ISSN (e): 2250 – 3005 || Volume, 09 || Issue, 6 || June– 2019 ||
International Journal of Computational Engineering Research (IJCER)

Relativistic effects on the propagation of ion - acoustic solitons in Inhomogeneous Plasma

Dr. L. B. Gogoi
Department of Mathematics, Duliajan College, Duliajan-786602, Assam, India.

ABSTRACT: *The propagation of ion-acoustic solitary waves in relativistic inhomogeneous plasma has been investigated. The fluid equations for ions have been treated by reductive perturbation analysis technique. In this formulation process we have used a space–time stretched coordinates. The system of equations has been reduced to a modified Korteweg–de–Vries (mKdV) equation. The soliton solutions are found to be affected by the relativistic factor* γ *.*

KEYWORDS*: Ion–acoustic solitons, inhomogeneous plasma, mKdV equation, relativistic factor.*

Date of Submission: 21-06-2019 Date of acceptance:05-07-2019

I. INTRODUCTION

The study of ion-acoustic solitary waves in different plasma models have been going on which begins several decades back with the works of Washimi and Taniuti[1]. This type of ion-acoustic solitary waves in plasma arises due to the balance of non-linearity and dispersion which are described by the Korteweg-de-Vries(KdV)[2] equation. Beginning with Washimi and Taniuti, a numerous works by many authors have been done during last decades in relativistic [3-5] and non-relativistic [6-8] plasmas. In relativistic plasma, speeds of the particles are very high which are comparable to those of speed of light. Therefore relativistic effects played an important rule in the formation as well as propagation of solitary waves. Malik et al investigate the effect of ion-drift velovity in the existence of ion-acoustic solitons in relativistic plasma. Nejoh[3] had analysed relativistic effects on large amplitude ion-acoustic waves in two fluid plasma. Kalita et al [9, 10] studied about the existence of relativistic solitons with electron inertia.

Almost all of the studies of the non-relativistic and relativistic environments, the plasma models are considered as uniform (homogeneous) in which ion-acoustic waves travelwithout change in amplitude, shape and speed. As homogeneity is a special kind of inhomogeneity and hence the plasma in all situations, is to some extent non-uniform (inhomogeneous) where solitons are altered as it propagates. The relativistic effects on the propagation of solitons in this type of inhomogeneous plasma are not so much considered till now except a few. G. C. Das et al[11] observed various forms of spiky and explosive solitary waves in relativistic space-plasma. Relativistic solitary waves in an inhomogeneous plasma with two dimensional particle-in-cell simulation have been studied by Sentoku et al[12] . An investigation of the form of obliquely incident wave on a plane parallel layered inhomogeneous relativistic inhomogeneous plasma was made by Bourdier[13] in 1983. To the best of our knowledge, further studies of solitary wave propagation in the relativistic inhomogeneous plasma have never been undertaken. Therefore, in this paper we investigate the effect of relativistic components in the propagation of ion-acoustic solitary waves in inhomogeneous plasma. The investigation is based on the inclusion of ion-drift velocity only and its other components are considered as non-relativistic. We have derived a modified KdV equation in relativistic inhomogeneous plasma with density gradient of the ions. The reductive perturbation analysis [14] of fluid equations is carried out by employing a set of 'stretched coordinates' appropriate for spatially inhomogeneous plasma.

II. BASIC EQUATIONS

We have considered unmagnetised, inhomogeneous and collisionless plasma having weakly relativistic effect together with thermal electrons. We also consider the ionization free environment of the plasma. The continuity and momentum equation for this plasma model with Poisson equation are as follows:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(\mathrm{nu}) = 0 \qquad (1)$$

$$\frac{\partial(\gamma u)}{\partial t} + u\,\frac{\partial(\gamma u)}{\partial x} + \frac{\partial \phi}{\partial x} = 0 \qquad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} - n_0 e^{\phi} + n = 0 \qquad (3)$$

where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \cong 1 + \frac{v^2}{2c^2}$ is relativistic factor for a weakly relativistic plasma. We have normalized the ion number density n by equilibrium ion density n, velocities including c by ion – acoustic speed $C_s = \left(\frac{K_b T_e}{m_i}\right)^{1/2}$, time t by the inverse of the characteristic ion plasma frequency $w_{pi}^{-1} = \left(m_i / 4\pi n_0 e^2\right)^{1/2}$, the space variable x by the Debye length $\lambda_{De} = \left(K_b T_e / 4\pi n_0 e^2\right)^{1/2}$, the electrostatic potentials ϕ by $K_b T_e / e$ where K_b is the Boltzman constant.

III. DERIVATION AND SOLUTION OF THE MODIFIED KDV EQUATION

Using a set of spatial stretched coordinates [15], which is appropriate for specially inhomogeneous plasma, along with the zeroth order fluid velocities as

$$\xi = \varepsilon^{1/2}\left(\frac{x}{\lambda_0} - t\right), \qquad \tau = \varepsilon^{3/2} x \qquad (4)$$

where ε is expansion parameter and λ_0 is the phase velocity of the ion-acoustic wave.

Since n_0 and λ_0 are independent of t, we have

$$\frac{\partial n_0}{\partial \xi} = \frac{\partial \lambda_0}{\partial \xi} = 0 \qquad (5)$$

Substituting equations (4) into equations (1) – (3) we get

$$-\frac{\partial n}{\partial \xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial \xi}(nu) + \varepsilon\frac{\partial}{\partial \tau}(nu) = 0 \qquad (6)$$

$$-\frac{\partial(\gamma u)}{\partial \xi} + \frac{u}{\lambda_0}\frac{\partial(\gamma u)}{\partial \xi} + \varepsilon u\frac{\partial(\gamma u)}{\partial \tau} + \frac{1}{\lambda_0}\frac{\partial \phi}{\partial \xi} + \varepsilon\frac{\partial \phi}{\partial \tau} = 0 \qquad (7)$$

and

$$\frac{\varepsilon}{\lambda_0^2}\frac{\partial^2 \phi}{\partial \xi^2} + \frac{2\varepsilon^2}{\lambda_0}\frac{\partial^2 \phi}{\partial \xi \partial \tau} - \frac{\varepsilon^2}{\lambda_0^2}\frac{\partial \lambda_0}{\partial \tau}\frac{\partial^2 \phi}{\partial \tau^2} + \varepsilon^5\frac{\partial^2 \phi}{\partial \tau^2} - n_0 e^{\phi} + n = 0 \qquad (8)$$

To employ the reductive perturbation technique [1], the plasma parameters n, u and ϕ are expressed as a power series in ϵ as

$$\left.\begin{aligned} n &= n_0 + \varepsilon n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3 + \ldots\ldots \\ u &= u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \ldots\ldots \\ \phi &= \phi_0 + \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \ldots\ldots \end{aligned}\right\} \qquad (9)$$

where n_0 u_0 and ϕ_0 are the plasma parameters in unperturbed state.

From equations (6) – (7), we get zeroth-order equations as

$$-\frac{\partial n_0}{\partial \xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial \xi}(n_0 u_0) = 0 \qquad (10)$$

$$-\gamma\frac{\partial u_0}{\partial\xi}+\frac{\gamma u_0}{\lambda_0}\frac{\partial u_0}{\partial\xi}+\frac{1}{\lambda_0}\frac{\partial\phi_0}{\partial\xi}=0 \quad (11)$$

$$-n_0e^{\phi_0}+n_0=0 \quad (12)$$

Using (5) in above equations, we get

$$\frac{\partial u_0}{\partial\xi}=0 \text{ and } \phi_0=0 \quad (13)$$

Now using (9) into equations (6) – (8) the lowest order coefficients in ε together with equations (5) and (13) we get

$$-\frac{\partial n_1}{\partial\xi}+\frac{n_0}{\lambda_0}\frac{\partial u_1}{\partial\xi}+\frac{n_0}{\lambda_0}\frac{\partial n_1}{\partial\xi}+\frac{\partial}{\partial\tau}\left(n_0u_0\right)=0 \quad (14)$$

$$-\gamma\frac{\partial u_1}{\partial\xi}+\frac{\gamma u_0}{\lambda_0}\frac{\partial u_1}{\partial\xi}+\gamma u_0\frac{\partial u_0}{\partial\tau}+\frac{1}{\lambda_0}\frac{\partial\phi_1}{\partial\xi}=0 \quad (15)$$

$$-n_0\phi_1+n_1=0 \quad (16)$$

Integrating these equations and using boundary conditions

$u_0,\phi_0\rightarrow 0,\ \ n_1=u_1=\phi_1$ and $n_0,\lambda_0\rightarrow 1$ as $|\xi|\rightarrow\propto$

$$\left.\begin{aligned} u_1 &= An_1-\xi B\\ \phi_1 &= \gamma u_1\left(\lambda_0-u_0\right)-\gamma\lambda_0u_0\frac{\partial u_0}{\partial\tau}\xi\\ &= \gamma \mathrm{n}_0u_1A-C\xi\\ n_1 &= n_0\phi_1 \end{aligned}\right\} \quad (17)$$

Where

$$A=\frac{\lambda_0-u_0}{n_0},\ \mathrm{B}=\frac{\lambda_0}{n_0}\frac{\partial}{\partial\tau}\left(n_0u_0\right),\ \mathrm{C}=\gamma\lambda_0u_0\frac{\partial u_0}{\partial\tau} \quad (18)$$

Using equation (17) with simple algebra, we get

$$\phi_1=\frac{ABn_0\gamma+C}{\gamma A^2{n_0}^2-1} \quad (19)$$

In equation (19) we see that the left hand side is a first order perturbation while the right hand side contains only zeroth-order quantities. Thus, in order to obtain nonsecular solution of φ_1, numerator and denominator of equation (19) must be equal to zero separately. These yields

$$\left(\lambda_0-u_0\right)^2={}^{1}\!/_{\gamma} \quad (20)$$

and

$$\lambda_0\frac{\partial u_0}{\partial\tau}+\frac{\lambda_0-u_0}{n_0}u_0\frac{\partial n_0}{\partial\tau}=0 \quad (21)$$

Which is a self consistent relation between n_0 and u_0.

For second order of ε, i.e. $O\left(\varepsilon^2\right)$, we obtain the following equations

$$-\frac{\partial n_2}{\partial\xi}+\frac{1}{\lambda_0}\frac{\partial}{\partial\xi}\left(n_0u_2+n_1u_1+n_2u_0\right)+\frac{\partial}{\partial\tau}\left(n_0u_1+n_1u_0\right)=0 \quad (22)$$

$$-\frac{\partial u_2}{\partial\xi}+\frac{u_0}{\lambda_0}\frac{\partial u_2}{\partial\xi}+\frac{u_1}{\lambda_0}\frac{\partial u_1}{\partial\xi}+u_1\frac{\partial u_0}{\partial\tau}+u_0\frac{\partial u_1}{\partial\tau}+\frac{1}{\lambda_0}\frac{\partial\phi_2}{\partial\xi}+\frac{\partial\phi_1}{\partial\tau}=0 \quad (23)$$

and

$$\frac{1}{{\lambda_0}^2}\frac{\partial^2\phi_1}{\partial\xi^2}-n_0\phi_2-\frac{1}{2}n_0{\phi_1}^2+n_2=0 \quad (24)$$

Using equation (20), we can eliminate all the second- order quantities from the above three equations exactly. Substituting for n_1 and u_1 in terms of ϕ_1 from equation (17) into equations (22) - (24), we get the following modified KdV equation as

$$\frac{\partial \phi_1}{\partial \tau} + P\phi_1 \frac{\partial \phi_1}{\partial \xi} + Q\frac{\partial^3 \phi_1}{\partial \xi^3} + R\phi_1 = 0 \qquad (25)$$

where

$$P = \frac{(3+\lambda_0)}{\lambda_0^{\,2}\gamma^{3/2}}, \quad \text{Q}=\frac{1}{\text{n}_0\lambda_0^{\,4}\gamma^{3/2}}, \; \text{R}=\frac{1}{n_0\lambda_0\gamma^{3/2}+1}\left(n_0\gamma^{3/2} + \frac{\partial n_0}{\partial \tau}\right) \qquad (26)$$

Eq. (25) is a modified form of the KdV(mKdV) equation. This equation contains a term with variable coefficients as well as an additional term containing density gradient. The nonlinear coefficient P and the dispersion coefficient Q depend on the density inhomogeneity and the relativistic factor γ. The additional term with coefficient R arises due to the density gradient in the plasma.

IV. SOLUTION OF MKDV EQUATION

In order to obtain the solitary wave solutions of Eq. (25), we use the transformation $\phi_1 = \psi \exp(-Rn_0)$ to get the well known KdV equation

$$\frac{\partial \psi}{\partial \tau} + P_1\psi \frac{\partial \psi}{\partial \xi} + Q\frac{\partial^3 \psi}{\partial \xi^3} = 0, \qquad (27)$$

where $P_1 = P\exp(-Rn_0)$.

To solve the Eq. (27), we introduce a new variable $X = \xi - U\tau$ with respect to a frame moving with velocity U. We have obtained the solution of this equation following the method of Kodama and Taniuty[16] as

$$\psi = \psi_m \,\text{Sech}^2\left(\frac{X}{\Omega}\right), \qquad (28)$$

where the amplitude $\psi_m = \frac{3U}{P_1}$ and the width $\Omega = \left(\frac{4Q}{U}\right)^{\frac{1}{2}}$.

V. RESULTS AND DISCUSSION

From the solution (28), we observe that the soliton amplitude is directly proportional to the relativistic factor γ. Therefore the increase (decrease) of relativistic factor γ increases (decreases) the amplitude of the solitary wave. Also the width of the solitary wave is inversely proportional to the relativistic factor γ. So the increase (decrease) of relativistic factor γ decreases (increases) the width of the solitary wave. From the additional term with coefficient C, we can conclude that the gradients have to be in opposite direction of relativistic factor γ . Along the density gradient the amplitude and the velocity of the soliton decreases whereas the width increases.

VI. CONCLUSION

We have presented a study of the nonlinear ion acoustic solitary waves in inhomogeneous relativistic plasma. By employing reductive perturbation method together with stretched variables for space and time, we have derived a mKdV equation with variable coefficients. For numerical results of the ion acoustic soliton propagation, the solution of mkdV equation is obtained as given by Kodama and Taniuty [16]. In the present plasma model, only rarefactive solitons are observed.

REFERENCES

[1]. H. Washimi and T. Tanuiti, Phys. Rev. Lett., 17(19), pp. 996, (1966).
[2]. D.J.Korteweg and G.de Vries, Philosophical Magazine, 39, pp. 422-443, (1895).
[3]. Y. Nejoh, Phys Plasmas 1, 2154, (1994).
[4]. H. K. Malik, S. Singh and R. P. Dahiya, Phys. Lett. A, 195, 369(1994)
[5]. H. K. Malik, IEEE Trans Plasma Sci, 23, 813(1995).
[6]. S. G. Tagore, Plasma Phys. 15, 1247(1973).
[7]. H. K. Malik and R. P. Dahiya, Phys. Plasmas, 1, 2872(1994).

[8]. L. L.Yadav, R. S. Tiwari and S. R. Sharma, Phys. Plasma, 1, 599(1994).
[9]. H. K. Malik, S. Singh and R. P. Dahiya, Phys. Plasma, 11137(1994).
[10]. B. C. Kalita, S. N. Barman and G. Goswami, Phys. Plasma, 3, 144,(1996).
[11]. B. C. Kalita, R. Das and H. K. Sarmah, Int. J. of Appl. Math. And Mech., 5(6), 100,(2009).
[12]. G. C. Das, S. Duorah, S. S. Singh and C. Uberoi, Planetary and Space Science, 44(5), 485, (1996).
[13]. Y. Sentoku, T. Zh. Esirkepov, K. Mima, k. Nishihara, F. Califano, F. Pegoraro, H. Sakagami, Y. Kitagawa, N. M. Naumova and S.V. Bulanov, Phys. Rev. Lett., 83, 3434(1999).
[14]. A. Bourdien, Phys. Of Fluids, 26, 1804(1983).
[15]. N. Asano and T. Taniuti J, Phys. Soc. Jpn. 27, pp. 1059(1969).
[16]. Y. Kodama and T. Taniuty, J. Phys. Soc. Jpn., Vol. 45, No. 1, pp. 298- 310(1978).

Dr. L. B. Gogoi" Relativistic effects on the propagation of ion - acoustic solitons in Inhomogeneous Plasma" International Journal of Computational Engineering Research (IJCER), vol. 09, no. 6, 2019, pp 53-57