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A New quotient mappings in Topological spaces

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ABSTRACT:
*There are many situations in topology where we build a topological space by starting with some simple spaces and doing some kind of " gluing" or "identifications". The situations may look different at first, but really they are instances of the same general construction.The present chapter introduces g*α-quotient map using and characterize their basic properties. Using these new types of function, several characterizations and its properties have been obtained.*
KEYWORDS: *g*α-closed set, g*α-continuous, g*α-irresolute, g*α-open set, g*α*-quotient map, strongly g*α-quotient,strongly g*α*-quotient*

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I. INTRODUCTION

By considering the concept of g-closed**[3]** sets many concepts of topology have been generalized and interesting results have been obtained by several mathematicians. In this paper we introduce g*α-quotient functions and we prove some of their basic properties.

II. PRELIMINARIES:

Throughout this paper (X,τ),(Y,σ) and (Z,η)(or X, Y, and Z) represent nonempty topological spaces on which no separation axiom is defined unless otherwise mentioned. For a subset A of a space the closure of A, interior of A and complement of A are denoted by cl(A),int (A) and A^c respectively.
We recall the following definitions which are useful in the sequel.

Definition 2.1.
1. A subset A of a space X is called g*-closed **[6]**if cl(A) ⊆ U whenever A ⊆ U and U is g-open.
2. A subset A of a space X is called g*α-closed**[5]**if αcl(A) ⊆ U whenever A ⊆ U and U is g*-open. The family of g*α-closed sets are denoted by G*α-C(X). The complement of above closed sets are called respectively their open sets.
3. ${}_{\alpha}T^{**}_{1/2}$-space**[5]** if every g*α-closed set in it is closed.

Definition 2.2. A map f :(X,τ)→(Y,σ) is said to be
(i) g*α-continuous**[5]**if f^{-1}(V) is g*α-closed in (X,τ) for every closed set V in (Y,σ).
(ii) g*α-irresolute**[5]**if is f^{-1}(V) g*α-closed in (X,τ) for each g*α-closed set V of (Y,σ).
(iii) a quotient map**[4]** provided a subset V of (Y,σ) is open if and only if f^{-1}(V)is open in (X,τ).

III. G*A-QUOTIENT MAPS:

Definition 3.1: A surjective function f: (X,τ) → (Y,σ) is said to be a g*α-quotient map if f is g*α-continuous and f^{-1}(V) is open in (X,τ) implies V is a g*α-open set in (Y,σ)

Example 3.2: Let X = {1,2,3,4} with τ = {∅, X, {1}, {2}, {1,2}}, Y = {1,2,3} with σ = {{∅,Y,{1},{2},{1,2}}.Define a map f: (X,τ)→(Y,σ) by f(1) = 2,f(2) = 1,f(3) = 3 = f(4). g*α-O(X) = { ∅, X, {1}, {2}, {3}, {4}, {1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {1,2,3}, {1,2,4}}. g*α-O(Y) = { ∅, Y, {1}, {2}, {1,2}.Then f is g*α-continuous and f^{-1}(V) is open in (X,τ) implies V is g*α-open set in (Y,σ).Hence f is g*α-quotient map.

Theorem 3.3: Every quotient map is g*α-quotient.

Proof: Let f: (X, τ) →(Y,σ) be a quotient map. Since every continuous map is g*α-continuous and every open set is g*α-open the proof follows from the definition.

But the converse need not be true**.**

Example 3.4: Let X = {1,2,3,4} with τ = {∅, X, {3}, {4}, {3,4}} Y = {1,2,3}. σ = {∅, Y, {1,3}}. Define f: (X,τ) → (Y,σ) by f(1) = 2 = f(2), f(2) = 3,f(3) = 1,f(4) = 3. g*α-O(X) = {∅, X, {1}, {2}, {3}, {4}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}, {1,3,4}, {2,3,4}}. g*α-O(Y) = {∅, Y, {1}, {3}, {1,3}}. Here f is g*α-quotient but not quotient map since f^{-1}({3}) = {4} which is open in X but the set {3} is not open in Y.

Prop 3.5: If a map f: (X, τ) → (Y, σ) is surjective, g*α-continuous and g*α-open then f is g*α-quotient map.

Proof: we only need to prove f^{-1}(V) is open in (X,τ) implies V is g*α-open set in (Y,σ). Let f^{-1}(V) be open in (X,τ). Then f(f^{-1}(V)) is g*α-open in (Y,σ) since f is g*α-open. Hence V is g*α-open in (Y,σ).

Definition 3.7:A surjective function f: (X,τ) → (Y,σ) is said to be g*α*-quotient map if f is g*α-irresolute and f^{-1} (V) is g*α-open in (X,τ) implies V is open in (Y,σ)

Example 3.8: Let X = {1,2,3,4} with τ = {∅, X, {1}, {3,4}, {1,3,4}} Y = {1,2,3}with σ = {∅, Y, {1,2}}. Define f: (X, τ)→(Y,σ) by f(1) = 1, f(2) = 3, f(3) = 2 = f(4). g*α-O(X) = {∅, X, {1}, {3}, {4}, {1,3}, {1,4}, {3,4}, {1,3,4} }. g*α-O(Y) = {∅, Y, {1}, {2}, {1,2}}. Here f is g*α-irresolute and f^{-1}(V) is g*α-open in (X,τ) implies V is open in (Y, σ)

Theorem 3.9: Every g*α*-quotient map is g*α-irresolute.

Proof proof is obviously true from the definition.

But the converse need not be true.

Example 3.10: Let X = {1,2,3,4} with τ = {∅, X, {1}, {1,2,4}}.Y = {1,2,3}with σ = {∅, Y, {2}, {2,3} } Define f: (X,τ) → (Y,σ) by f(1) = 2, f(3) = 1,f(2) = 3 = f(4). g*α-O(X) = { ∅, X, {1}, {2}, {4}, {1,2}, {1,3}, {1,4}, {2,4}, {1,2,3}, {1,2,4},{1,3,4}}. g*α-O(Y) = {∅,Y, {2}, {3}, {1,2}, {2,3}}. Here f is g*α-irresolute but not g*α*quotient map since f^{-1}({1,2}) = {1,3} is g*α-open in (X,τ) but {1,2} is not open in (Y,σ).

4.Strong form of g*α-quotient map:

Definition 4.1: Let f: (X,τ) →(Y,σ) be a surjective map. Then f is called strongly g*α-quotient if a set U is open in Y iff f^{-1}(U) is a g*α-open set in X.

Example 4.2 Let X = {1,2,3,4} with τ = {∅, X, {1}, {1,4}, {1,2,4}}. Y = {1,2,3}with σ = {∅, Y, {1}, {1,2}, {1,3} } Define f: (X,τ) → (Y,σ) by f(1) = 1, f(2) = 2, f(3) = 3 = f(4). g*α-O(X) = {∅, X, {1}, {2}, {4}, {1,2}, {1,3}, {1,4}, {2,4}, {1,2,3},{1,2,4},{1,3,4}}. g*α-O(Y) = {∅, Y, {1}, {1,2}, {1,3}}. The map f is a strongly g*α-quotient map.

Theorem 4.3 Every strongly g*α-quotient map is g*α-open map.

Proof: Let f: (X,τ) → (Y,σ) be a strongly g*α-quotient map. Let V be any open set in (X,τ). Since every open set is g*α-open, V is g*α-open in (X,τ).Then f^{-1}(f(V)) is g*α-open in (X,τ).Since f is strongly g*α-quotient, then f(V) is open in (Y,σ) and hence f(V) is g*α-open in (Y,σ).This shows that f is a g*α-open map.

Here the reverse implication is not true.

Example 4.4: Let X = {1,2,3,4} with τ = {∅, X, {1}, {2,3}, {1,2,3}}. Y = {1,2,3}with σ = {∅, Y, {1,2},{3}}. Define f: (X,τ) → (Y,σ) by f(1) = 1 = f(2), f(3) = 3,f(4) = 2. g*α-O(X) = { ∅, X, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}}. g*αO(Y) = { ∅, Y, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}}. Here f is a g*α-open map but not a strongly g*α-quotient map since the set {1,2} is open in (Y,σ) but f^{-1}({1,2}) = {1,2,4} is not g*α-open in (X,τ)

Theorem 4.5: Every strongly g*α-quotient map is g*α-quotient

Proof: Let V be any open set in (Y,σ).Since f is strongly g*α-quotient, f^{-1}(V) is a g*α-open set in (X,τ).Hence f is g*α-continuous. Let f^{-1}(V) be open in (X,τ).Then f^{-1}(V) is g*α-open in (X,τ).Since f is strongly g*α-quotient, V is open in (Y,σ).Hence f is g*α-quotient.

Converse need not be true here

Example 4.6: : Let X = {1,2,3,4} with τ = {∅, X, {2}, {3,4}, {2,3,4}}. Y = {1,2,3}with σ = {∅, Y, {2,3}} Define f: (X, τ) → (Y,σ) by f(1) = 1, f(2) = 2, f(3) = 3 = f(4). g*α-O(X) ={∅, X, {2}, {3}, {4}, {2,3}, {2,4}, {3,4}, {2,3,4}}. g*α-O(Y) = {∅,Y,{2}, {3}, {2,3}}.Here f is a g*α-quotient but not a strongly g*α-quotient map since the set f^{-1}({2}) ={2} is g*α-open in (X,τ) but {2} is not open in (Y,σ).

Theorem 4.7: Every g*α*-quotient map is strongly g*α-quotient

Proof: Let f:(X, τ) → (Y, σ) be g*α*-quotient map. Let U be an open set inY. Then U is g*α-open in Y. Since f is g*α-irresolute, f^{-1}(U) is g*α-open in X. If f^{-1}(U) is g*α-open in X, then f is g*α*-quotient map implies that U is open set in Y. Hence f is strongly g*α-quotient.

corallary 4.8: Every g*α*-quotient map is g*α-quotient.

Proof: By theorem 4.7, Every g*α*-quotient map is strongly g*α-quotient and by theorem 4.5 every strongly g*α-quotient map is g*α-quotient, every g*α*-quotient map is g*α-quotient.

Definition 4.9: Let f: (X, τ) → (Y, σ) be a surjective map. Then f is called strongly g*α*-quotient if a set U is g*α-open in Y iff f^{-1}(U) is g*α-open in X.

Example 4.10: : Let X = {1,2,3,4} with τ = {∅, X, {1}, {1,2,4}}.Y = {1,2,3}with σ = {∅,Y,{2}}. Define f: (X,τ) → (Y,σ) by f(1) = 2 = f(3),f(2) = 1,f(4) = 3. g*α-O(X) ={∅, X, {1}, {2}, {4}, {1,2}, {1,3}, {1,4}, {2,4},{1,2,3},{1,2,4},{1,3,4}}. g*α-O(X) ={∅, X, {1}, {2}, {4}, {1,2}, {1,3}, {1,4}, {2,4}, {1,2,3}, {1,2,4}, {1,3,4}}. g*αO(Y) = {∅, Y, {1}, {2}, {3}, {1,2}, {2,3}}. Here f is strongly g*α*-quotient.

Theorem 4.11: Every g*α*-quotient map is strongly g*α*-quotient

Proof: Let f:(X, τ) →(Y, σ) be g*α*-quotient map. Let U be a g*α-open set inY. Since f is g*α-irresolute, f^{-1}(U) is g*α-open. Let f^{-1}(U) be g*α-open in X. since f is g*α*-quotient, it follows that U is open. Hence U is g*α-open and hence f is a strongly g*α*-quotient map.

Theorem 4.12: Every strongly g*α*-quotient map is g*α-quotient

Proof: Let f: (X, τ) → (Y, σ) be strongly g*α*-quotient map.Let V be an open set inY.Then V is a g*α-open set. Since f is strongly g*α*-quotient, f^{-1}(V) is g*α-open.Hence f is g*α-continuous.Let f^{-1}(V) be an open set in X.Then f^{-1}(V) is g*α-open in X. Hence V is g*α-open and f is g*α-quotient map.

Theorem 4.13: Let f:(X,τ) → (Y, σ) be a strongly g*α*-quotient map and Y is a ${}_{\alpha}T^{**}_{1/2}$-space then f is a strongly g*α-quotient map.

Proof: Let U be an open set inY. Then U is g*α-open set in Y. Since f is strongly g*α*-quotient, f^{-1}(U) is g*α-open in X. Let f^{-1}(U) be g*α-open in X. Then U is g*α-open in Y. Since Y is a ${}_{\alpha}T^{**}_{1/2}$- space U is open in Y. Hence f is strongly g*α-quotient map.

From the above results we have the following diagram where A→B represrents A implies B but not conversely.

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