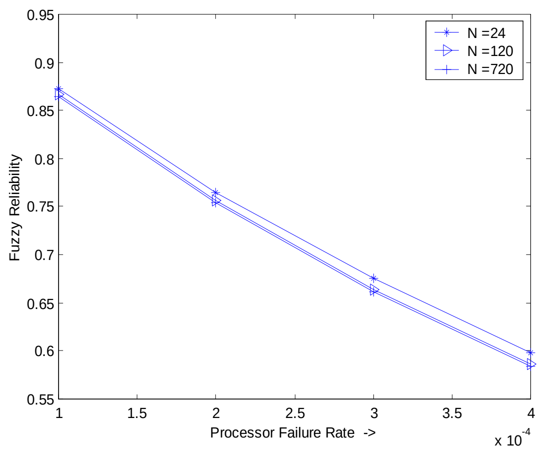


Figure 4 Fuzzy reliability of star graph multiprocessor with processor failure rate

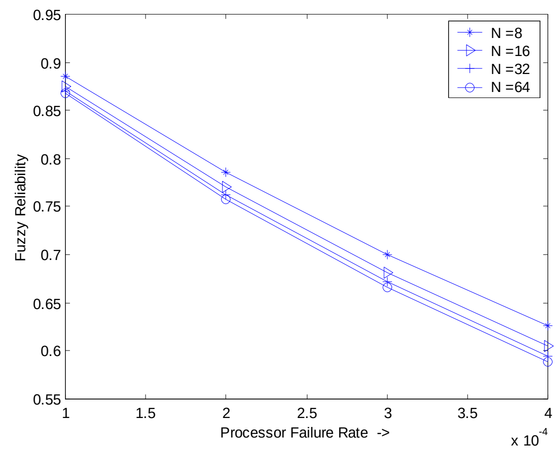
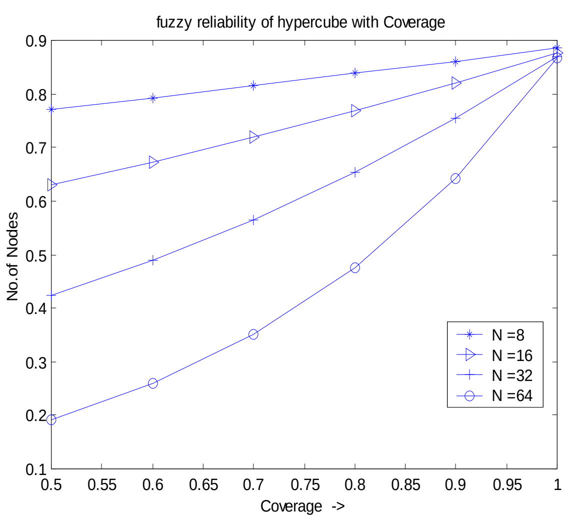


Figure 6Fuzzy reliability of Hypercube multiprocessor with Coverage

Figure 5 Fuzzy reliability of hypercube multiprocessor with processor failure rate



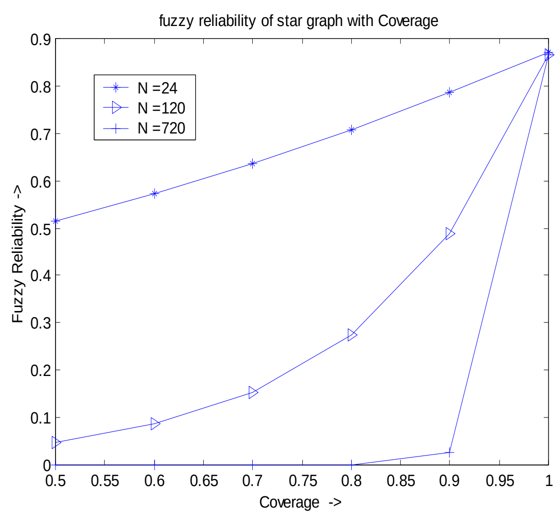


Figure 7 Fuzzy reliability of Star based multiprocessor with Coverage

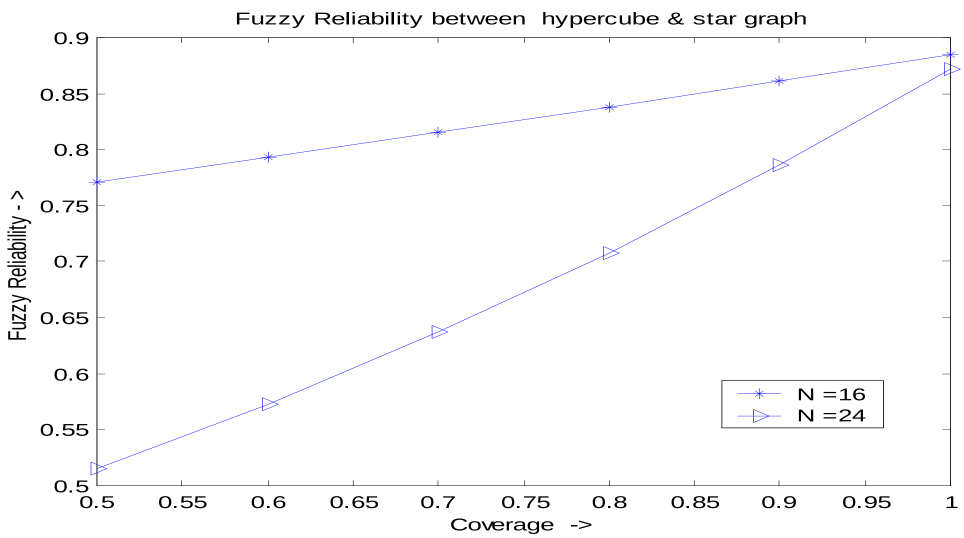


Figure 8 Comparision of Fuzzy reliability between Hypercube and Star graph

Again the fuzzy reliability of hypercube decreases with increase in dimensions. The rate of decrease is more from dimension 3-4 as compared with the decrease from 4-5. Fig.8 compares the fuzzy reliability of both hypercube and star based network. It is observed that with increase in the system coverage, the fuzzy reliability of hypercube improves more compared to star based network.

VI. Conclusion

In this paper the reliability aspect of multiprocessor systems have been discussed. The paper develops fuzzy reliability model for grace fully degrading multiprocessor system. The fuzzy reliability model has been evaluated for two classes of multiprocessor interconnection system viz: hypercube and star graph. Results for fuzzy reliability of various processor failure rate and coverage have been presented. It is observed that fuzzy reliability improves more in hypercube than star graph with increase in the system coverage.

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