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Restrained Triple Connected Two Domination Number of Central, Total, and Line Graphs of Path and Cycle

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ABSTRACT

Let G = (V,E) be a simple graph. A restrained two dominating set S is said to be a restrained triple connected two dominating set, if <S> is triple connected. The minimum cardinality taken over all restrained triple connected two dominating sets is called the restrained triple connected two domination number of G and is denoted by $\gamma_{2rtc}(G)$. In this paper, we study the restrained triple connected two domination number for central graph, total graph and line graph of path and cycle.

KEYWORDS: *restrained triple connected two domination number, central graph, total graph, line graph.*

AMS Subject Classification: 05C69.

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I. INTRODUCTION

We begin with finite, connected and undirected graph G (V, E) without loops and parallel edges, where V denotes its vertex set and E denotes its edge set. The vertices and edges are commonly addressed as graph elements. A subset S of V of a nontrivial graph G is called a dominating set of G if every vertex in V − S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G. A subset S of V of a nontrivial graph G is called a restrained dominating set of G if every vertex in V − S is adjacent to at least one vertex in S as well as another vertex in V - S. The restrained domination number $\gamma_r(G)$ of G is the minimum cardinality taken over all restrained dominating sets in G. A subset S of V is said to be a restrained 2-dominating set of G if every vertex of V - S is adjacent to at least two vertices in S and every vertex of V - S is adjacent to a vertex in V - S. The minimum cardinality taken over all restrained two dominating sets is called the restrained two domination number and is denoted by $\gamma_{r2}(G)$.

A graph G is said to be triple connected if any three vertices lie on a path in G. The central graph C(G) of a graph G is a graph obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G. The total graph T (G) of a graph G is the graph whose vertex set is V(G) ∪ E(G) and two vertices are adjacent whenever they are either adjacent or incident in G. With every non empty ordinary graph there is associated a graph L(G), called the line graph of G whose points are in one-to-one correspondence with the lines of G and such that two points are adjacent in L(G) if and only if the corresponding lines of G are adjacent.

II. MAIN RESULTS

Theorem 2.1: The Restrained Triple Connected Two Domination Number of the Central graph of a path of order p is 2p – 3.

Proof: Let the path P_p have vertex set $\{v_i: 1\leq i \leq p\}$ and edge set $\{v_i v_{i+1}: 1\leq i \leq p-1\}$. By the definition of central graph add new vertices subdividing each edge exactly once. Let the new set of vertices be $\{u_i: 1\leq i \leq p-1\}$. The vertex set and edge set of C (P_p) is given by, $V(C(P_p)) = \{v_i: 1\leq i \leq p\} \cup \{u_i: 1\leq i \leq p-1\}$ and $E(C(P_p)) = \{v_iu_i: 1\leq i \leq p-1\} \cup \{u_iv_{i+1}: 1\leq i \leq p-1\} \cup \{v_iv_j: 1\leq i \leq p-2, i+2\leq j \leq p\}$.In C($P_p$) we see that vertex v_i is adjacent with all vertices except v_{i+1} and v_{i-1} for v_iu_i $1\leq i \leq p-1$. Now the new set of vertices $\{u_i: 1\leq i \leq p-1\}$ are adjacent to v_i and v_{i+1}. Hence the degree of each vertex in this set is two and by the definition of γ_{2rtc} – set all the vertices in the set $\{u_i: 1\leq i \leq p-1\}$ must be included in the γ_{2rtc} – set. Therefore, $S = \{v_1, u_1, v_2 u_2,\ldots\ldots\ldots\ldots,v_p u_p\}- \{v_1,v_p\}$ forms a γ_{2rtc} – set. Hence $\gamma_{2rtc}(C(P_p)) = 2p – 1 – 2 = 2p – 3$.

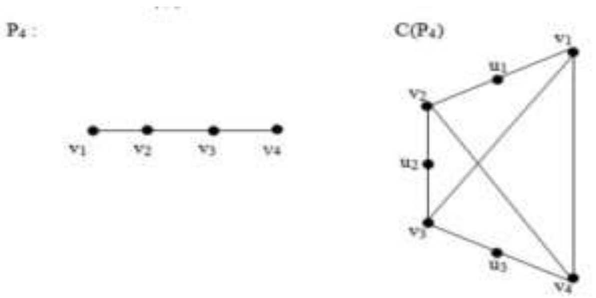

Figure 1: Central graph of path P_4

Theorem 2.2: The Restrained Triple Connected Two Domination Number of the Central graph of a cycle of order p is 2p.
Proof: Let the cycle C_p have vertex set $\{v_i: 1\leq i \leq p\}$ and edge set $\{v_i v_{i+1}: 1\leq i \leq p-1\} \cup \{v_1v_p\}$. By the definition of central graph add new vertices subdividing each edge exactly once. Let the new set of vertices be $\{u_i: 1\leq i \leq p\}$. The vertex set and edge set of $C(C_p)$ is given by, $V(C(C_p)) = \{v_i: 1\leq i \leq p\} \cup \{u_i: 1\leq i \leq p\}$ and $E(C(P_p)) = \{v_iu_i: 1\leq i \leq p\} \cup \{u_iv_{i+1}: 1\leq i \leq p-1\} \cup \{u_n,v_1\} \cup \{v_iv_j: i = 1, 3 \leq j \leq p-1\} \cup \{v_iv_j: 2 \leq i \leq p - 2, i+ 2 \leq j \leq p\}$. In $C(C_p))$ we see that vertex v_i is adjacent with all vertices except v_{i+1} and v_{i-1} for v_iu_i $1\leq i \leq p$. Now the new set of vertices $\{u_i: 1\leq i \leq p-1\}$ are adjacent to v_i and v_{i+1}. Hence the degree of each vertex in this set is two and by the definition of γ_{2rtc} – set all the vertices in the set $\{u_i: 1\leq i \leq p-1\}$ must be included in the γ_{2rtc} – set. Therefore, $S = \{v_1, u_1, v_2\ u_2,\ldots\ldots\ldots\ldots,v_p\ u_p\}$ forms a γ_{2rtc} – set. Hence $\gamma_{2rtc}\ (C(C_p)) = 2p$.

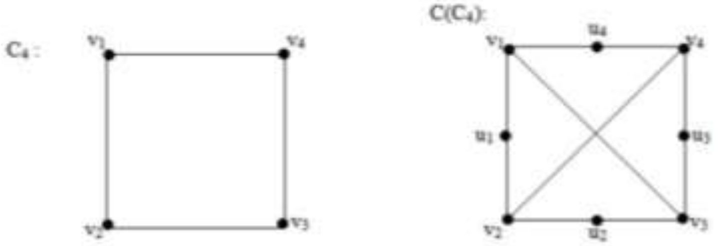

Figure 2: Central graph of cycle C_4

Theorem 2.3: The Restrained Triple Connected Two Domination Number of the Total graph of a path of order p is p.
Proof: Let the path P_p have vertex set $\{v_i: 1\leq i \leq p\}$ and edge set $\{v_i v_{i+1}: 1\leq i \leq p-1\}$. By the definition of total graph each edge $\{ e_i = v_i v_{i+1}: 1\leq i \leq p-1\}$ in P_p is subdivided by the vertices $\{u_i: 1\leq i \leq p-1\}$in $T(P_p)$. The vertex set and edge set of $T(P_p)$ is given by, $V(T(P_p)) = \{v_i: 1\leq i \leq p\} \cup \{u_i: 1\leq i \leq p-1\}$ where $\{u_i: 1\leq i \leq p-1\}$ are the vertices of $T(P_p)$ corresponding to the edge $\{v_iv_{i+1}: 1\leq i \leq p-1\}$of P_p and $E(T(P_p)) = \{v_iu_i: 1\leq i \leq p-1\} \cup \{u_iv_{i+1}: 1\leq i \leq p-1\} \cup \{u_iu_{i+1}: 1\leq i \leq p-1\} \cup\{v_iv_{i+1}: 1\leq i \leq p-1\}$. Therefore, $S = \{v_i: 1\leq i \leq p\}$ forms a minimum restrained 2 – dominating set and the induced subgraph <S> is triple connected. Hence $\gamma_{2rtc}\ (T(P_p)) = p$.

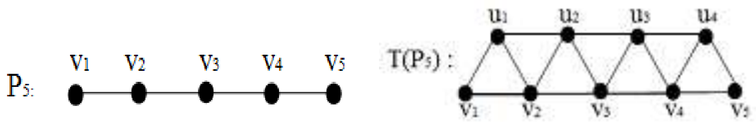

Figure 3: Total graph of path P_5

Theorem 2.4: The Restrained Triple Connected Two Domination Number of the Total graph of a cycle of order p is p – 1.
Proof: Let the cycle C_p have vertex set $\{v_i: 1\leq i \leq p\}$ and edge set $\{v_i v_{i+1}: 1\leq i \leq p-1\} \cup \{v_1,v_p\}$. By the definition of total graph each edge $\{e_i = v_i v_{i+1}: 1\leq i \leq p-1\} \cup \{v_1,v_p\}$ in C_p is subdivided by the vertices $\{u_i: 1\leq i \leq p\}$in $T(C_p)$. The vertex set and edge set of $T(C_p)$ is given by, $V(T(C_p)) = \{v_i: 1\leq i \leq p\} \cup \{u_i: 1\leq i \leq p\}$ and $E(T(P_p)) = \{v_iu_i: 1\leq i \leq p\} \cup \{u_iv_{i+1}: 1\leq i \leq p-1\} \cup \{u_p,v_1\} \cup \{v_iv_{i+1}: 1\leq i \leq p-1\}\cup\{v_1,v_p\} \cup \{u_iu_{i+1}: 1\leq i \leq p-1\}\cup\{u_1,u_p\}$.Therefore, $S = \{v_i: 1\leq i \leq p-1\}$ forms a minimum restrained 2 – dominating set and the induced subgraph <S> is triple connected.. Hence $\gamma_{2rtc}\ (T(P_p)) = p-1$.

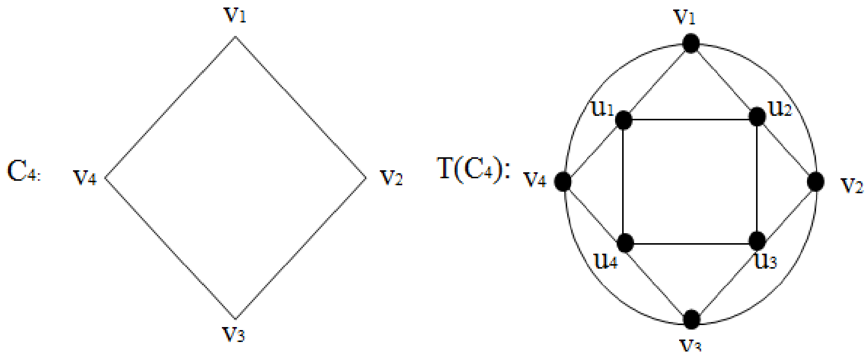

Figure 4: Total graph of cycle C_4

Theorem 2.5: The Restrained Triple Connected Two Domination Number of the Line graph of a path of order p is p – 1.
Proof: Let the path P_p have vertex set $\{v_i: 1\leq i \leq p\}$ and edge set $\{v_i\, v_{i+1}: 1\leq i \leq p-1\}$. By the definition of line graph the edges $\{v_i\, v_{i+1}: 1\leq i \leq p-1\}$in P_p are considered as the vertices $\{u_i: 1\leq i \leq p-1\}$ in $L(P_p)$ and two vertices of L(G) are joined by an edge if and only if the corresponding edges of G are adjacent in G. Hence $L(P_p)$ is a path with p – 1 vertices and p – 2 edges. The vertex set and edge set of L (P_p) is given by, $V(L(P_p)) = \{u_i: 1\leq i \leq p-1\}$ and $E(L(P_p)) = \{e_i^{'}: 1\leq i \leq p-2\}$.Therefore, S = $\{u_i: 1\leq i \leq p-1\}$ forms a minimum restrained 2 – dominating set and the induced subgraph <S> is triple connected. Hence γ_{2rtc} $(L(P_p))$ = p - 1.

Figure 5: Line graph of path P_4

Theorem 2.6: The Restrained Triple Connected Two Domination Number of the Line graph of a cycle of order p is p.

Proof: Let the path C_p have vertex set $\{v_i: 1\leq i \leq p\}$ and edge set $\{v_i\, v_{i+1}: 1\leq i \leq p-1\} \cup \{v_1 v_p\}$. By the definition of line graph the edges $\{v_i\, v_{i+1}: 1\leq i \leq p-1\} \cup \{v_1, v_p\}$ in C_p are considered as the vertices $\{u_i: 1\leq i \leq p\}$ in $L(C_p)$ and two vertices of L(G) are joined by an edge if and only if the corresponding edges of G are adjacent in G. Hence $L(C_p)$ is a cycle with p vertices and p edges. The vertex set and edge set of $L(CP_p)$ is given by, $V(L(C_p)) = \{u_i: 1\leq i \leq p\}$ and $E(L(C_p)) = \{e_i^{'}: 1\leq i \leq p\}$.Therefore, S = $\{u_i: 1\leq i \leq p\}$ forms a minimum restrained 2 – dominating set and the induced subgraph <S> is triple connected. Hence γ_{2rtc} $(L(P_p))$ = p .

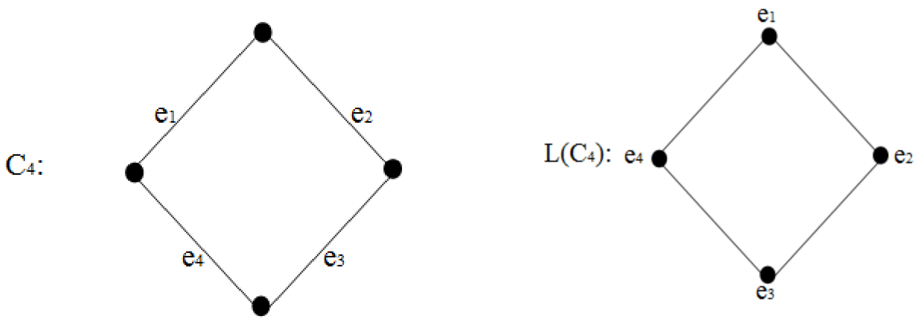

Figure 6: Line graph of cycle C_4

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