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A Novel Distance Minimizer Method of Ranking for Fuzzy Number: A Case Study

Sanjay Kumar[1], A.K. Nishad[2], Abhishekh[*]
[1]Department of Mathematics, Institute of Science, Banaras Hindu University, Varanasi, India
[2] School of Basic and Applied Sciences, Shobhit University, Gangoh, India
[*]Department of Mathematics, Vishwavidyalaya Engineering College, Ambikapur, Chhattisgarh, India
Corresponding Author: Abhishekh

ABSTRACT: This paper presents a new ranking method for fuzzy numbers (FN) by using distance minimizer between any two fuzzy numbers with the help of membership functions. Till now there are several ranking methods have been proposed but every method has some short coming to overcome these types of drawback. Here, we propose a new ranking method for fuzzy numbers which satisfy the general axioms of ranking function and applicable for normal as well as generalized fuzzy numbers. The developed method has been illustrated by examples and compared with some existing ranking methods to show its suitability.
Keywords: Fuzzy sets, Trapezoidal fuzzy number, Triangular fuzzy numbers (TFNs), Membership function, Ranking function.

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I. INTRODUCTION

Decision making problems need the processing of information for getting an optimal solution for a specific situation. But in general the information available is often imprecise, vague and many times contain uncertainty and thus such situation demand handling by non traditional methods. Firstly the fuzzy set theory was developed by Zadeh [1, 2] and further improved methodology by Zimmerman [3, 4] immersed as a potential tool in theory of optimization to deal with imprecision and vagueness in parameters. Such situations of optimization problems in general involve the fuzzy numbers.

While defining arithmetic operations on fuzzy number (FN), its ranking is needed to have comparison among fuzzy numbers. Unlike to ranking of fuzzy numbers one has also to take the cognition of membership functions in ranking of fuzzy numbers. Thus this raking becomes an interesting property of fuzzy number and various workers have proposed several methods of ranking in literature. In order to develop a standard for ranking methods, Wang and Kerre [5] proposed six axioms which a reasonable ranking method is desired to satisfy. Yager and Filev [6], Filev and Yager [7] introduced a general approach to defuzzification based upon the basic defuzzification distribution and distance metric method (DMM) transformations. Two commonly used methods for defuzzification are the center of area (COA) method and the mean of maximum (MOM) method has been studied by Larkin [8] and Liu [9]. Further, Asady and Zendehnam [10] studied distances and ordering in a family of fuzzy numbers. A significant work on ranking method was carried out by Asady [11], Abbasbandy [12], Abbasbandy and Hajjari [13]. Theory of ranking methods for fuzzy numbers was further enriched by Su [14] and gave more accurate result. Recently, Rao and Shanker [15] also introduced the ranking of fuzzy number with higher accuracy. Some authors in literature have used application of fuzzy sets in various aspects such as Abhishekh et al. [23, 25] presented type 2 and score function based forecasting model under fuzzy environment. In addition, Abhishekh and Kumar [24, 26] introduced weighted and higher order intuitionistic fuzzy time series model. Recently, Gautam et al. [27, 28] have been studied high-order fuzzy time series forecasting model and decision making problems using the theory of fuzzy soft set. Also, Abhishekh and Nishad [22] gave a ranking method based on LR-intuitionistic fuzzy number. In the present study, we have studied the various aspects of some ranking methods on six axioms and have proposed a general method for ranking of

[*] Corresponding author- Abhishekh
Email: abhibhu1989@gmail.com

fuzzy numbers satisfying the six standard axioms for its application to decision making problems and some comparison to existing researchers to show it superiority.
The remainder of this paper is presented as follows: Section 2, the basic concept of fuzzy sets, fuzzy number and equality of fuzzy number are briefly presented. Section 3 contains the ranking axiom along with proposed ranking function. Numerical illustration to the validation and the effectiveness of the proposed method is given in Section 4. Comparisons of fuzzy numbers with different researchers are presented in section 5. Finally, Section 6 concludes the paper.

II. BASIC PRELIMINARIES

In this section, we describe some foundations for fuzzy numbers and ranking functions to comparing fuzzy numbers and proposed new definitions needed for the study carried out in the subsequent sections.

Definition 2.1 If X is a collection of objects denoted generically by x, then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs such that

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) | x \in X\}, \qquad (1)$$

where, $\mu_{\widetilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\widetilde{A}$ that maps X to the membership space M (M is generally taken to be $[0, 1]$).

Definition 2.2 A fuzzy set $\widetilde{A}$ on real line $\mathbb{R}$ with membership function $\mu_{\widetilde{A}}(x) : \mathbb{R} \to [0, 1]$ is called a fuzzy number, if it holds following axioms

i. $\widetilde{A}$ is a normal set.
ii. $\widetilde{A}$ is convex fuzzy set.
iii. $\mu_{\widetilde{A}}(x)$ is upper semi continuous.
iv. $\widetilde{A}$ is bounded.

Definition 2.3 A fuzzy number $\widetilde{A} = (\alpha, \beta, \gamma)$ is said to be triangular fuzzy number if, its membership function is given as

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{(x-\alpha)}{(\beta-\alpha)}, & \text{if } \alpha \leq x \leq \beta \\ \frac{(\gamma-x)}{(\gamma-\beta)}, & \text{if } \beta \leq x \leq \gamma \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

Definition 2.4 A triangular fuzzy number $\widetilde{A} = (\alpha, \beta, \gamma)$ is said to be non-negative fuzzy number iff $\alpha \geq 0$.

Definition 2.5 The two triangular fuzzy numbers $\widetilde{m} = (\alpha_1, \beta_1, \gamma_1)$ and $\tilde{n} = (\alpha_2, \beta_2, \gamma_2)$ are equal iff $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2$.

III. RANKING OF FUZZY NUMBERS

Ranking of intuitionistic fuzzy numbers is an important arithmetic property of fuzzy numbers which provides comparison of fuzzy numbers. This property of ranking is used in many decision making problems being studied in uncertain environment. In view of belonging for a fuzzy numbers various ranking methods are being developed. Thus in order to standardize these methods, Wang and Kerre (2001) proposed six axioms as follows:

i. For any arbitrary finite subset $\widetilde{A}$ of X and $\tilde{a} \in \widetilde{A}$, $\tilde{a} \gtrsim \tilde{b}$ by M on $\widetilde{A}$.
ii. For an arbitrary finite subset $\widetilde{A}$ of X and $(\tilde{a}, \tilde{b}) \in \widetilde{A}^2$, $\tilde{a} \gtrsim \tilde{b}$, and $\tilde{b} \gtrsim \tilde{a}$ by M on $\widetilde{A}$, we should have $\tilde{a} \sim \tilde{b}$ by M on $\widetilde{A}$.
iii. For an arbitrary finite subset $\widetilde{A}$ of X and $(\tilde{a}, \tilde{b}, \tilde{c}) \in \widetilde{A}^3$, $\tilde{a} \gtrsim \tilde{b}$ and $\tilde{b} \gtrsim \tilde{c}$ by M on $\widetilde{A}$, we should have $\tilde{a} \gtrsim \tilde{c}$ by M on $\widetilde{A}$.
iv. For an arbitrary finite subset $\widetilde{A}$ of X and $(\tilde{a}, \tilde{b}) \in \widetilde{A}^2$ inf sup $(\tilde{a}) \geq$ sup supp $(\tilde{b})$, we should have $\tilde{a} \geq \tilde{b}$ by M on $\widetilde{A}$.
v. Let X and X' be two arbitrary finite sets of fuzzy quantities in which M can be applied and $\tilde{a}$ *and* $\tilde{b}$ are in X∩X'. We obtain the ranking order $\tilde{a} > \tilde{b}$ by M on X' iff $\tilde{a} > \tilde{b}$ by M on X.
vi. Let $\tilde{a}$, $\tilde{b}$, $\tilde{a} + \tilde{c}$, $\tilde{b} + \tilde{c}$ be element of X. If $\tilde{a} \gtrsim \tilde{b}$ by M on $(\tilde{a}, \tilde{b})$, then $\tilde{a} + \tilde{c} \gtrsim \tilde{b} + \tilde{c}$ by M on $(\tilde{a} + \tilde{c}, \tilde{b} + \tilde{c})$.

3.1 Proposed Ranking Method

In this section, to overcome the above mentioned problems to some extent, we are going to propose a ranking method for fuzzy sets associated with a metric D in space of fuzzy sets F. For this, following definitions are used.

Definition 3.11 Let ∇: F→{-1,1} be a function that is defined as follows : ∀ A ∈ F , ∇(A) =

$$f(x) = \begin{cases} 1, & I(A) \geq 0 \\ -1, & I(A) < 0 \end{cases} \quad (3)$$

Definition 3.12 The weighted average of a fuzzy set Ã is defined by

$$I(\tilde{A}) = \int_0^1 (c \cdot L_{\tilde{A}}(\alpha) + (1-c)\, R_{\tilde{A}}(\alpha))\, d\alpha \quad (4)$$

where $L_{\tilde{A}}(\alpha)$ and $R_{\tilde{A}}(\alpha)$ are left and right spreads of the fuzzy set, respectively and c ∈ [0, 1] denotes "optimism/pessimism" coefficient in bringing out operations on fuzzy sets.

Definition 3.13 The weighted width of a fuzzy set Ã is defined by

$$D(\tilde{A}) = \int_0^1 (R_{\tilde{A}}(\alpha) - L_{\tilde{A}}(\alpha))\, f(\alpha)\, d\alpha \quad (5)$$

Here f(α) is considered as weighting function, which is a non-negative and increasing function in [0,1] and it satisfies following requirement : f(0) = 0, f(1) = 1 and $\int_0^1 f(\alpha) d\alpha = \frac{1}{2}$. We define f($\alpha$) = α for further uses.

Definition 3.14 For arbitrary fuzzy sets A and B, the quantity is defined as

$$TRD(\tilde{A}, \tilde{B}) = \sqrt{[I(\tilde{A}) - I(\tilde{B})]^2 + [D(\tilde{A}) - D(\tilde{B})]^2} \quad (6)$$

is called the TRD distance between the fuzzy sets Ã and $\tilde{B}$. TRD distance satisfies the conditions of non-negativity, symmetry and transitivity i.e.,

TRD (Ã, $\tilde{B}$) ≥ 0,
TRD (Ã, $\tilde{B}$) = TRD ($\tilde{B}$, Ã)
TRD (Ã, $\tilde{B}$) + TRD ($\tilde{B}$, $\tilde{C}$) ≥ TRD (Ã, $\tilde{C}$)

Furthermore, we consider $\tilde{A}_0$ as a fuzzy origin. Since $\tilde{A}_0$ ∈ F, so for each Ã ∈ F

$$TRD(\tilde{A}, \tilde{A}_0) = \sqrt{[I(\tilde{A})]^2 + [D(\tilde{A})]^2} \quad (7)$$

Definition 3.15 For each Ã ∈ F and with "optimistic/pessimistic" coefficient equal to 0.5, TRD (Ã, $\tilde{A}_0$) = ∇(Ã) TRD (Ã, $\tilde{A}_0$)

where ∇(Ã) is defined as

$$\nabla(\tilde{A}) = \begin{cases} 1 & when\ I(\tilde{A}) \geq 0 \\ -1 & when\ I(\tilde{A}) < 0 \end{cases} \quad (8)$$

Definition 3.16 For each arbitrary fuzzy set Ã, $\tilde{B} \in F$, define the ranking of Ã and $\tilde{B}$ by TRD on F i.e.,

TRD (Ã, $\tilde{A}_0$) > TRD ($\tilde{B}$, $\tilde{A}_0$) if and only if Ã > $\tilde{B}$
TRD (A, $\tilde{A}_0$) < TRD ($\tilde{B}$, $\tilde{A}_0$) if and only if Ã < $\tilde{B}$
TRD (Ã, $\tilde{A}_0$) = TRD ($\tilde{B}$, $\tilde{A}_0$) if and only if Ã ~ $\tilde{B}$.

Consider a fuzzy number $\tilde{A} = (a, b, c; w)$ given by

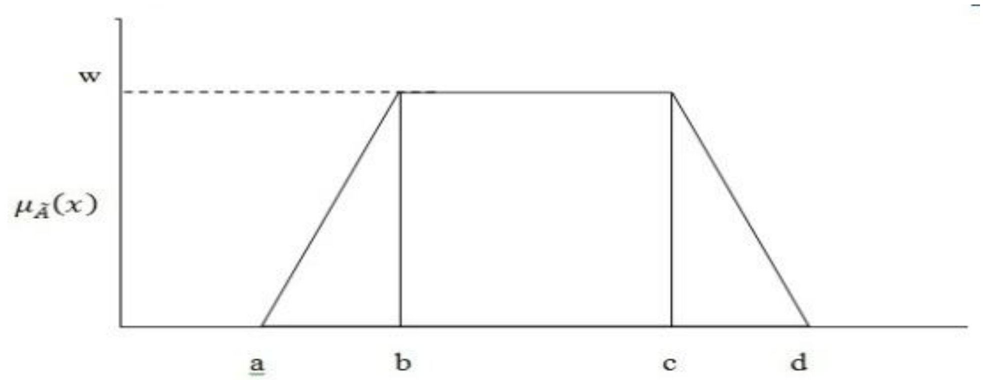

Fig. 1 Trapezoidal fuzzy number

Let the element of fuzzy set $\tilde{A}$ is generically denoted by x. Then membership function of $\tilde{A}$ is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)}, & when\ a \leq x \leq b \\ w, & when\ b \leq x \leq c \\ \frac{w(x-d)}{(c-d)}, & when\ c \leq x \leq d \end{cases} \quad (9)$$

Let the α-cut A^α of fuzzy no. defined as given as $A^\alpha = [L_{\tilde{A}}(\alpha),\ R_{\tilde{A}}(\alpha)]$,
where $L_{\tilde{A}}(\alpha) = \frac{\alpha(c-d)}{w} + a$ and $R_{\tilde{A}}(\alpha) = \frac{\alpha(b-a)}{w}$ + d for the left and right spread respectively.
Now we calculate I(Ã) and D(Ã) as

$$I(\tilde{A}) = \int_0^1 (c' L_{\tilde{A}}(\alpha) + (1-c')R_{\tilde{A}}(\alpha))\, d\alpha,$$

where c' denotes "optimistic/pessimistic" coefficient,

$$I(\tilde{A}) = \int_0^1 \left[c' \frac{\alpha(b-a)}{w} + a + (1-c')\frac{\alpha(c-d)}{w} + d\right] d\alpha$$

$$I(\tilde{A}) = \left[c'\left(\frac{\alpha(b-a)}{2w} + a\right) + (1-c')\left(\frac{(c-d)}{2w} + d\right)\right] \quad (10)$$

We take weight function $f(\alpha) = \alpha$ for further uses

$$D(\tilde{A}) = \int_0^1 \left\{\frac{\alpha(c-d)}{w} + d - \frac{\alpha(b-a)}{w} + a\right\} \alpha . d\alpha$$

$$D(\tilde{A}) = \left[\frac{(a+c)-(b+d)}{3w} + \frac{(d-a)}{2}\right] \quad (11)$$

If we consider $\tilde{A}_0$ as a fuzzy origin, since $\tilde{A}_0 \in F$, so for each $\tilde{A} \in F$,

$$TRD\ (\tilde{A}\ ,\ \tilde{A}_0) = \sqrt{I(\tilde{A})^2 + D(\tilde{A})^2} \quad (12)$$

If in this formula (Eqs. 10 and 11), we take $b = c$, then above ranking method for trapezoidal fuzzy set transformed into the ranking method for triangular fuzzy sets.
So, in case of triangular fuzzy sets I(Ã) and D(Ã) are given as

$$I(\tilde{A}) = \left[c'\left(\frac{\alpha(b-a)}{2w} + a\right) + (1-c')\left(\frac{(b-d)}{2w} + d\right)\right] \quad (13)$$

and $D(\tilde{A}) = \left[\frac{a-d)}{3w} + \frac{(d-a)}{2}\right]$ (14)

and if $\tilde{A}_0$ as a fuzzy origin, since $\tilde{A}_0 \in F$, so for each $\tilde{A} \in F$

$$TRD\ (\tilde{A}, \tilde{A}_0) = \sqrt{[I(\tilde{A})]^2 + [D(\tilde{A})]^2} \quad (15)$$

Definition 3.17 Ranking function (index) approach
An efficient approach for comparing the fuzzy numbers are defined by a ranking function $\mathfrak{R}$ from the set of all fuzzy numbers $F(\mathbb{R})$ in $\mathbb{R}$ to real number i.e. $\mathfrak{R} : F(\mathbb{R}) \to \mathbb{R}$, called a ranking function or ranking index, which maps each fuzzy number into the real number.
If $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers and their coresponding crisp values are $\mathfrak{R}(\tilde{A})$ and $\mathfrak{R}(\tilde{B})$ respectively, then the ranking of fuzzy numbers are as follows:

i. $\tilde{A} \precsim \tilde{B}$, if and only if $\mathfrak{R}(\tilde{A}) \le \mathfrak{R}(\tilde{B})$
ii. $\tilde{A} \approx \tilde{B}$, if and only if $\mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$
iii. $\tilde{A} \succsim \tilde{B}$, if and only if $\mathfrak{R}(\tilde{A}) \ge \mathfrak{R}(\tilde{B})$.

IV. NUMERICAL ILLUSTRATIONS

Example 4.1 Consider the following three triangular fuzzy sets Ã = (0.2, 0.3, 0.5; 0.8), $\tilde{B}$ = (0.17, 0.32, 0.58; 0.6), and $\tilde{C}$ = (0.25, 0.40, 0.70; 0.5), so that on applying our proposed ranking method we get
I(Ã) = 0.331, D(Ã) = 0.020, I($\tilde{B}$) = 0.365, D($\tilde{B}$) = 0.113, I($\tilde{C}$) = 0.115, D($\tilde{C}$) = 0.15, and TRD $(\tilde{A}, \tilde{A}_0) = \sqrt{[I(\tilde{A})]^2 + [D(\tilde{A})]^2}$ = 0.331,

TRD $(\tilde{B}, \tilde{A}_0) = \sqrt{[I(\tilde{B})]^2 + [D(\tilde{B})]^2}$ = 0.383,

and TRD $(\tilde{C}, \tilde{A}_0) = \sqrt{[I(\tilde{C})]^2 + [D(\tilde{C})]^2}$ = 0.189.

Clearly, TRD $(\tilde{A}, \tilde{A}_0)$ = TRD $(\tilde{A}, \tilde{A}_0)$,
TRD $(\tilde{B}, \tilde{A}_0)$ = TRD $(\tilde{B}, \tilde{A}_0)$,
TRD $(\tilde{C}, \tilde{A}_0)$ = TRD $(\tilde{C}, \tilde{A}_0)$,
$\Rightarrow$ TRD $(\tilde{B}, \tilde{A}_0)$ > TRD $(\tilde{A}, \tilde{A}_0)$ > TRD $(\tilde{C}, \tilde{A}_0)$
i.e., $\tilde{B} > \tilde{A} > \tilde{C}$.

Example 4.2 Let the following three trapezoidal number Ã=(0, 0.3, 0.6, 0.8; 0.6), $\tilde{B}$= (0.1, 0.4, 0.7, 0.9; 0.5) ,$\tilde{C}$ = (0, 0.3, 0.8, 1; 0.7), so that on applying the proposed ranking method for the above fuzzy set, we calculate
I(Ã) = 0.883, D(Ã)= 0.122, I($\tilde{B}$) = 0.550, D($\tilde{B}$) = 0.066, I($\tilde{C}$) = 0.535, D($\tilde{C}$) = 0.261, and
TRD $(\tilde{A}, \tilde{A}_0) = \sqrt{[I(\tilde{A})]^2 + [D(\tilde{A})]^2}$ = 0.795,

TRD $(\tilde{B},\tilde{A}_0) = \sqrt{[I(\tilde{B})]^2 + [D(\tilde{B})]^2} = 0.554$,

TRD $(\tilde{C},\tilde{A}_0) = \sqrt{[I(\tilde{C})]^2 + [D(\tilde{C})]^2} = 0.596$.

TRD $(\tilde{A}, \tilde{A}_0)$ = TRD $(\tilde{A}, \tilde{A}_0)$,
TRD $(\tilde{B}, \tilde{A}_0)$ = TRD $(\tilde{B}, \tilde{A}_0)$
and TRD $(\tilde{C}, \tilde{A}_0)$ = TRD $(\tilde{C}, \tilde{A}_0)$
$\Rightarrow$TRD $(\tilde{B}, \tilde{A}_0)$ < TRD $(\tilde{C}, \tilde{A}_0)$ < TRD $(\tilde{A}, \tilde{A}_0)$,
i.e., $\tilde{A} > \tilde{C} > \tilde{B}$.

V. COMPARISON OF PROPOSED RANKING METHOD WITH EXISTING METHODS

5.1 Allahviranloo et al. [17] ranking method:

By Allahviranloo et al. method, we compare the fuzzy sets when they are normal. If the two fuzzy sets are sub-normal, then this method gives same ranking for fuzzy numbers as that for normal fuzzy sets, which is not appropriate i.e.,
Let the three fuzzy numbers $\tilde{A}$ = (0.2, 0.3, 0.5), $\tilde{B}$ = (0.17, 0.32, 0.58),
$\tilde{C}$ = (0.25, 0.40, 0.70).
By the proposed method by Allahviranloo et al. [17] ranking method we get $\tilde{A} > \tilde{C} > \tilde{B}$
Further if we take the following subnormal fuzzy number $\tilde{A}$ = (0.2, 0.3, 0.5; 0.8), $\tilde{B}$ = (0.17, 0.32, 0.58; 0.6), $\tilde{C}$ = (0.25, 0.40, 0.70; 0.5), for these fuzzy numbers Allahviranloo et al. ranking method gives the same result as given above which is not appropriate. But our proposed ranking method gives $\tilde{B} > \tilde{A} > \tilde{C}$ as shown in example 5.1. So that our proposed ranking method is more appropriate than Allahviranloo et al. ranking method.

5.2 In this sub-section, we compare the proposed method with others existing methods by Allahviranloo et al. [17,18], Saneifard[19, 20] and Saneifard and Ezatti [19] and consider the following sets in Allahviranloo et al. [17,18]. A comparisons by different researcher are placed in Table 1.
Set 1: $\tilde{A} = (0.4, 0.5, 1.0), \tilde{B} = (0.4, 0.7, 1.0), \tilde{C} = (0.4, 0.9, 1.0)$.
Set 2: $\tilde{A} = (0.3, 0.4, 0.7, 0.9), \tilde{B} = (0.3, 0.7, 0.9), \tilde{C} = (0.5, 0.7, 0.9)$.
Set 3: $\tilde{A} = (0.3, 0.5, 0.7), \tilde{B} = (0.3, 0.5, 0.8, 0.9), \tilde{C} = (0.3, 0.5, 0.9)$.
Set 4: $\tilde{A} = (0.0, 0.4, 0.7, 0.8), \tilde{B} = (0.2, 0.5, 0.9), \tilde{C} = (0.1, 0.6, 0.8)$.

VI. CONCLUSION

In our proposed method, we consider height of fuzzy sets while ranking the fuzzy set and gives weight to height along with skewness and spreads of fuzzy sets. Therefore, by proposed method, we can rank both normal fuzzy sets as well as sub-normal fuzzy sets. If we take two sets of same fuzzy numbers with different height, the proposed method will give different ranking to both (which is shown in the examples). The proposed method is applicable in decision making problem, fuzzy optimization problem, financial problem, multiple-attribute decision making (MADM) problem, fuzzy time series and many other soft computing techniques.

Table 5.1 A comparison of fuzzy numbers with proposed along with existing methods

Authors	Fuzzy Numbers	Set 1	Set 2	Set 3	Set 4
Proposed method	$\tilde{A}$	0.756	0.403	0.604	0.312
	$\tilde{B}$	0.855	0.509	0.873	0.370
	$\tilde{C}$	0.995	0.899	0.803	0.370
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \sim \tilde{C}$
R. Saneifard	$\tilde{A}$	0.566	0.566	0.500	0.500
	$\tilde{B}$	0.700	0.666	0.666	0.516
	$\tilde{C}$	0.830	0.700	0.700	0.550
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Sing distance method with	$\tilde{A}$	1.200	1.150	1.000	0.095
P = 1	$\tilde{B}$	1.400	1.300	1.250	1.050
	$\tilde{C}$	1.600	1.400	1.100	1.050
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \sim \tilde{C}$
Sing distance method with	$\tilde{A}$	0.886	0.875	0.725	0.785
P = 2	$\tilde{B}$	1.019	0.952	0.941	0.795
	$\tilde{C}$	1.160	1.003	0.816	0.838
Results				$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Distance minimization	$\tilde{A}$	0.600	0.575	0.500	0.475
method	$\tilde{B}$	0.700	0.650	0.625	0.525
	$\tilde{C}$	0.900	0.700	0.550	0.525
		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \sim \tilde{C}$

Abbasbandy and Hajjari	$\tilde{A}$	0.533	0.558	0.500	0.525
(magnitude method)	$\tilde{B}$	0.700	0.633	0.641	0.508
Results	$\tilde{C}$	0.866	0.700	0.516	0.575
		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{B} \prec \tilde{A} \prec \tilde{C}$
Chen	$\tilde{A}$	0.337	0.431	0.375	0.520
	$\tilde{B}$	0.500	0.562	0.425	0.570
	$\tilde{C}$	0.667	0.625	0.550	0.625
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Chu and Tsao	$\tilde{A}$	0.299	0.284	0.250	0.244
	$\tilde{B}$	0.350	0.324	0.315	0.262
Results	$\tilde{C}$	0.399	0.350	0.274	0.261
		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Yao and Wu	$\tilde{A}$	0.600	0.575	0.500	0.475
	$\tilde{B}$	0.700	0.650	0.625	0.525
	$\tilde{C}$	0.800\	0.700	0.550	0.525
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \sim \tilde{C}$
	$\tilde{B}$	0.860	0.814	0.803	0.725
	$\tilde{C}$	0.926	0.860	0.745	0.724
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Cheng CV uniform	$\tilde{A}$	0.027	0.032	0.013	0.069
distribution	$\tilde{B}$	0.021	0.024	0.030	0.038
	$\tilde{C}$	0.022	0.009	0.027	0.043
Results		$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{B} \prec \tilde{C} \prec \tilde{A}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Cheng CV proportional	$\tilde{A}$	0.018	0.026	0.008	0.047
distribution	$\tilde{B}$	0.012	0.014	0.023	0.023
	$\tilde{C}$	0.013	0.005	0.017	0.025
Results		$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{B} \prec \tilde{C} \prec \tilde{A}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$

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