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Minimizing Utilization Time for Specially Structured Two Stage Flow Shop Scheduling Problem with Job-Weightage, Transportation Time and Jobs in a String of Disjoint Job Blocks

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ABSTRACT
Scheduling plays an important role in production management. It is helpful in improving the quality and increasing the rate of production, fulfilling the customer demands in time, to minimize the flow time or production cost etc. In flow shop scheduling the emphasis is on minimization of elapsed time but minimization of elapsed time may not always lead to minimization of utilization time. This paper provides a simple heuristic algorithm to optimize the utilization time of machines for specially structured n-job and 2-machine flow shop scheduling problem with jobs in a string of disjoint job blocks including job weightage and transportation time. An algorithm has been developed to minimize the utilization time of machines and is validated with the help of numerical examples.
Keywords: Disjoint Job Blocks, Jobs in a String, Specially Structured Flow Shop Scheduling**,** Transportation time, Weightage of jobs, Utilization Time.

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I. INTRODUCTION

Scheduling problems exist in many experimental and real industrial world situations. Scheduling involves determination of the order of processing a set of tasks on machines. In today's manufacturing and distribution systems, scheduling have significant role to meet customer requirements as quickly as possible while maximizing the profits. Flow shop scheduling deals with allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives. The resources and tasks in a system can take different forms. The resources may be machines in a workshop, runways at an airport, crews at a construction site, processing units in a computing establishment etc. The tasks may be operation in a production process, take offs and landings at an airport, phases in a construction work, executions of computer programs etc. The objectives can also take different forms. The common objectives in a flow shop scheduling problems are to minimize some performance measures such as make-span, mean flow time, mean tardiness, mean setup time, number of tardy jobs and mean number of setups. Johnson [1] studied flow shop scheduling in his research paper titled "Optimal two and three stage production schedule with set up times included". Flow shop scheduling has been studied intensively for more than 60 years by various researchers in management, engineering, computer science, operations research etc. Gupta, J.N.D. [2] developed an algorithm to find the optimal sequence for specially structured flow shop scheduling problem. A heuristic algorithm to minimize the rental cost for specially structured two stage flow shop scheduling problem was given by Gupta, D., Sharma, S. & Bala, S. [11].
The concept of job block is significant in scheduling systems where the priority of one job over the other is to be taken into account. The ordering of jobs is also useful when service is done in batches in industries or in service centers. Maggu, P. L. and Das, G. [3] established the basic concept of equivalent job for job block in job sequencing. The string of disjoint job blocks consist of two disjoint job blocks such that in one job block the order of jobs is fixed and in second job block the order of jobs is arbitrary. Heydari [9] studied flow shop scheduling problem with processing of jobs in a string of disjoint job blocks. Singh T.P., Kumar, V. and Gupta, D. [10] studied n×2 flow-shop scheduling problem in which processing time, set up time each associated with probabilities along with jobs in a string of disjoint job blocks. Gupta et al. [14] gave a heuristic method to minimize the utilization time for specially structured two stage flow shop scheduling problem having jobs in a string of disjoint job blocks.

Miyazaki and Nishiyama [4] considered flow-shop scheduling problems to minimize the weighted mean flow time of jobs. In scheduling problems the weight of a job shows its relative importance for processing in comparison with other jobs. The scheduling problems with weights arise when inventory costs for jobs are involved. Maggu et al. [6] developed an algorithm for n-job, 2-machine flow shop scheduling problem to optimize weighted mean flow time of jobs. Deepak, Bala & Singla [12] gave an algorithm to minimize rental cost for specially structured two stage flow shop scheduling problem including setup time and weightage of jobs.

Most of the published literature on sequencing and scheduling up to the year 1980 does not take into consideration the transportation time i.e. the moving time for a job from one machine to another machine during the processing of jobs. The earliest scheduling paper that explicitly considers the transportation factor is probably the one by Maggu and Dass [5]. In this paper they consider a two machine flow shop make span problem with unlimited buffer spaces on both machines in which there are a sufficient number of transporters so that whenever a job is completed on the first machine, it can be transported with a job dependent transportation time, to the second machine. Kise [7] studied scheduling problem with only one transporter with a capacity of one i.e. it can transport only one job at a time. Chung et al. [8] studied machine scheduling problems models for two types of transportation situations. The first situation investigated transporting a semi-finished job from one machine to another for further processing and the second situation considered the case of delivering a finished job to the customer or warehouse. Gupta, D., Sharma, S., and Bala, S. [13] gave a heuristic algorithm to minimize the utilization time and rental cost of machines for n×2 specially structured flow shop problem involving transportation time.

In the present paper we consider n-job and 2-machine specially structured flow shop scheduling problem including job weightage, transportation time and processing of jobs in a string of disjoint job blocks. The objective of the study is to obtain an optimal sequence of jobs to minimize the utilization time of machines. An algorithm is proposed to solve the specially structured flow shop scheduling problem and is validated with the help of numerical examples.

II. PRACTICAL SITUATION

In our day to day functioning in service centres and factories many applied and experimental situations exist regarding flow shop scheduling. For optimal utilization of available resources there must be a proper scheduling system and this makes scheduling a highly important aspect of industrial units. Specially structured flow shop scheduling problem has been considered as there are many practical situations where the processing times are not random but follow well defined structural relationship to one another.

Most machine scheduling models assume that either there are an infinite number of transporters for delivering jobs or jobs are delivered instantaneously from one location to another without transportation time involved. During the completion of processing of jobs in many production and distribution units, semi-finished tasks are transferred from one machine to another through various modes such as automated guided vehicles and conveyors, and finished jobs are delivered to consumers or storehouses by vehicles such as trains or trucks. Machine scheduling models that take into account the job transportation time are certainly more practical than those scheduling models that do not take these factors into consideration. Regarding weightage of jobs the practical example may be taken in a paper manufacturing unit where different types of paper are produced with relative importance.

III. NOTATIONS

The following notations have been used throughout the paper:

σ: Sequence of n- jobs obtained by applying Johnson's algorithm.

σ_k: Sequence of jobs obtained by applying the proposed algorithm, k = 1, 2, 3, ------.

M_j: Machine j, j= 1, 2.

a_{ij}: Processing time of i^{th} job on machine M_j.

p_{ij}: Probability associated to the processing time a_{ij}.

A_{ij}: Expected processing time of i^{th} job on machine M_j.

$T_{i,1\to2}$: Transportation time of i^{th} job from first machine to second machine.

W_i: Weight of i^{th} job according to its relative importance for processing in the sequence.

G_i: Weighted flow time of i^{th} job on machine M_1.

H_i: Weighted flow time of i^{th} job on machine M_2.

$t_{ij}(\sigma_k)$: Completion time of i^{th} job of sequence σ_k on machine M_j.

$T(\sigma_k)$: Total elapsed time for jobs 1, 2, --------, n for sequence σ_k.

$U_j(\sigma_k)$: Utilization time for which machine M_j is required for sequence σ_k.

$A_{ij}(\sigma_k)$: Expected processing time of i^{th} job on machine M_j for sequence σ_k.

α: Fix order job block.

β: Job block with arbitrary order.
β_k: Job block with jobs in an optimal order obtained by applying the proposed algorithm, k = 1, 2, 3, ------.
S: String of job blocks α and β i.e. S = (α, β)
S′: Optimal string of job blocks α and β_k.

IV. ASSUMPTIONS

The assumptions for the proposed algorithm are stated below:

a) Pre-emption is not allowed. Once a job is started on a machine the process on that machine cannot be stopped unless the job is completed.
b) Jobs are independent to each other and are processed thorough two machines M_1 and M_2 in order M_1 M_2.
c) Each job has two operations and each job is processed through each of the machine once and only once.
d) Weighted flow times must satisfy the structural conditions $min_i\{G_i\} \geq max_i\{H_i\}$ or $max_i\{G_i\} \leq min_i\{H_i\}$.
e) Each machine can perform only one task at a time.
f) A job is not available to the next machine until and unless processing on the current machine is completed.
g) $\sum_{i=1}^{n} p_{ij} = 1$; where $0 \leq p_{ij} \leq 1$.
h) Jobs i_1, i_2, ---------------, i_h are to be processed as a job block (i_1, i_2, ----------------, i_h) showing priority of job i_1 over i_2 etc. in that order in case of a fixed order job block.

V. DEFINITION

Completion time of i^{th} job on machine M_j is given by,
$t_{ij} = \max(t_{i-1,j}, t_{i,j-1} + t_{i,1\to 2}) + A_{ij}$; $j \geq 2$, where A_{ij} = Expected processing time of i^{th} job on machine M_j.

VI. PROBLEM FORMULATION

Let n- jobs (i = 1, 2, ------, n) be processed on two machines M_j (j = 1, 2) in the order M_1M_2. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij}; where $0 \leq p_{ij} \leq 1$ and $\sum_{i=1}^{n} p_{ij} = 1$. Let A_{ij} be the expected processing time of i^{th} job on j^{th} machine. Let $T_{i,1\to 2}$ be the transportation time of i^{th} job from machine M_1 to machine M_2. Let W_i be the weight of i^{th} job according to its priority for processing in the sequence. The mathematical model of the problem in matrix form is given below:

Table -1

Jobs	Machine M_1		Transportation time	Machine M_2		Weight
i	a_{i1}	p_{i1}	$T_{i,1\to 2}$	a_{i2}	p_{i2}	W_i
1	a_{11}	p_{11}	$T_{1,1\to 2}$	a_{12}	p_{12}	W_1
2	a_{21}	p_{21}	$T_{2,1\to 2}$	a_{22}	p_{22}	W_2
3	a_{31}	p_{31}	$T_{3,1\to 2}$	a_{32}	p_{32}	W_3
-	-	-	-	-	-	-
n	a_{n1}	p_{n1}	$T_{n,1\to 2}$	a_{n2}	p_{n2}	W_n

Consider two job blocks α and β such that the job block α consist of s- jobs with fixed order of jobs and β consist of p- jobs in which order of jobs is arbitrary such that s + p = n. Let the job blocks α and β form a disjoint set in the sense that the two blocks have no job in common i.e. α ∩ β =∅. We define S = (α, β) and β_k be the job block with jobs in an optimal order obtained by applying the proposed algorithm. Our aim is to find an optimal string S' of job blocks α and β_k i.e. to find an optimal sequence σ_k of jobs which minimizes the elapsed time and hence minimizes the utilization times of machines given that S = (α, β).
Mathematically, the problem is stated as:
Minimize T (σ_k) and hence Minimize U_2 (σ_k), given that S = (α, β).

7. Proposed Algorithm

Step 1: Calculate the expected processing times A_{ij} given by $A_{ij} = a_{ij} \times p_{ij}$.
Step 2: Compute A'_{i1} and A'_{i2} for respective machines M_1 and M_2 as:

$$A'_{i1} = A_{i1} + T_{i,1\to 2} \text{ and}$$
$$A'_{i2} = A_{i2} + T_{i,1\to 2}$$

Step 3: Compute the weighted flow times G_i and H_i for respective machines M_1 and M_2 as:
(1) If $min_j\{A'_{ij}\} = A'_{i1}$; then $G_i = \frac{A'_{i1} + W_i}{W_i}$ and $H_i = \frac{A'_{i2}}{W_i}$
(2) If $min_j\{A'_{ij}\} = A'_{i2}$; then $G_i = \frac{A'_{i1}}{W_i}$ and $H_i = \frac{A'_{i2} + W_i}{W_i}$; (Here j= 1, 2)
Step 4: Take equivalent job α for the job block (r, m) and calculate the weighted flow time

G_α and H_α on the guidelines of Maggu and Das [3] as follows:

$$G_\alpha = G_r + G_m - \min(G_m, H_r)$$
$$H_\alpha = H_r + H_m - \min(G_m, H_r)$$

If a job block has three or more than three jobs then to find the expected flow times we use the property that the equivalent job for a job-block is associative i.e. $((i_1, i_2), i_3) = (i_1, (i_2, i_3))$.

Step 5: Check the structural conditions that $min_i\{G_i\} \geq max_i\{H_i\}$ or $max_i\{G_i\} \leq min_i\{H_i\}$ for the job block β. If the structural conditions hold good obtain the new job block β_k having jobs in an optimal order from the job block β (disjoint from job block α) by treating job block β as sub flow shop scheduling problem of the main problem. For finding β_k follow the following steps:

(a): Obtain the job J_1 (say) having maximum weighted flow time on 1st machine and job J_r (say) having minimum weighted flow time on 2nd machine. If $J_1 \neq J_r$ then put J_1 on the first position and J_r at the last position and go to 5(c) otherwise go to 5(b).

(b): Take the difference of weighted flow time of job J_1 on M_1 from job J_2 (say) having next maximum weighted flow time on machine M_1. Call this difference as G_1'. Also take the difference of weighted flow time of job J_r on machine M_2 from job J_{r-1} (say) having next minimum weighted flow time on M_2. Call this difference as G_2'. If $G_1' \leq G_2'$ then put J_r on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{r-1} on the last position. Now follow step 5(c).

(c): Arrange the remaining (p - 2) jobs, if any between 1st job J_1(or J_2) & last job J_r (or J_{r-1}) in any order; thereby due to structural conditions we get the job blocks $\beta_1, \beta_2 \ldots \beta_m$, where m= (p – 2)!; with jobs in optimal order and each having same elapsed time. Let $\beta_k = \beta_1$ (say).

Step 6: Obtain the weighted flow times G_{β_k} and H_{β_k} for the job block β_k on the guidelines of Maggu and Das [3] as defined in step 4. Now, reduce the given problem to a new problem by replacing s-jobs by job block α with weighted flow times G_α and H_α and remaining p-jobs by a disjoint job block β_k with weighted flow times G_{β_k} and H_{β_k}. The new reduced problem can be represented as:

Table: 2

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
α	G_α	H_α
β_k	G_{β_k}	H_{β_k}

Step 7: For finding optimal string S′ follow the following steps:

(*a*) Obtain the job I_1 (say) having maximum weighted flow time on 1st machine and job I'_1 (say) having minimum weighted flow time on 2nd machine. If $I_1 \neq I'_1$ then put I_1 on the first position and I'_1 at last position to obtain S′ otherwise go to step 7(b).

(b) Take the difference of weighted flow time of job I_1 on M_1 from job I_2 (say) having next maximum weighted flow time on machine M_1. Call this difference as H_1'. Also take the difference of weighted flow time of job I'_1 on machine M_2 from job I'_2 (say) having next minimum weighted flow time on M_2. Call this difference as H_2'. If $H_1' \leq H_2'$ then put I'_1 on the second position and I_2 at the first position otherwise put I_1 on first position and I'_2 at the second position to obtain the optimal string S′.

Step 8: Compute the in - out table for sequence σ_k of jobs in the optimal string S′.

Step 9: Compute the total elapsed time T (σ_k) and the utilization time U_2 (σ_k), given by

$$U_2(\sigma_k) = T(\sigma_k) - A_{11}(\sigma_k) - T_{1,1\to2}$$

VII. NUMERICAL ILLUSTRATION

Illustration-1: To minimize the utilization time for six jobs to be processed in a string of disjoint blocks on two machines as job block α = (3, 5) with fixed order of jobs and job block β = (1, 2, 4, 6) with arbitrary order of jobs such that α ∩ β =∅. The processing times and setup times with respective probabilities are given in the following table:

Table: 3

Jobs	Machine M_1		Transportation time	Machine M_2		Weight
i	a_{i1}	p_{i1}	$T_{i,1\to2}$	a_{i2}	p_{i2}	W_i
1	54	0.1	3	10	0.1	2
2	30	0.2	4	8	0.1	2
3	43	0.2	4	7	0.2	3
4	36	0.1	5	6	0.2	2
5	56	0.2	6	4	0.3	4
6	61	0.2	4	23	0.1	3

Solution: Step 1: The expected processing times and expected setup times for machines M_1 and M_2 are as follow:

Table: 4

Jobs	Machine M_1	Transportation time	Machine M_2	Weight
i	A_{i1}	$T_{i,1\to2}$	A_{i2}	W_i
1	5.4	3	1.0	2
2	6.0	4	0.8	2
3	8.6	4	1.4	3
4	3.6	5	1.2	2
5	11.2	6	1.2	4
6	12.2	4	2.3	3

Step 2: The processing times $A_{i1}^{'} = A_{i1} + T_{i,1\to2}$ and $A_{i2}^{'} = A_{i2} + T_{i,1\to2}$ for machines M_1 and M_2 are given in the following table:

Table: 5

Jobs	Machine M_1	Machine M_2	Weight
i	$A_{i1}^{'}$	$A_{i2}^{'}$	W_i
1	8.4	4.0	2
2	10.0	4.8	2
3	12.6	5.4	3
4	8.6	6.2	2
5	17.2	7.2	4
6	16.2	6.3	3

Step 3: The weighted flow times G_i and H_i for machines M_1 and M_2 are as follow:

Table: 6

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
1	4.2	3.0
2	5.0	3.4
3	4.2	2.8
4	4.3	4.1
5	4.3	2.8
6	5.4	3.1

Step 4: Weighted flow times G_α and H_α for the job-block α = (3, 5) are calculated on the guidelines of Maggu and Das [3] as follows:

$G_\alpha = G_r + G_m - \min(G_m, H_r)$ (Here r = 3 & m = 5)

$= 4.2 + 4.3 - \min(4.3, 2.8)$

$= 8.5 - 2.8 = 5.7$

$H_\alpha = H_r + H_m - \min(G_m, H_r)$

$= 2.8 + 2.8 - \min(4.3, 2.8)$

$= 5.6 - 2.8 = 2.8$

Step 5: The structural conditions $min_i\{G_i\} \geq max_i\{H_i\}$ hold good and so using step 6 we get β_k = (6, 2, 4, 1).

Step 6: The equivalent job for job-block is associative i.e. ((i_1, i_2), i_3) = (i_1, (i_2, i_3)) and so we have β_k = (6, 2, 4, 1) = ((6, 2), 4, 1) = (α_1, 4, 1) = (α_2, 1), where α_1 = (6, 2) and α_2 = (α_1, 4). Therefore, the weighted flow times G_{β_k} and H_{β_k} for the job block β_k are calculated as:

$G_{\alpha_1} = 5.4 + 5.0 - \min(5.0, 3.1) = 10.4 - 3.1 = 7.3$

$H_{\alpha_1} = 3.1 + 3.4 - \min(5.0, 3.1) = 6.5 - 3.1 = 3.4$

$G_{\alpha_2} = 7.3 + 4.3 - \min(4.3, 3.4) = 11.6 - 3.4 = 8.2$

$H_{\alpha_2} = 3.4 + 4.1 - \min(4.3, 3.4) = 7.5 - 3.4 = 4.1$

$G_{\beta_k} = 8.2 + 4.2 - \min(4.2, 4.1) = 12.4 - 4.1 = 8.3$

$H_{\beta_k} = 4.1 + 3.0 - \min(4.2, 4.1) = 7.1 - 4.1 = 3.0$

The reduced problem is defined below:

Table: 7

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
α	5.7	2.8
β_k	8.3	3.0

Step 7: The optimal string S′ is given by S′ = (β_k, α). Hence, the optimal sequence σ_k of jobs as per string S′ is σ_k = 6 – 2 − 4 – 1 – 3 −5.
The in-out table for optimal sequence σ_k is:

Table: 8

Jobs	Machine M_1	Transportation time	Machine M_2
i	In-Out	$T_{i,1\to 2}$	In-Out
6	0.0 – 12.2	4	16.2 – 18.5
2	12.2 – 18.2	4	22.2 – 23.0
4	18.2 – 21.8	5	26.8 – 28.0
1	21.8 – 27.2	3	30.2 – 31.2
3	27.2 – 35.8	4	39.8 – 41.2
5	35.8 – 47.0	6	53.0 – 54.2

Therefore, the total elapsed time = T (σ_k) = 54.2 units.
Utilization time of machine M_2 = U_2 (σ_k) = (54.2 – 16.2) units = 38.0 units.
Remarks: If we solve the same problem by Johnson's [1] method by treating job block β as sub flow shop scheduling problem of the main problem we get the optimal job block as
β′ = (4, 2, 6, 1). The expected flow time $G_{\beta'}$ and $H_{\beta'}$ for the job block β′= (4, 2, 6, 1) on the guidelines of Maggu and Das [3] are calculated below:
Now, β′ = (4, 2, 6, 1) = ((4, 2), 6, 1) = (α′, 6, 1) = (α'', 1); where α′ = (4, 2) and α''= (α′, 6).

$$G_{\alpha'} = 4.3 + 5.0 - \min(5.0, 4.1) = 9.3 - 4.1 = 5.2$$
$$H_{\alpha'} = 4.1 + 3.4 - \min(5.0, 4.1) = 7.5 - 4.1 = 3.4$$
$$G_{\alpha''} = 5.2 + 5.4 - \min(5.4, 3.4) = 10.6 - 3.4 = 7.2$$
$$H_{\alpha''} = 3.4 + 3.1 - \min(5.4, 3.4) = 6.5 - 3.4 = 3.1$$
$$G_{\beta'} = 7.2 + 4.2 - \min(4.2, 3.1) = 11.4 - 3.1 = 8.3$$
$$H_{\beta'} = 3.1 + 3.0 - \min(4.2, 3.1) = 6.1 - 3.1 = 3.0$$

The reduced problem is defined below:

Table: 9

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
α	5.7	2.8
β′	8.3	3.0

By Johnson's algorithm the optimal string S′ is given by S′ = (β′, α).
Therefore, the optimal sequence σ for the original problem corresponding to optimal string S′ is given by σ = 4 – 2 – 6 − 1 – 3 – 5. The in – out flow table for the optimal sequence σ is:

Table: 10

Jobs	Machine M_1	Transportation time	Machine M_2
i	In - Out	$T_{i,1\to 2}$	In - Out
4	0.0 – 3.6	5	8.6 – 9.8
2	3.6 – 9.6	4	13.6 – 14.4
6	9.6 – 21.8	4	25.8 – 28.1
1	21.8 – 27.2	3	30.2 – 31.2
3	27.2 – 35.8	4	39.8 – 41.2
5	35.8 – 47.0	6	53.0 – 54.2

Therefore, the total elapsed time = T (σ) = 54.2 units.
Utilization time of machine M_2 = U_2(σ) = (54.2 − 8.6) units.
= 45.6 units.

Illustration-2: To minimize the utilization time for five jobs to be processed in a string of disjoint blocks on two machines as job block α = (2, 4) with fixed order of jobs and job block β = (1, 3, 5) with arbitrary order of jobs such that α ∩ β =∅. The processing times and setup times with respective probabilities are given in the following table:

Table: 11

Jobs	Machine M_1		Transportation time	Machine M_2		Weight
i	a_{i1}	p_{i1}	$T_{i,1\to 2}$	a_{i2}	p_{i2}	W_i
1	34	0.2	2	6	0.3	2
2	42	0.3	3	9	0.2	4
3	74	0.1	4	23	0.1	3
4	27	0.2	3	3	0.2	2
5	50	0.2	5	5	0.2	3

Solution: Step 1: The expected processing times and expected setup times for machines M_1 and M_2 are as follow:

Table: 12

Jobs	Machine M_1	Transportation time	Machine M_2	Weight
i	A_{i1}	$T_{i,1\to 2}$	A_{i2}	W_i
1	6.8	2	1.8	2
2	12.6	3	1.8	4
3	7.4	4	2.3	3
4	5.4	3	0.6	2
5	10.0	5	1.0	3

Step 2: The processing times$A'_{i1} = A_{i1} + T_{i,1\to 2}$ and $A'_{i2} = A_{i2} + T_{i,1\to 2}$ for machines M_1 and M_2 are given in the following table:

Table: 13

Jobs	Machine M_1	Machine M_2	Weight
i	A'_{i1}	A'_{i2}	W_i
1	8.8	3.8	2
2	15.6	4.8	4
3	11.4	6.3	3
4	8.4	3.6	2
5	15.0	6.0	3

Step 3: The weighted flow times G_i and H_i for machines M_1 and M_2 are as follow:

Table: 14

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
1	4.4	2.9
2	3.9	2.2
3	3.8	3.1
4	4.2	2.8
5	5.0	3.0

Step 4: Weighted flow times G_α and H_α for the job-block $\alpha = (2, 4)$ are calculated on the guidelines of Maggu and Das [3] as follows:

$G_\alpha = G_r + G_m - \min(G_m, H_r)$ (Here r = 2 & m = 4)

$= 3.9 + 4.2 - \min(4.2, 2.2)$

$= 8.1 - 2.2 = 5.9$

$H_\alpha = H_r + H_m - \min(G_m, H_r)$

$= 2.2 + 2.8 - \min(4.2, 2.2)$

$= 5.0 - 2.2 = 2.8$

Step 5: The structural conditions $\min_i\{G_i\} \geq \max_i\{H_i\}$ hold good and so using step 6 we get $\beta_k = (5, 3, 1)$.
Step 6: We know that the equivalent job for job-block is associative i.e. $((i_1, i_2), i_3) = (i_1, (i_2, i_3))$ and so we have $\beta_k = (5, 3, 1) = ((5, 3), 1) = (\alpha_1, 1)$; where $\alpha_1 = (5, 3)$.
Therefore, the weighted flow times G_{β_k} and H_{β_k} for the job block β_k are calculated as:

$G_{\alpha_1} = 5.0 + 3.8 - \min(3.8, 3.0) = 8.8 - 3.0 = 5.8$

$H_{\alpha_1} = 3.0 + 3.1 - \min(3.8, 3.0) = 6.1 - 3.0 = 3.1$

$G_{\beta_k} = 5.8 + 4.4 - \min(4.4, 3.1) = 10.2 - 3.1 = 7.1$

$H_{\beta_k} = 3.1 + 2.9 - \min(4.4, 3.1) = 6.0 - 3.1 = 2.9$

The reduced problem is defined below:

Table: 15

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
α	5.9	2.8
β_k	7.1	2.9

Step 7: The optimal string S′ is given by S′ = (β_k, α). Hence, the optimal sequence σ_k of jobs as per string S′ is σ_k = 5 – 3 − 1 – 2 −4. The in-out table for optimal sequence σ_k is:

Table: 16

Jobs	Machine M_1	Transportation time	Machine M_2
i	In-Out	$T_{i,1\to 2}$	In-Out
5	0.0 − 10.0	5	15.0 − 16.0
3	10.0 − 17.4	4	21.4 − 23.7
1	17.4 − 24.2	2	26.2 − 28.0
2	24.2 − 36.8	3	39.8 − 41.6
4	36.8 − 42.2	3	45.2 − 45.8

Therefore, the total elapsed time = T (σ_k) = 45.8 units.
Utilization time of machine M_2 = U_2 (σ_k) = (45.8 – 15.0) units = 30.8 units.
Remarks: If we solve the above problem by Johnson's [1] method by treating job block β as sub flow shop scheduling problem of the main problem we get the optimal job block as
β′ = (3, 5, 1). The expected flow time $G_{\beta'}$ and $H_{\beta'}$ for the job block β′ = (3, 5, 1) on the guidelines of Maggu and Das [3] are calculated below:
Now, β′ = (3, 5, 1) = ((3, 5), 1) = (α′, 1); where α′ = (3, 5).
$G_{\alpha'}$ = 3.8 + 5.0 – min (5.0, 3.1) = 8.8 – 3.1 = 5.7
$H_{\alpha'}$ = 3.1 + 3.0 – min (5.0, 3.1) = 6.1 – 3.1 = 3.0
$G_{\beta'}$ = 5.7 + 4.4 – min (4.4, 3.0) = 10.1 – 3.0 = 7.1
$H_{\beta'}$ = 3.0 + 2.9 – min (4.4, 3.0) = 5.9 – 3.0 = 2.9
The reduced problem is defined below:

Table: 17

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
α	5.9	2.8
β′	7.1	2.9

By Johnson's algorithm the optimal string S′ is given by S′ = (β′, α).
Therefore, the optimal sequence σ for the original problem corresponding to optimal string S′ is given by σ = 3 – 5 – 1 – 2 – 4. The in – out flow table for the optimal sequence σ is:

Table: 18

Jobs	Machine M_1	Transportation time	Machine M_2
i	In - Out	$T_{i,1\to 2}$	In - Out
3	0.0 − 7.4	4	11.4 − 13.7
5	7.4 − 17.4	5	22.4 − 23.4
1	17.4 − 24.2	2	26.2 – 28.0
2	24.2 − 36.8	3	39.8 − 41.6
4	36.8 − 42.2	3	45.2 – 45.8

Therefore, the total elapsed time = T (σ) = 45.8 units.
Utilization time of machine M_2 = U_2 (σ) = (45.8 − 11.4) units = 34.4 units.

VIII. CONCLUSION

We see that the proposed algorithm is more efficient as it optimizes both the make-span and the utilization time for a specially structured two stage flow shop scheduling problem as compared to the algorithm proposed by Johnson [1].

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