Secondary Creep Stresses In Thick-Walled Composite Cylinders Under Internal And External Pressures

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ABSTRACT: Secondary creep stresses were obtained in thick-walled composite cylinders in the presence of both internal and external pressure. The cylinder is made of composite material with silicon carbide (SiC) particles as reinforcement in the aluminum matrix. The particle size of SiC is varied and its effect was observed on secondary creep stresses. It was found that the radial stress is negligibly affected with the increase in the size of SiC particles. On the other hand, the tangential and effective stresses are found to increase near the inner radius and decrease near the outer radius with the increase in particle size. The tangential stress showed the increasing trend throughout the entire radial distance. Whereas, the effective stress decreases throughout the entire radial distance.

KEYWORDS: Creep, Cylinder, Plane Stress, Threshold’s Law

I. INTRODUCTION

Creep in cylinder plays an important role when the cylinder is subjected to high temperature and mechanical loading. The operating lifespan of cylinders decreases with increase in creep [1,2,3]. Thick-walled cylinders made of metal and concrete are widely used in components which are subjected to high pressure and temperature such as boilers, aerospace industry, gun barrels, nuclear reactor, hydraulic cylinder etc. [4,5,6,7,8,9,10,11]. Creep analysis of thick-walled cylinder made of isotropic monolithic material and subjected to internal pressure has been investigated by many researchers [12,13,14]. In all these analyses it was assumed that the strains are infinitesimal and the deformation is referred with respect to original dimensions of the cylinder. Bhatnagar and Gupta, 1969 [15] obtained solution using constitutive equation of anisotropy creep theory and Norton’s law for orthotropic thick walled cylinders subjected to internal pressure. Bhatnagar and Arya, 1974 [16] have done large strain creep analysis of thick walled cylinder and found that the presence of external pressure in addition to internal pressure would be beneficial for design purpose and increases the life of cylinder. Bhatnagar el al., 1984 [17] have done analysis of an internally pressurized, homogeneous, orthotropic cylinder subjected to steady state creep condition and observed that orthotropy of certain type results in lower values of stress than for isotropic case thus reducing creep rates. Under severe environments such as high temperature and/or thermal gradients, the conventional materials alone may not survive. Therefore, to utilize the energy resources efficiently we need advanced materials such as composites in comparison with conventional materials when subjected to severe thermo-mechanical loads. Various types of composites included in this category are metal matrix composites (MMCs) such as aluminium/aluminium alloys reinforced with silicon carbide and functionally graded materials (FGMs). Other advanced composite materials may include Ceramics composites and carbon-carbon composites [18]. These materials can provide high resistance towards failure. Fukui and Yamanaka, 1992 [19] have studied the elastic problem of thick-walled tubes of a functionally graded material (FGM) under internal pressure in the case of plane strain. Fukui et al, 1993 [20], also analyzed the effects of the composition gradient in the radial direction on thermal stress for thick-walled tubes of functionally graded material (FGM) under uniform thermal loading. You et al., 2007 [21] have investigated steady state creep of thick-walled cylindrical vessels made of functionally graded materials subjected to internal pressure using Norton’s Law. Singh and Gupta, 2013 [22] done modeling creep in a thick composite cylinder subjected to internal and external pressures for plane strain condition using Threshold’s stress based law. This approach is employed to calculate stresses and creep strain rates in the thick-walled cylindrical vessels and also examined how variations of material parameters along the radial direction affect the stresses in the vessels. In the present work a mathematical model has been developed to investigate the effect of particle size on the secondary creep stresses in composite thick cylinder subjected to both internal and external pressure using Threshold’s stress based law.
II. SECONDARY STAGE CREEP

In aluminium based composites, undergoing steady state creep, the effective strain rate, $\dot{\varepsilon}_e$, is related to the effective stress, $\sigma_e$, through well documented threshold stress, $\sigma_0$, based creep law given by, Mishra and Pandey, 1990[23], Pandey et al. 1992[24].

$$\dot{\varepsilon}_e = [M(\sigma_e - \sigma_0)]^n$$

(1)

Where, M and $\sigma_0$ are known as creep parameters and are dependent on the type of material, temperature (T), reinforcement size ($P$) and the reinforcement content ($V$). In present work, the stress exponent $n$ is taken as 5 and values of M and $\sigma_0$ have been extracted from the experimental study of Pandey et al, 1992.[24]

### Table 1: Creep parameters used, Pandey et al, 1992 [24].

<table>
<thead>
<tr>
<th>P (µm)</th>
<th>T (°C)</th>
<th>V (Vol. %)</th>
<th>M (s$^{-1/5}$/MPa)</th>
<th>$\sigma_0$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>350</td>
<td>10</td>
<td>0.00435</td>
<td>19.83</td>
</tr>
<tr>
<td>14.5</td>
<td></td>
<td></td>
<td>0.00872</td>
<td>16.50</td>
</tr>
<tr>
<td>45.9</td>
<td></td>
<td></td>
<td>0.00939</td>
<td>16.29</td>
</tr>
</tbody>
</table>

III. MATHEMATICAL ANALYSIS AND SOLUTION

Considering a thick-walled cylinder made of Al-SiCp composite having internal radius “a” and external radius “b” and subjected to internal pressure “p = 85.25MPa” and external pressure “q = 21.31MPa”. The equilibrium equation of cylinder is given by, Gupta and Pathak, 2001[1]

$$r \frac{d\sigma_r}{dr} = \sigma_0 - \sigma_r$$

(2)

where $\sigma_r$, $\sigma_0$, are the radial and tangential stresses. The generalized constitutive equations for creep in an isotropic composite, are given by, Gupta et al. 2005[25]

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{2\sigma_e} [2\sigma_r - \sigma_0 - \sigma_z]$$

(3)

$$\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}_e}{2\sigma_e} [2\sigma_\theta - \sigma_r - \sigma_z]$$

(4)

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{2\sigma_e} [2\sigma_z - \sigma_r - \sigma_\theta]$$

(5)

Following Von-Mises yield criterion by Dieter, 1988[26], the effective stress is given by,

$$\sigma_e = \frac{1}{\sqrt{2}}[(\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 + (\sigma_r - \sigma_\theta)^2]^{1/2}$$

(6)

For,

Plane Stress when, $\sigma_z = 0$ the equations become,

$$\dot{\varepsilon}_r = \frac{d\varepsilon_r}{dr} = \frac{\dot{\varepsilon}_e}{2\sigma_e} [2\sigma_r - \sigma_0]$$

(7)

$$\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}_e}{r} = \frac{\dot{\varepsilon}_e}{2\sigma_e} [2\sigma_\theta - \sigma_r]$$

(8)

$$\dot{\varepsilon}_z = -\frac{\dot{\varepsilon}_e}{2\sigma_e} [\sigma_r + \sigma_\theta]$$

(9)

$$\sigma_e = \frac{1}{\sqrt{2}}[\sigma_\theta^2 + \sigma_r^2 + (\sigma_r - \sigma_\theta)^2]^{1/2}$$

(10)

Equilibrium equation along with constitutive Eqs. have been solved to obtain creep stresses namely radial, tangential and effective stresses are obtained.

IV. RESULTS AND DISCUSSION

Figure:4(a)-(c) shows the effect of variation of particle size from 1.7µm to 45.9µm on the creep stresses in a thick cylinder made of composite material subjected to both internal and external pressure. The radial stress shown in figure 4(a) remains negative throughout the radius having its maximum value of 85.25MPa at inner radius and 21.31MPa at outer radius, as per imposed boundary conditions. Table 2 shows the variation of radial stress for different particle size.
Table 2: Variation of Radial Stress for different particle size

<table>
<thead>
<tr>
<th>r (mm)</th>
<th>σ_r MPa (1.7 µm)</th>
<th>σ_r MPa (14.5 µm)</th>
<th>σ_r MPa (45.9 µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.40</td>
<td>-85.25</td>
<td>-85.25</td>
<td>-85.25</td>
</tr>
<tr>
<td>31.75</td>
<td>-62.99</td>
<td>-62.86</td>
<td>-62.86</td>
</tr>
<tr>
<td>38.10</td>
<td>-45.96</td>
<td>-45.84</td>
<td>-45.83</td>
</tr>
<tr>
<td>44.45</td>
<td>-32.43</td>
<td>-32.36</td>
<td>-32.35</td>
</tr>
<tr>
<td>50.80</td>
<td>-21.31</td>
<td>-21.31</td>
<td>-21.31</td>
</tr>
</tbody>
</table>

The tangential stress shown in figure 4(b) remains tensile and increases from inner radius to outer radius. It is found that the tangential stress with 1.7µm at inner radius is 18.54MPa and at outer radius is 58.33MPa. However the tangential stress with 45.9µm at inner radius is 19.52MPa and at outer radius is 57.72MPa. The tangential stress with smaller particle size is on lower side at inner radius and goes on increasing towards outer radius. Table 3 shows the variation of tangential stress for different particle size.

Table 3: Variation of Tangential Stress for different particle size

<table>
<thead>
<tr>
<th>r (mm)</th>
<th>σ_θ MPa (1.7 µm)</th>
<th>σ_θ MPa (14.5 µm)</th>
<th>σ_θ MPa (45.9 µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.40</td>
<td>18.34</td>
<td>19.47</td>
<td>19.52</td>
</tr>
<tr>
<td>31.75</td>
<td>33.56</td>
<td>33.88</td>
<td>33.90</td>
</tr>
<tr>
<td>38.10</td>
<td>44.81</td>
<td>44.72</td>
<td>44.72</td>
</tr>
<tr>
<td>44.45</td>
<td>52.70</td>
<td>52.33</td>
<td>52.31</td>
</tr>
<tr>
<td>50.80</td>
<td>58.33</td>
<td>57.75</td>
<td>57.72</td>
</tr>
</tbody>
</table>
It is clear from figure 4(a) and 4(b) that variation of particle size from 1.7µm to 45.9µm has marginal effect on radial and tangential stresses in thick walled composite cylinders subjected to both internal and external pressure. However, in case of effective stress as shown in figure 4(c) it has been observed that, the effective stress for coarser particle size is marginally higher in comparison with finer particle size at inner radius and it goes on decreasing from inner radius towards outer radius. At outer radius the effective stress for coarser particle size is marginally less than finer particle size. The effective stress with 1.7µm at inner radius is found as 94.97MPa and at outer radius it is 71.42MPa whereas with 45.9µm the stress at inner radius is 95.51MPa and at outer radius is 70.83MPa. Table 4 shows the variation of effective stress for different particle size.

![Figure 4(c). Variation of creep stress in cylinder.](image)

### Table 4: Variation of Effective Stress for different particle size

<table>
<thead>
<tr>
<th>r (mm)</th>
<th>$\sigma_e$ MPa (1.7µm)</th>
<th>$\sigma_e$ MPa (14.5µm)</th>
<th>$\sigma_e$ MPa (45.9µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.40</td>
<td>94.97</td>
<td>95.48</td>
<td>95.51</td>
</tr>
<tr>
<td>31.75</td>
<td>84.89</td>
<td>85.01</td>
<td>85.02</td>
</tr>
<tr>
<td>38.10</td>
<td>78.62</td>
<td>78.44</td>
<td>78.43</td>
</tr>
<tr>
<td>44.45</td>
<td>74.42</td>
<td>74.02</td>
<td>74.00</td>
</tr>
<tr>
<td>50.80</td>
<td>71.42</td>
<td>70.86</td>
<td>70.83</td>
</tr>
</tbody>
</table>

### V. CONCLUSION

The following conclusions have been drawn from the present study,
1. The radial stress is negligibly affected with the increase in the size of SiC particles.
2. The tangential stress and effective stresses increases near the inner radius and decreases near the outer radius with the increase in the size of SiC particles.
3. The tangential stress increases throughout the radius of the cylinder. However, the von Mises effective stress decreases throughout.

### REFERENCES

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